Computer Algebra Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/48-1.2.3.4-f-x-^m-d+e-x^n-^q-a+b-x^n+c-x^-2-n-^p

Nasser M. Abbasi

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [156]. This is test number [48].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (156)	0.00 (0)
Mathematica	94.23 (147)	5.77 (9)
Maple	87.82 (137)	12.18 (19)
Fricas	82.69 (129)	17.31 (27)
Mupad	78.21 (122)	21.79 (34)
Giac	70.51 (110)	29.49 (46)
Sympy	50.00 (78)	50.00 (78)
Maxima	42.31 (66)	57.69 (90)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

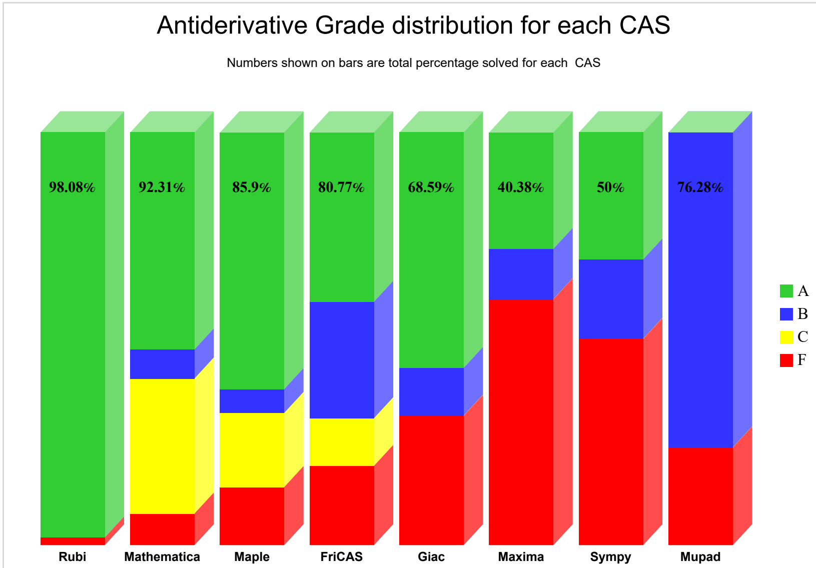
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

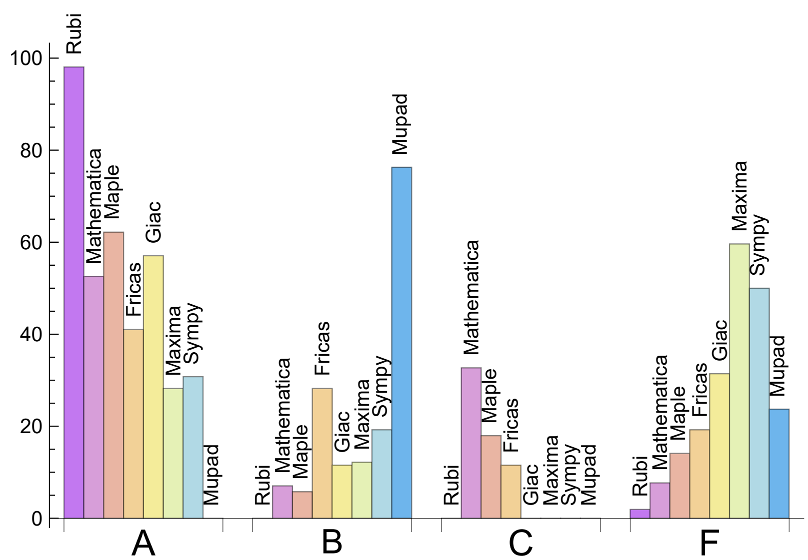
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.077	0.000	0.000	1.923
Maple	62.179	5.769	17.949	14.103
Giac	57.051	11.538	0.000	31.410
Mathematica	52.564	7.051	32.692	7.692
Fricas	41.026	28.205	11.538	19.231
Sympy	30.769	19.231	0.000	50.000
Maxima	28.205	12.179	0.000	59.615
Mupad	0.000	76.282	0.000	23.718

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	9	100.00	0.00	0.00
Maple	19	100.00	0.00	0.00
Fricas	27	70.37	29.63	0.00
Mupad	34	0.00	100.00	0.00
Giac	46	78.26	13.04	8.70
Sympy	78	11.54	80.77	7.69
Maxima	90	68.89	0.00	31.11

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.24
Rubi	0.26
Giac	0.45
Maple	1.72
Mathematica	2.23
Fricas	5.71
Sympy	10.21
Mupad	10.21

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	209.28	1.00	153.50	1.00
Maxima	215.95	10.05	35.50	1.06
Sympy	216.04	9.03	93.50	1.02
Giac	221.92	3.24	68.00	1.00
Maple	258.39	2.34	44.00	0.95
Mathematica	436.46	1.95	80.00	0.99
Fricas	1431.50	9.27	240.00	1.45
Mupad	3286.97	17.19	295.00	3.15

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

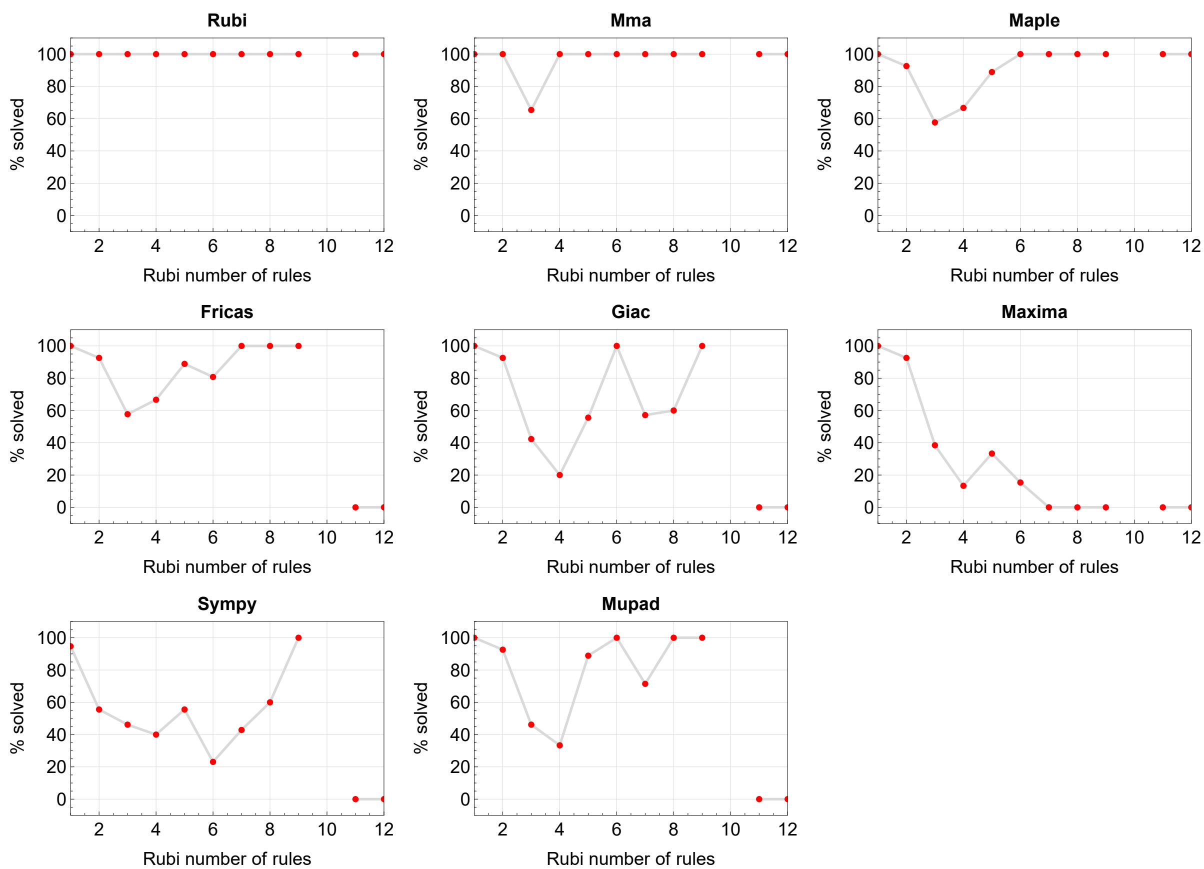


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

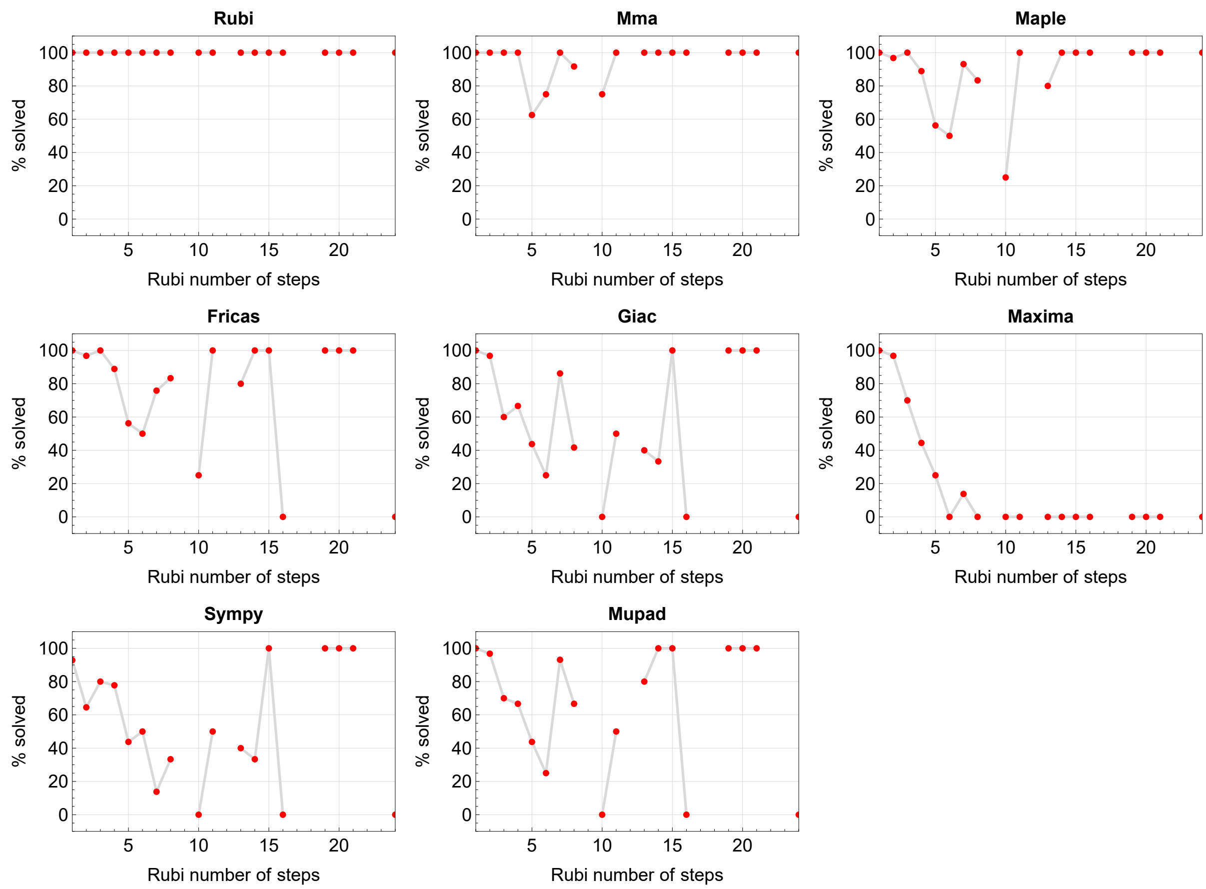


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

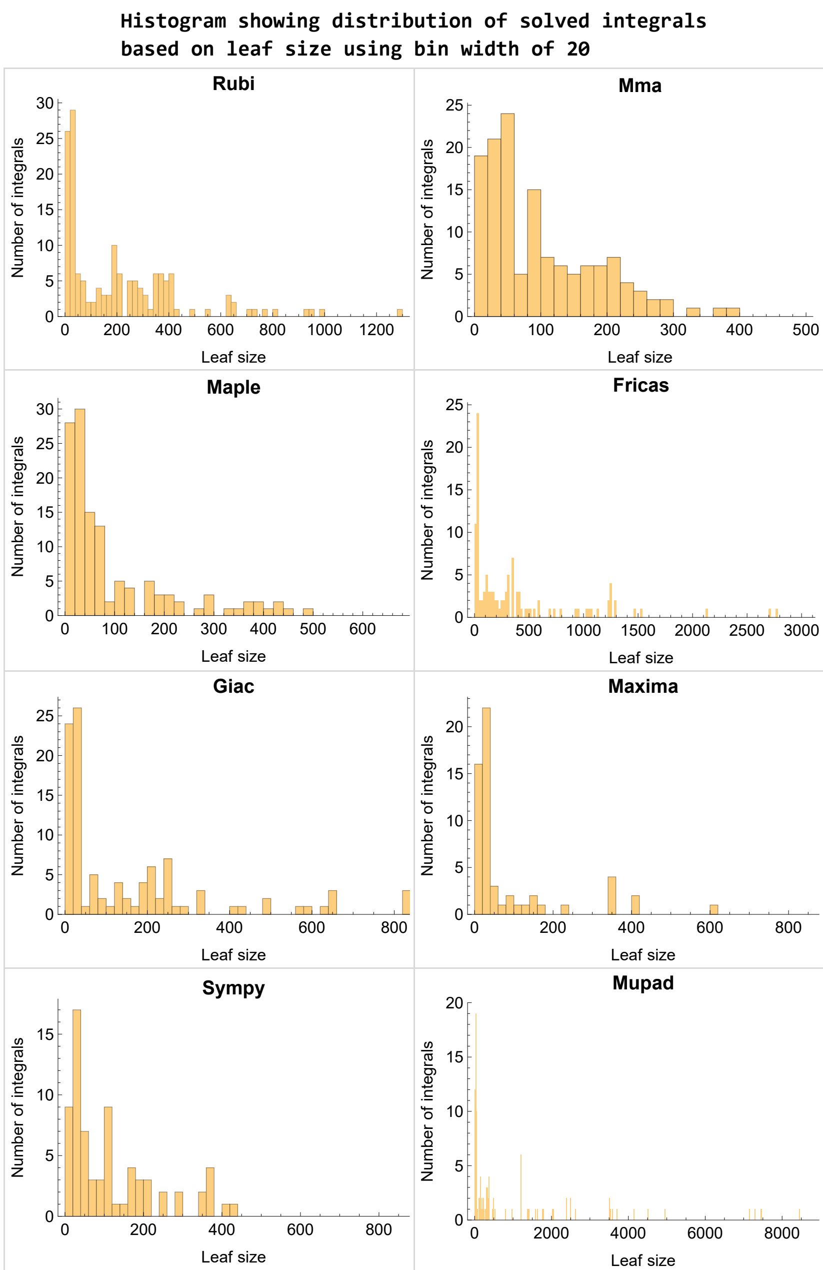


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

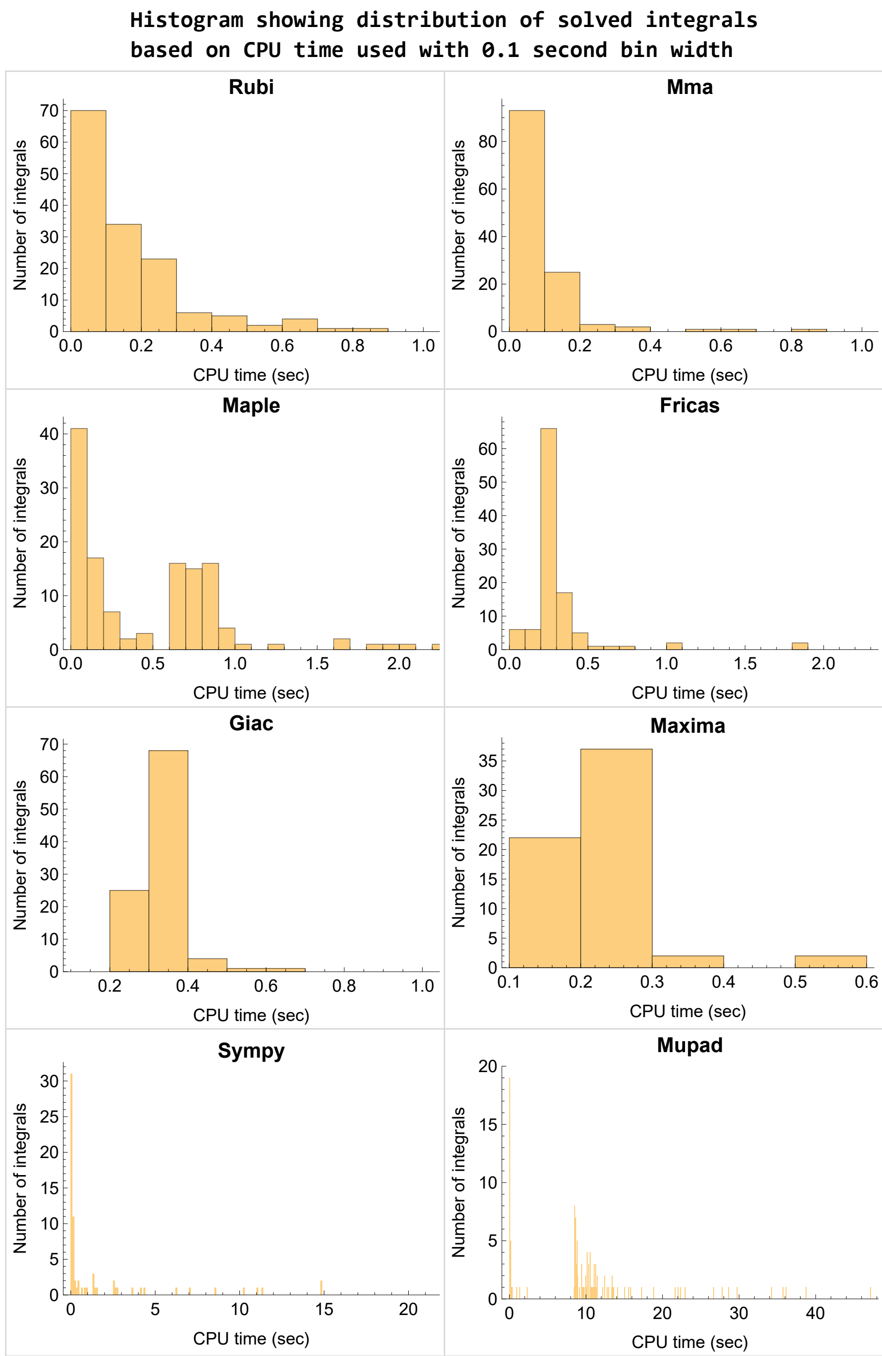


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

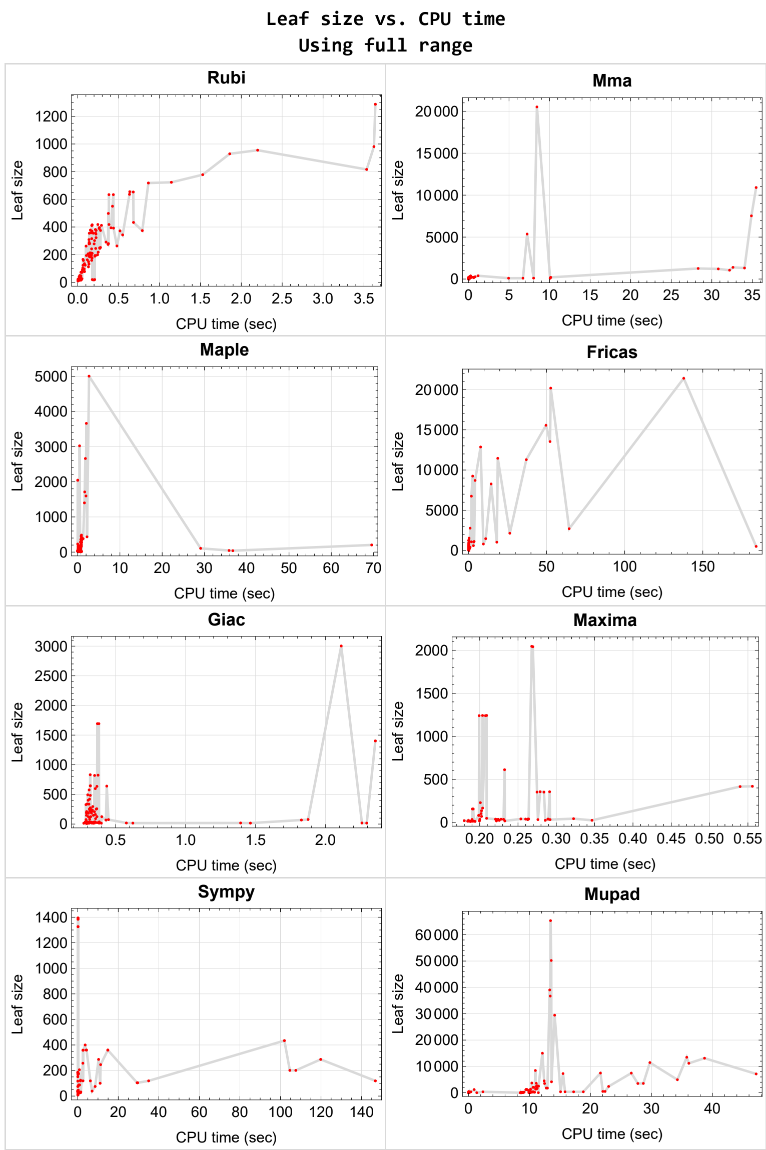


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{86, 155, 156\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {143}

Maple {140}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

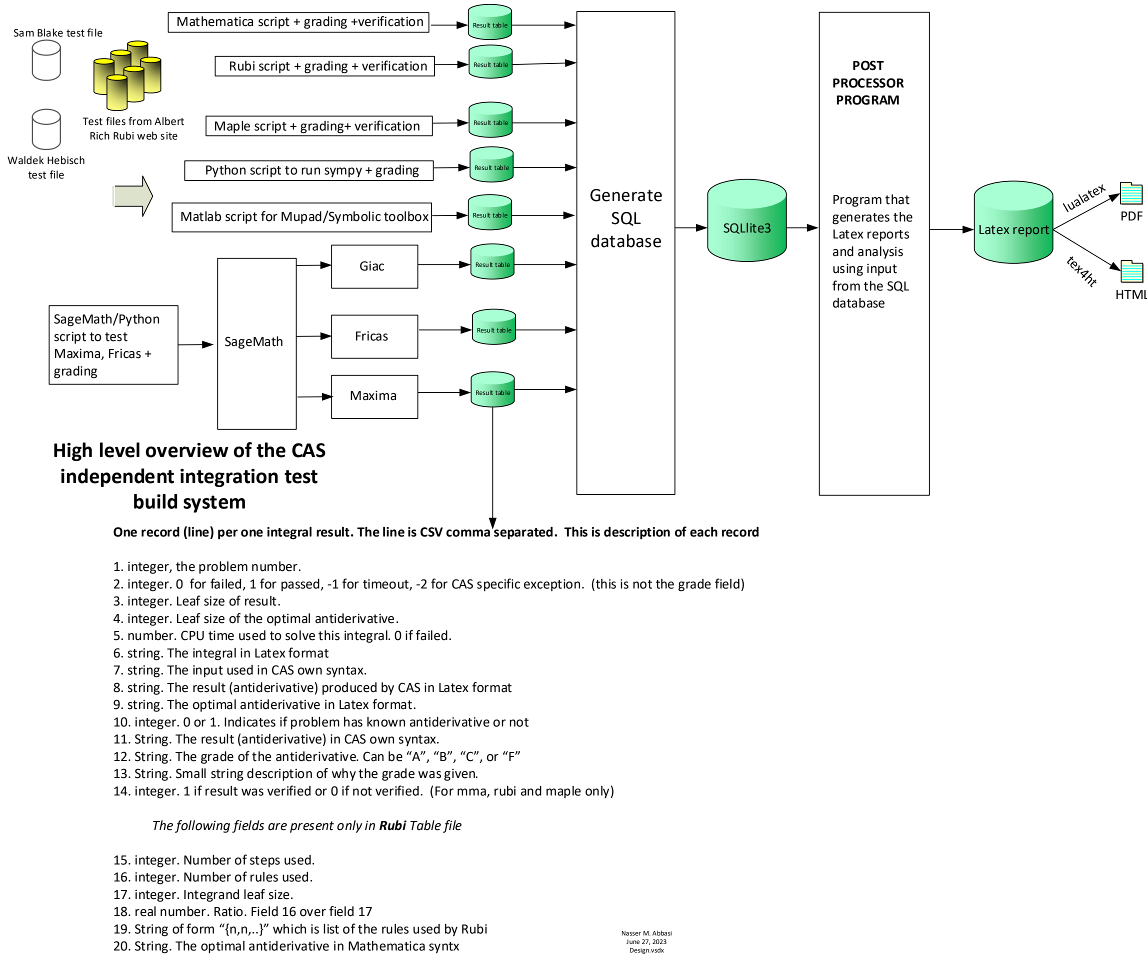
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 20, 21, 22, 32, 44, 46, 53, 55, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 87, 88, 89, 96, 100, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 141, 144, 149, 152, 153, 154 }

B grade { 93, 94, 95, 97, 98, 99, 101, 102, 103, 142, 143 }

C grade { 12, 13, 14, 15, 16, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 48, 49, 50, 51, 52, 54, 56, 57, 58, 59, 60, 79, 80, 81, 82, 83, 84, 85, 138, 139, 140 }

F normal fail { 90, 91, 92, 145, 146, 147, 148, 150, 151 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 46, 48, 50, 53, 55, 57, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 84, 85, 93, 94, 95, 97, 98, 99, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 144 }

B grade { 79, 80, 81, 82, 83, 96, 100, 104, 128 }

C grade { 6, 7, 8, 14, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 31, 43, 45, 47, 49, 51, 52, 54, 56, 58, 59, 60, 140 }

F normal fail { 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 44, 48, 53, 55, 57, 61, 62, 63, 64, 65, 66, 67, 105, 106, 107, 108, 113, 114, 115, 116, 121, 122, 123, 124, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144 }

B grade { 8, 14, 15, 16, 17, 18, 19, 43, 45, 46, 47, 49, 50, 51, 70, 71, 72, 73, 74, 75, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 109, 110, 111, 112, 117, 118, 119, 120, 125, 126, 127, 128 }

C grade { 35, 36, 37, 38, 39, 40, 41, 42, 52, 54, 56, 58, 59, 60, 79, 80, 81, 82 }

F normal fail { 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

F(-1) timedout fail { 68, 69, 76, 77, 78, 83, 84, 85 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 20, 21, 22, 23, 24, 32, 33, 53, 57, 93, 97, 101, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 121, 122, 123, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144 }

B grade { 94, 95, 96, 98, 99, 100, 102, 103, 104, 110, 111, 112, 118, 119, 120, 124, 126, 127, 128 }

C grade { }

F normal fail { 14, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

F(-1) timedout fail { }

F(-2) exception fail { 6, 7, 8, 9, 10, 11, 12, 13, 44, 48, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 44, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140 }

B grade { 25, 26, 27, 28, 29, 30, 31, 46, 50, 93, 94, 95, 96, 97, 98, 99, 100, 104 }

C grade { }

F normal fail { 15, 16, 17, 18, 19, 35, 36, 37, 38, 39, 40, 41, 42, 79, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 153, 154 }

F(-1) timedout fail { 14, 43, 45, 47, 49, 51 }

F(-2) exception fail { 87, 88, 144, 152 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144 }

C grade { }

F normal fail { }

F(-1) timedout fail { 35, 36, 37, 38, 39, 40, 41, 42, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 52, 53, 54, 55, 56, 57, 58, 59, 60, 105, 106, 107, 113, 114, 115, 121, 122, 123 }

B grade { 10, 11, 44, 93, 94, 95, 97, 98, 99, 101, 102, 103, 109, 110, 111, 117, 118, 119, 124, 125, 126, 127, 129, 130, 133, 134, 137, 138, 140, 144 }

C grade { }

F normal fail { 79, 80, 81, 82, 83, 84, 85, 141, 149 }

F(-1) timedout fail { 9, 12, 13, 14, 15, 16, 17, 18, 19, 42, 43, 45, 46, 47, 48, 49, 50, 51, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 87, 88, 89, 90, 91, 92, 96, 100, 104, 108, 112, 116, 120, 128, 131, 132, 135, 136, 139, 142, 143, 148, 151, 152, 153, 154 }

F(-2) exception fail { 145, 146, 147, 150, 155, 156 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	164	165	166	166	187	182	158
N.S.	1	1.00	1.01	1.01	1.02	1.02	1.15	1.12	0.97
time (sec)	N/A	0.117	0.046	0.875	0.204	0.247	0.035	0.306	0.052

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	135	134	135	135	151	147	130
N.S.	1	1.00	1.00	0.99	1.00	1.00	1.12	1.09	0.96
time (sec)	N/A	0.082	0.028	0.676	0.202	0.260	0.029	0.291	0.034

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	104	103	102	102	117	112	102
N.S.	1	1.00	1.01	1.00	0.99	0.99	1.14	1.09	0.99
time (sec)	N/A	0.061	0.021	0.644	0.202	0.267	0.028	0.312	0.027

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	69	69	75	76	70
N.S.	1	1.00	1.00	0.96	0.95	0.95	1.03	1.04	0.96
time (sec)	N/A	0.039	0.016	0.674	0.202	0.260	0.027	0.288	0.023

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	36	36	39	40	38
N.S.	1	1.00	1.00	0.88	0.86	0.86	0.93	0.95	0.90
time (sec)	N/A	0.018	0.008	0.101	0.200	0.255	0.020	0.295	0.026

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	176	67	0	465	175	191	165
N.S.	1	1.00	0.94	0.36	0.00	2.47	0.93	1.02	0.88
time (sec)	N/A	0.139	0.112	0.709	0.000	0.276	0.416	0.297	0.172

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	199	88	0	697	206	211	187
N.S.	1	1.00	0.93	0.41	0.00	3.27	0.97	0.99	0.88
time (sec)	N/A	0.150	0.131	0.651	0.000	0.299	0.818	0.292	10.843

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	209	114	0	941	246	236	221
N.S.	1	1.00	0.86	0.47	0.00	3.89	1.02	0.98	0.91
time (sec)	N/A	0.170	0.189	0.645	0.000	0.325	11.306	0.333	0.174

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	126	136	0	430	0	125	3586
N.S.	1	1.00	0.95	1.03	0.00	3.26	0.00	0.95	27.17
time (sec)	N/A	0.138	0.049	0.270	0.000	0.522	0.000	0.366	11.115

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	93	98	0	305	434	91	2624
N.S.	1	1.00	0.96	1.01	0.00	3.14	4.47	0.94	27.05
time (sec)	N/A	0.079	0.076	0.160	0.000	0.364	101.949	0.357	11.310

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	66	0	216	287	68	1632
N.S.	1	1.00	0.99	0.92	0.00	3.00	3.99	0.94	22.67
time (sec)	N/A	0.047	0.040	0.122	0.000	0.326	10.297	0.430	11.273

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	80	75	0	240	0	75	4149
N.S.	1	1.00	1.03	0.96	0.00	3.08	0.00	0.96	53.19
time (sec)	N/A	0.081	0.028	0.097	0.000	0.414	0.000	0.449	13.612

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	130	126	0	385	0	124	7282
N.S.	1	1.00	1.16	1.12	0.00	3.44	0.00	1.11	65.02
time (sec)	N/A	0.131	0.036	0.125	0.000	0.677	0.000	0.401	15.505

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	723	723	88	70	0	13535	0	0	13112
N.S.	1	1.00	0.12	0.10	0.00	18.72	0.00	0.00	18.14
time (sec)	N/A	1.145	0.037	0.079	0.000	52.228	0.000	0.000	38.703

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	718	88	67	0	8705	0	0	11453
N.S.	1	1.00	0.12	0.09	0.00	12.12	0.00	0.00	15.95
time (sec)	N/A	0.864	0.033	0.062	0.000	4.249	0.000	0.000	29.746

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	634	634	59	49	0	8268	0	0	7457
N.S.	1	1.00	0.09	0.08	0.00	13.04	0.00	0.00	11.76
time (sec)	N/A	0.436	0.033	0.061	0.000	14.503	0.000	0.000	26.698

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	634	634	61	47	0	6748	0	0	7469
N.S.	1	1.00	0.10	0.07	0.00	10.64	0.00	0.00	11.78
time (sec)	N/A	0.382	0.023	0.056	0.000	1.872	0.000	0.000	21.637

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	653	653	85	71	0	11285	0	0	11174
N.S.	1	1.00	0.13	0.11	0.00	17.28	0.00	0.00	17.11
time (sec)	N/A	0.679	0.033	0.102	0.000	37.035	0.000	0.000	36.131

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	655	655	89	68	0	11459	0	0	13466
N.S.	1	1.00	0.14	0.10	0.00	17.49	0.00	0.00	20.56
time (sec)	N/A	0.636	0.032	0.099	0.000	18.760	0.000	0.000	35.790

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	38	37	37	42	37	39
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.91	0.80	0.85
time (sec)	N/A	0.037	0.028	0.048	0.290	0.279	0.063	0.383	10.137

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	25	24	24	32	24	26
N.S.	1	1.00	1.00	0.81	0.77	0.77	1.03	0.77	0.84
time (sec)	N/A	0.022	0.008	0.045	0.286	0.284	0.057	0.367	0.027

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	32	32	37	32	34
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.026	0.009	0.042	0.292	0.296	0.058	0.325	0.035

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	44	33	38	34	41	35	36
N.S.	1	1.00	1.07	0.80	0.93	0.83	1.00	0.85	0.88
time (sec)	N/A	0.035	0.017	0.054	0.289	0.326	0.069	0.315	10.153

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	45	25	24	28	36	24	26
N.S.	1	1.00	1.45	0.81	0.77	0.90	1.16	0.77	0.84
time (sec)	N/A	0.029	0.012	0.059	0.347	0.302	0.062	0.301	10.004

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	47	46	0	266	31	645	332
N.S.	1	1.00	0.11	0.11	0.00	0.64	0.07	1.54	0.79
time (sec)	N/A	0.382	0.012	0.046	0.000	0.317	0.082	0.319	0.396

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	48	44	0	300	32	820	309
N.S.	1	1.00	0.13	0.12	0.00	0.79	0.08	2.15	0.81
time (sec)	N/A	0.216	0.012	0.046	0.000	0.327	0.084	0.350	10.566

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	46	41	0	288	24	635	330
N.S.	1	1.00	0.12	0.11	0.00	0.76	0.06	1.68	0.87
time (sec)	N/A	0.155	0.011	0.040	0.000	0.316	0.079	0.366	10.547

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	55	44	0	237	22	824	281
N.S.	1	1.00	0.13	0.11	0.00	0.58	0.05	2.00	0.68
time (sec)	N/A	0.173	0.010	0.043	0.000	0.293	0.086	0.374	10.351

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	57	44	0	299	26	640	319
N.S.	1	1.00	0.14	0.11	0.00	0.73	0.06	1.56	0.78
time (sec)	N/A	0.172	0.011	0.045	0.000	0.299	0.084	0.436	10.351

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	47	40	0	313	31	832	313
N.S.	1	1.00	0.11	0.10	0.00	0.75	0.07	2.00	0.75
time (sec)	N/A	0.177	0.013	0.055	0.000	0.299	0.098	0.318	0.255

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	47	38	0	299	32	645	332
N.S.	1	1.00	0.11	0.09	0.00	0.72	0.08	1.54	0.79
time (sec)	N/A	0.246	0.012	0.057	0.000	0.298	0.090	0.319	10.538

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	37	33	32	32	37	32	34
N.S.	1	1.00	1.03	0.92	0.89	0.89	1.03	0.89	0.94
time (sec)	N/A	0.025	0.008	0.046	0.276	0.294	0.063	0.304	0.029

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	55	33	38	34	41	35	36
N.S.	1	1.00	1.41	0.85	0.97	0.87	1.05	0.90	0.92
time (sec)	N/A	0.034	0.011	0.052	0.260	0.286	0.067	0.298	8.860

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	55	33	0	34	41	35	36
N.S.	1	1.00	1.41	0.85	0.00	0.87	1.05	0.90	0.92
time (sec)	N/A	0.038	0.008	0.053	0.000	0.292	0.066	0.283	0.024

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	396	103	434	0	169	400	0	0
N.S.	1	1.00	0.26	1.10	0.00	0.43	1.01	0.00	0.00
time (sec)	N/A	0.265	8.031	2.217	0.000	0.092	3.679	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	101	398	0	137	257	0	0
N.S.	1	1.00	0.28	1.12	0.00	0.38	0.72	0.00	0.00
time (sec)	N/A	0.218	6.722	0.894	0.000	0.099	2.534	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	98	362	0	104	124	0	0
N.S.	1	1.00	0.31	1.15	0.00	0.33	0.39	0.00	0.00
time (sec)	N/A	0.171	4.947	0.893	0.000	0.092	1.411	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	98	333	0	72	119	0	0
N.S.	1	1.00	0.35	1.20	0.00	0.26	0.43	0.00	0.00
time (sec)	N/A	0.138	10.065	0.855	0.000	0.095	1.300	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	102	382	0	125	119	0	0
N.S.	1	1.00	0.35	1.32	0.00	0.43	0.41	0.00	0.00
time (sec)	N/A	0.140	10.072	1.293	0.000	0.096	6.245	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	129	401	0	190	119	0	0
N.S.	1	1.00	0.42	1.30	0.00	0.61	0.39	0.00	0.00
time (sec)	N/A	0.138	10.086	1.000	0.000	0.105	34.982	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	166	437	0	268	119	0	0
N.S.	1	1.00	0.48	1.25	0.00	0.77	0.34	0.00	0.00
time (sec)	N/A	0.223	10.121	0.898	0.000	0.101	146.823	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	200	484	0	346	0	0	0
N.S.	1	1.00	0.51	1.24	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.263	10.144	0.906	0.000	0.109	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	433	88	67	0	12866	0	0	50213
N.S.	1	1.00	0.20	0.15	0.00	29.71	0.00	0.00	115.97
time (sec)	N/A	0.682	0.054	0.073	0.000	7.770	0.000	0.000	13.574

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	66	0	216	287	68	3704
N.S.	1	1.00	0.99	0.92	0.00	3.00	3.99	0.94	51.44
time (sec)	N/A	0.052	0.040	0.154	0.000	0.421	119.858	1.827	10.386

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	59	51	0	15561	0	0	29445
N.S.	1	1.00	0.16	0.14	0.00	41.50	0.00	0.00	78.52
time (sec)	N/A	0.279	0.034	0.066	0.000	49.644	0.000	0.000	14.129

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	179	168	0	1535	0	1402	4501
N.S.	1	1.00	0.97	0.91	0.00	8.34	0.00	7.62	24.46
time (sec)	N/A	0.149	0.106	0.119	0.000	0.408	0.000	2.356	12.415

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	61	47	0	9245	0	0	36707
N.S.	1	1.00	0.16	0.13	0.00	24.65	0.00	0.00	97.89
time (sec)	N/A	0.225	0.033	0.062	0.000	2.697	0.000	0.000	13.406

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	80	74	0	240	0	77	8454
N.S.	1	1.00	1.03	0.95	0.00	3.08	0.00	0.99	108.38
time (sec)	N/A	0.083	0.026	0.123	0.000	0.755	0.000	1.875	10.971

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	85	73	0	21400	0	0	39028
N.S.	1	1.00	0.22	0.19	0.00	54.59	0.00	0.00	99.56
time (sec)	N/A	0.438	0.091	0.112	0.000	137.794	0.000	0.000	13.320

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	89	177	0	2772	0	3003	15013
N.S.	1	1.00	0.45	0.89	0.00	13.93	0.00	15.09	75.44
time (sec)	N/A	0.242	0.030	0.151	0.000	1.058	0.000	2.112	12.123

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	86	68	0	20184	0	0	65350
N.S.	1	1.00	0.22	0.17	0.00	51.23	0.00	0.00	165.86
time (sec)	N/A	0.407	0.049	0.119	0.000	52.641	0.000	0.000	13.454

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	46	34	0	104	170	208	56
N.S.	1	1.00	0.17	0.12	0.00	0.37	0.61	0.75	0.20
time (sec)	N/A	0.201	0.022	0.066	0.000	0.277	0.105	0.348	0.061

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	32	32	37	32	34
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.028	0.010	0.070	0.263	0.285	0.063	0.346	0.027

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	55	46	0	545	27	253	248
N.S.	1	1.00	0.15	0.13	0.00	1.54	0.08	0.71	0.70
time (sec)	N/A	0.190	0.015	0.078	0.000	0.296	1.354	0.361	0.094

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	44	39	0	41	42	31	20
N.S.	1	1.00	0.88	0.78	0.00	0.82	0.84	0.62	0.40
time (sec)	N/A	0.026	0.016	0.076	0.000	0.262	0.063	0.344	8.478

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	57	44	0	417	26	253	208
N.S.	1	1.00	0.16	0.12	0.00	1.17	0.07	0.71	0.59
time (sec)	N/A	0.142	0.012	0.065	0.000	0.305	1.350	0.312	0.001

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	44	33	38	34	41	38	36
N.S.	1	1.00	1.07	0.80	0.93	0.83	1.00	0.93	0.88
time (sec)	N/A	0.035	0.012	0.081	0.263	0.262	0.074	0.352	8.504

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	47	38	0	111	168	210	58
N.S.	1	1.00	0.17	0.14	0.00	0.40	0.60	0.75	0.21
time (sec)	N/A	0.147	0.015	0.091	0.000	0.260	0.105	0.326	8.549

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	49	40	0	169	76	81	56
N.S.	1	1.00	0.55	0.45	0.00	1.90	0.85	0.91	0.63
time (sec)	N/A	0.057	0.013	0.105	0.000	0.257	0.111	0.314	0.058

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	47	38	0	419	32	258	479
N.S.	1	1.00	0.13	0.10	0.00	1.13	0.09	0.70	1.29
time (sec)	N/A	0.175	0.014	0.102	0.000	0.294	1.543	0.365	0.041

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	283	286	0	1027	0	295	2490
N.S.	1	1.00	1.01	1.02	0.00	3.67	0.00	1.05	8.89
time (sec)	N/A	0.376	0.151	0.671	0.000	18.156	0.000	0.336	11.488

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	218	208	0	798	0	221	2051
N.S.	1	1.00	1.00	0.95	0.00	3.66	0.00	1.01	9.41
time (sec)	N/A	0.251	0.109	0.709	0.000	9.547	0.000	0.331	10.700

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	178	164	0	596	0	185	1367
N.S.	1	1.00	1.01	0.93	0.00	3.39	0.00	1.05	7.77
time (sec)	N/A	0.175	0.113	0.822	0.000	3.159	0.000	0.327	10.179

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	132	130	0	405	0	147	966
N.S.	1	1.00	0.89	0.87	0.00	2.72	0.00	0.99	6.48
time (sec)	N/A	0.132	0.077	0.743	0.000	1.066	0.000	0.341	9.718

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	107	105	0	305	0	125	801
N.S.	1	1.00	0.86	0.85	0.00	2.46	0.00	1.01	6.46
time (sec)	N/A	0.092	0.045	0.690	0.000	0.431	0.000	0.322	9.902

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	105	104	0	305	0	125	521
N.S.	1	1.00	0.85	0.85	0.00	2.48	0.00	1.02	4.24
time (sec)	N/A	0.064	0.047	0.681	0.000	0.453	0.000	0.305	10.160

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	159	152	160	0	504	0	162	2399
N.S.	1	1.01	0.96	1.01	0.00	3.19	0.00	1.03	15.18
time (sec)	N/A	0.171	0.111	0.769	0.000	184.148	0.000	0.355	11.089

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	194	207	0	0	0	206	2388
N.S.	1	1.00	1.01	1.07	0.00	0.00	0.00	1.07	12.37
time (sec)	N/A	0.217	0.109	0.871	0.000	0.000	0.000	0.325	22.980

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	252	277	0	0	0	274	3530
N.S.	1	1.00	1.00	1.10	0.00	0.00	0.00	1.09	14.01
time (sec)	N/A	0.278	0.138	0.742	0.000	0.000	0.000	0.319	28.669

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	338	360	0	2703	0	576	3503
N.S.	1	1.00	0.99	1.05	0.00	7.88	0.00	1.68	10.21
time (sec)	N/A	0.549	0.231	0.749	0.000	64.455	0.000	0.311	12.481

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	269	290	0	2139	0	483	2495
N.S.	1	1.00	0.98	1.06	0.00	7.81	0.00	1.76	9.11
time (sec)	N/A	0.371	0.186	0.742	0.000	26.472	0.000	0.319	11.200

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	207	228	0	1465	0	420	2037
N.S.	1	1.00	0.84	0.93	0.00	5.96	0.00	1.71	8.28
time (sec)	N/A	0.271	0.152	0.846	0.000	11.045	0.000	0.314	10.578

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	159	188	0	1120	0	339	1585
N.S.	1	1.00	0.82	0.97	0.00	5.77	0.00	1.75	8.17
time (sec)	N/A	0.211	0.143	0.858	0.000	3.638	0.000	0.297	11.246

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	148	187	0	1059	0	328	1768
N.S.	1	1.00	0.81	1.02	0.00	5.79	0.00	1.79	9.66
time (sec)	N/A	0.165	0.141	0.713	0.000	3.324	0.000	0.288	12.946

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	151	197	0	1079	0	336	1782
N.S.	1	1.00	0.80	1.04	0.00	5.71	0.00	1.78	9.43
time (sec)	N/A	0.210	0.129	0.741	0.000	1.860	0.000	0.309	12.751

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	249	246	281	0	0	0	401	3510
N.S.	1	1.00	0.99	1.13	0.00	0.00	0.00	1.62	14.15
time (sec)	N/A	0.267	0.162	0.955	0.000	0.000	0.000	0.303	27.771

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	287	346	0	0	0	493	4948
N.S.	1	1.00	0.99	1.19	0.00	0.00	0.00	1.69	17.00
time (sec)	N/A	0.346	0.218	0.998	0.000	0.000	0.000	0.300	34.253

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	370	455	0	0	0	598	7144
N.S.	1	1.00	0.99	1.22	0.00	0.00	0.00	1.61	19.20
time (sec)	N/A	0.517	0.265	0.818	0.000	0.000	0.000	0.354	47.165

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	981	981	10904	5004	0	920	0	0	0
N.S.	1	1.00	11.12	5.10	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	3.622	35.469	2.698	0.000	0.116	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	778	778	7531	3661	0	734	0	0	0
N.S.	1	1.00	9.68	4.71	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	1.528	34.884	2.061	0.000	0.127	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	636	636	1314	2662	0	598	0	0	0
N.S.	1	1.00	2.07	4.19	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.633	34.034	1.835	0.000	0.107	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	550	550	1051	1711	0	490	0	0	0
N.S.	1	1.00	1.91	3.11	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.425	32.189	1.669	0.000	0.099	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	955	955	1258	3023	0	0	0	0	0
N.S.	1	1.00	1.32	3.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.200	28.320	0.451	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	929	929	1207	1400	0	0	0	0	0
N.S.	1	1.00	1.30	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.859	30.801	1.605	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1287	1287	1392	1597	0	0	0	0	0
N.S.	1	1.00	1.08	1.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.639	32.614	1.972	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	26	28	28	0	28	28
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08
time (sec)	N/A	0.011	0.604	0.349	0.258	0.323	0.000	0.348	8.786

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	358	358	249	0	0	0	0	0	0
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.164	0.316	0.000	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	262	189	0	0	0	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.102	0.197	0.000	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	136	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.065	0.096	0.000	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	302	302	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.206	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	412	412	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	201	15	14	1234	1326	216	1203
N.S.	1	1.00	12.56	0.94	0.88	77.12	82.88	13.50	75.19
time (sec)	N/A	0.038	0.119	0.789	0.192	0.273	0.156	0.323	9.422

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	233	17	1240	1240	1384	246	1210
N.S.	1	1.00	12.94	0.94	68.89	68.89	76.89	13.67	67.22
time (sec)	N/A	0.201	0.117	0.171	0.208	0.261	0.144	0.323	9.433

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	233	17	1240	1240	1394	246	1210
N.S.	1	1.00	12.94	0.94	68.89	68.89	77.44	13.67	67.22
time (sec)	N/A	0.187	0.110	0.217	0.199	0.270	0.154	0.321	9.605

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	2042	2041	1297	0	1693	1395
N.S.	1	1.00	0.96	88.78	88.74	56.39	0.00	73.61	60.65
time (sec)	N/A	0.020	0.077	0.025	0.269	0.293	0.000	0.380	11.021

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	201	17	16	1238	1326	218	1208
N.S.	1	1.00	11.17	0.94	0.89	68.78	73.67	12.11	67.11
time (sec)	N/A	0.044	0.108	0.817	0.189	0.258	0.146	0.314	0.931

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	233	19	1242	1242	1384	246	1214
N.S.	1	1.00	11.65	0.95	62.10	62.10	69.20	12.30	60.70
time (sec)	N/A	0.208	0.120	0.172	0.204	0.259	0.141	0.324	9.465

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	233	19	1242	1242	1394	246	1214
N.S.	1	1.00	11.65	0.95	62.10	62.10	69.70	12.30	60.70
time (sec)	N/A	0.178	0.113	0.220	0.209	0.281	0.140	0.322	9.572

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	2046	2045	1299	0	1693	1401
N.S.	1	1.00	0.96	81.84	81.80	51.96	0.00	67.72	56.04
time (sec)	N/A	0.022	0.063	0.025	0.268	0.332	0.000	0.370	11.073

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	172	13	13	154	175	13	154
N.S.	1	1.00	11.47	0.87	0.87	10.27	11.67	0.87	10.27
time (sec)	N/A	0.010	0.008	0.612	0.187	0.252	0.052	0.291	0.105

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	182	15	156	156	182	15	156
N.S.	1	1.00	11.38	0.94	9.75	9.75	11.38	0.94	9.75
time (sec)	N/A	0.033	0.006	0.655	0.192	0.252	0.046	0.308	0.111

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	186	15	156	156	185	15	156
N.S.	1	1.00	11.62	0.94	9.75	9.75	11.56	0.94	9.75
time (sec)	N/A	0.034	0.004	0.703	0.191	0.253	0.048	0.294	0.108

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	229	189	0	189	229
N.S.	1	1.00	1.00	10.95	10.90	9.00	0.00	9.00	10.90
time (sec)	N/A	0.021	0.012	0.015	0.201	0.269	0.000	0.291	9.130

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	10	12	11	11	10	12	11
N.S.	1	1.00	0.91	1.09	1.00	1.00	0.91	1.09	1.00
time (sec)	N/A	0.003	0.004	0.898	0.194	0.287	0.077	0.293	8.633

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	16	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	0.94	0.88
time (sec)	N/A	0.012	0.006	0.033	0.187	0.266	0.162	0.623	8.592

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	16	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	0.94	0.88
time (sec)	N/A	0.015	0.006	0.039	0.189	0.264	0.215	0.382	0.032

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	33	24	23	19	0	19	121
N.S.	1	1.00	1.74	1.26	1.21	1.00	0.00	1.00	6.37
time (sec)	N/A	0.017	0.055	0.393	0.226	0.286	0.000	0.304	8.893

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	15	15	14	350	359	14	358
N.S.	1	1.00	0.94	0.94	0.88	21.88	22.44	0.88	22.38
time (sec)	N/A	0.003	0.010	0.886	0.185	0.279	2.546	0.341	2.374

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	352	352	360	16	360
N.S.	1	1.00	1.00	0.94	19.56	19.56	20.00	0.89	20.00
time (sec)	N/A	0.012	0.012	0.431	0.284	0.302	4.194	1.463	15.833

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	352	352	360	16	360
N.S.	1	1.00	1.00	0.94	19.56	19.56	20.00	0.89	20.00
time (sec)	N/A	0.015	0.010	0.222	0.275	0.285	14.853	2.294	18.831

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	416	394	0	21	496
N.S.	1	1.00	0.96	0.96	18.09	17.13	0.00	0.91	21.57
time (sec)	N/A	0.018	0.062	0.016	0.540	0.345	0.000	0.305	22.399

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	12	14	13	13	10	14	13
N.S.	1	1.00	0.92	1.08	1.00	1.00	0.77	1.08	1.00
time (sec)	N/A	0.003	0.004	0.889	0.191	0.275	0.091	0.272	0.026

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	14	18	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	0.74	0.95	0.89
time (sec)	N/A	0.013	0.006	0.034	0.200	0.279	0.152	0.576	8.565

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	14	18	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	0.74	0.95	0.89
time (sec)	N/A	0.016	0.006	0.041	0.186	0.270	0.218	0.350	0.034

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	34	26	25	21	0	21	199
N.S.	1	1.00	1.62	1.24	1.19	1.00	0.00	1.00	9.48
time (sec)	N/A	0.019	0.054	0.391	0.221	0.291	0.000	0.313	8.928

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	16	17	16	354	359	16	358
N.S.	1	1.00	0.89	0.94	0.89	19.67	19.94	0.89	19.89
time (sec)	N/A	0.003	0.011	0.856	0.190	0.290	2.746	0.331	10.652

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	356	356	360	18	360
N.S.	1	1.00	1.00	0.95	17.80	17.80	18.00	0.90	18.00
time (sec)	N/A	0.013	0.014	0.424	0.291	0.296	4.361	1.393	15.096

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	356	356	360	18	360
N.S.	1	1.00	1.00	0.95	17.80	17.80	18.00	0.90	18.00
time (sec)	N/A	0.016	0.013	0.211	0.279	0.296	14.848	2.260	17.239

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	24	419	397	0	23	496
N.S.	1	1.00	0.92	0.96	16.76	15.88	0.00	0.92	19.84
time (sec)	N/A	0.018	0.056	0.016	0.556	0.307	0.000	0.307	22.029

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	10	10	8	11	8
N.S.	1	1.00	0.90	0.90	1.00	1.00	0.80	1.10	0.80
time (sec)	N/A	0.003	0.006	0.632	0.188	0.294	0.060	0.296	8.581

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	15	15	14	17	13	12	15	13
N.S.	1	0.94	0.94	0.88	1.06	0.81	0.75	0.94	0.81
time (sec)	N/A	0.016	0.005	0.600	0.222	0.268	0.084	0.308	0.037

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	15	15	14	17	13	12	15	13
N.S.	1	0.94	0.94	0.88	1.06	0.81	0.75	0.94	0.81
time (sec)	N/A	0.019	0.007	0.638	0.233	0.258	0.094	0.295	8.653

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	19	18	47	17	39	17	28
N.S.	1	1.00	1.27	1.20	3.13	1.13	2.60	1.13	1.87
time (sec)	N/A	0.022	0.009	0.770	0.209	0.258	7.081	0.295	8.678

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	14	13	13	81	87	13	12
N.S.	1	1.00	0.93	0.87	0.87	5.40	5.80	0.87	0.80
time (sec)	N/A	0.003	0.014	0.676	0.184	0.287	0.451	0.292	9.900

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	81	81	87	15	14
N.S.	1	1.00	1.00	0.94	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.014	0.020	0.679	0.199	0.286	0.661	0.397	1.366

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	81	81	87	15	14
N.S.	1	1.00	1.00	0.94	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.016	0.025	0.710	0.199	0.272	0.928	0.300	11.460

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	203	612	105	0	20	107
N.S.	1	1.00	1.00	9.67	29.14	5.00	0.00	0.95	5.10
time (sec)	N/A	0.021	0.009	69.556	0.232	0.335	0.000	0.313	8.772

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	20	28	104	20	39
N.S.	1	1.00	0.95	1.05	1.00	1.40	5.20	1.00	1.95
time (sec)	N/A	0.004	0.006	0.880	0.180	0.293	29.582	0.311	8.813

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	33	33	201	23	49
N.S.	1	1.00	1.00	0.96	1.32	1.32	8.04	0.92	1.96
time (sec)	N/A	0.013	0.046	0.132	0.222	0.325	104.634	0.284	8.763

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	33	33	0	23	49
N.S.	1	1.00	1.00	0.96	1.32	1.32	0.00	0.92	1.96
time (sec)	N/A	0.016	0.071	0.259	0.224	0.322	0.000	0.300	8.737

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	40	39	38	0	27	56
N.S.	1	1.00	0.96	1.48	1.44	1.41	0.00	1.00	2.07
time (sec)	N/A	0.019	0.112	36.713	0.264	0.301	0.000	0.329	8.978

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	22	32	104	22	42
N.S.	1	1.00	0.95	1.05	1.00	1.45	4.73	1.00	1.91
time (sec)	N/A	0.003	0.052	0.901	0.185	0.271	29.298	0.336	8.698

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	37	37	201	25	52
N.S.	1	1.00	1.00	0.96	1.37	1.37	7.44	0.93	1.93
time (sec)	N/A	0.013	0.038	0.140	0.221	0.278	107.588	0.299	8.611

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	37	37	0	25	52
N.S.	1	1.00	1.00	0.96	1.37	1.37	0.00	0.93	1.93
time (sec)	N/A	0.017	0.068	0.261	0.228	0.262	0.000	0.281	8.584

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	28	45	43	42	0	29	59
N.S.	1	1.00	0.97	1.55	1.48	1.45	0.00	1.00	2.03
time (sec)	N/A	0.019	0.092	35.813	0.323	0.294	0.000	0.342	8.888

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	17	20	19	26	46	19	23
N.S.	1	1.00	0.89	1.05	1.00	1.37	2.42	1.00	1.21
time (sec)	N/A	0.003	0.011	0.674	0.185	0.249	0.306	0.295	8.562

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	97	31	35	31	75	22	31
N.S.	1	1.00	4.04	1.29	1.46	1.29	3.12	0.92	1.29
time (sec)	N/A	0.010	0.064	0.714	0.224	0.250	8.591	0.285	8.609

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	97	31	35	31	0	22	31
N.S.	1	1.00	4.04	1.29	1.46	1.29	0.00	0.92	1.29
time (sec)	N/A	0.014	0.060	0.761	0.231	0.262	0.000	0.283	8.582

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	B	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	26	26	111	106	40	36	100	26	34
N.S.	1	1.00	4.27	4.08	1.54	1.38	3.85	1.00	1.31
time (sec)	N/A	0.052	0.110	29.093	0.254	0.280	11.068	0.328	8.643

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	196	196	158	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.168	0.651	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	374	374	5363	0	0	0	0	0	0
N.S.	1	1.00	14.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.790	7.233	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	816	816	20515	0	0	0	0	0	0
N.S.	1	1.00	25.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.533	8.441	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	34	33	119	0	31
N.S.	1	1.00	1.00	0.77	0.72	0.70	2.53	0.00	0.66
time (sec)	N/A	0.039	0.090	0.030	0.190	0.258	2.605	0.000	8.818

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	245	245	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	210	210	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.177	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	206	206	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.127	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.106	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	263	263	218	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.479	0.576	0.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	212	212	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.163	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	210	210	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.159	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	498	498	391	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.374	1.188	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	323	323	273	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.825	0.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	181	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.070	0.339	0.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	33	0	33	33
N.S.	1	1.00	1.06	1.00	1.06	1.06	0.00	1.06	1.06
time (sec)	N/A	0.017	1.119	0.124	0.233	0.280	0.000	0.350	8.701

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	33	31	33	46	0	33	33
N.S.	1	1.00	1.06	1.00	1.06	1.48	0.00	1.06	1.06
time (sec)	N/A	0.016	0.927	0.142	0.239	0.311	0.000	0.489	9.018

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [83] had the largest ratio of [.4230999999999999976]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	22	0.045
2	A	2	1	1.00	22	0.045
3	A	2	1	1.00	22	0.045
4	A	2	1	1.00	22	0.045
5	A	2	1	1.00	20	0.050
6	A	8	8	1.00	22	0.364
7	A	8	8	1.00	22	0.364
8	A	8	8	1.00	22	0.364
9	A	7	6	1.00	25	0.240
10	A	6	6	1.00	25	0.240
11	A	5	5	1.00	25	0.200
12	A	7	6	1.00	25	0.240
13	A	7	6	1.00	25	0.240
14	A	14	8	1.00	25	0.320
15	A	14	8	1.00	25	0.320
16	A	13	7	1.00	23	0.304
17	A	13	7	1.00	22	0.318
18	A	14	8	1.00	25	0.320
19	A	14	8	1.00	25	0.320
20	A	7	6	1.00	23	0.261
21	A	4	4	1.00	23	0.174
22	A	5	5	1.00	23	0.217
23	A	7	6	1.00	23	0.261
24	A	5	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	15	9	1.00	23	0.391
26	A	15	9	1.00	23	0.391
27	A	14	8	1.00	23	0.348
28	A	13	7	1.00	21	0.333
29	A	13	7	1.00	20	0.350
30	A	14	8	1.00	23	0.348
31	A	15	9	1.00	23	0.391
32	A	5	5	1.00	21	0.238
33	A	7	6	1.00	21	0.286
34	A	8	7	1.00	18	0.389
35	A	6	4	1.00	24	0.167
36	A	5	4	1.00	24	0.167
37	A	4	4	1.00	24	0.167
38	A	3	3	1.00	24	0.125
39	A	3	3	1.00	24	0.125
40	A	3	3	1.00	24	0.125
41	A	4	4	1.00	24	0.167
42	A	5	4	1.00	24	0.167
43	A	8	5	1.00	25	0.200
44	A	5	5	1.00	25	0.200
45	A	7	4	1.00	25	0.160
46	A	4	3	1.00	23	0.130
47	A	7	4	1.00	22	0.182
48	A	7	6	1.00	25	0.240
49	A	8	5	1.00	25	0.200
50	A	5	4	1.00	25	0.160
51	A	8	5	1.00	25	0.200
52	A	20	7	1.00	23	0.304
53	A	5	5	1.00	23	0.217
54	A	21	7	1.00	23	0.304
55	A	4	3	1.00	21	0.143
56	A	19	6	1.00	20	0.300
57	A	7	6	1.00	23	0.261
58	A	20	7	1.00	23	0.304
59	A	11	8	1.00	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	21	9	1.00	23	0.391
61	A	7	6	1.00	25	0.240
62	A	7	6	1.00	25	0.240
63	A	7	6	1.00	23	0.261
64	A	7	6	1.00	22	0.273
65	A	7	6	1.00	25	0.240
66	A	7	7	1.00	25	0.280
67	A	7	6	1.01	25	0.240
68	A	7	6	1.00	25	0.240
69	A	7	6	1.00	25	0.240
70	A	7	6	1.00	25	0.240
71	A	7	6	1.00	25	0.240
72	A	7	6	1.00	23	0.261
73	A	7	6	1.00	22	0.273
74	A	7	6	1.00	25	0.240
75	A	8	7	1.00	25	0.280
76	A	7	6	1.00	25	0.240
77	A	7	6	1.00	25	0.240
78	A	7	6	1.00	25	0.240
79	A	11	7	1.00	29	0.241
80	A	10	7	1.00	29	0.241
81	A	8	7	1.00	29	0.241
82	A	8	7	1.00	27	0.259
83	A	16	11	1.00	26	0.423
84	A	16	11	1.00	29	0.379
85	A	24	12	1.00	29	0.414
86	N/A	0	0	1.00	26	0.000
87	A	13	4	1.00	26	0.154
88	A	10	4	1.00	26	0.154
89	A	7	4	1.00	24	0.167
90	A	6	3	1.00	26	0.115
91	A	8	3	1.00	26	0.115
92	A	10	3	1.00	26	0.115
93	A	1	1	1.00	19	0.053
94	A	2	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	2	2	1.00	26	0.077
96	A	2	2	1.00	30	0.067
97	A	1	1	1.00	21	0.048
98	A	2	2	1.00	26	0.077
99	A	2	2	1.00	28	0.071
100	A	2	2	1.00	32	0.062
101	A	1	1	1.00	18	0.056
102	A	3	3	1.00	23	0.130
103	A	3	3	1.00	25	0.120
104	A	3	3	1.00	29	0.103
105	A	1	1	1.00	19	0.053
106	A	2	2	1.00	24	0.083
107	A	2	2	1.00	26	0.077
108	A	2	2	1.00	30	0.067
109	A	1	1	1.00	19	0.053
110	A	2	2	1.00	24	0.083
111	A	2	2	1.00	26	0.077
112	A	2	2	1.00	30	0.067
113	A	1	1	1.00	21	0.048
114	A	2	2	1.00	26	0.077
115	A	2	2	1.00	28	0.071
116	A	2	2	1.00	32	0.062
117	A	1	1	1.00	21	0.048
118	A	2	2	1.00	26	0.077
119	A	2	2	1.00	28	0.071
120	A	2	2	1.00	32	0.062
121	A	1	1	1.00	18	0.056
122	A	4	3	0.94	23	0.130
123	A	4	3	0.94	25	0.120
124	A	4	3	1.00	29	0.103
125	A	1	1	1.00	18	0.056
126	A	3	3	1.00	23	0.130
127	A	3	3	1.00	25	0.120
128	A	3	3	1.00	29	0.103
129	A	1	1	1.00	19	0.053

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	2	2	1.00	24	0.083
131	A	2	2	1.00	26	0.077
132	A	2	2	1.00	30	0.067
133	A	1	1	1.00	21	0.048
134	A	2	2	1.00	26	0.077
135	A	2	2	1.00	28	0.071
136	A	2	2	1.00	32	0.062
137	A	1	1	1.00	18	0.056
138	A	1	1	1.00	23	0.043
139	A	1	1	1.00	25	0.040
140	A	2	2	1.00	29	0.069
141	A	4	2	1.00	29	0.069
142	A	5	3	1.00	29	0.103
143	A	6	3	1.00	29	0.103
144	A	3	3	1.00	59	0.051
145	A	5	3	1.00	31	0.097
146	A	5	3	1.00	29	0.103
147	A	5	3	1.00	27	0.111
148	A	5	3	1.00	26	0.115
149	A	8	5	1.00	29	0.172
150	A	5	3	1.00	29	0.103
151	A	5	3	1.00	29	0.103
152	A	10	4	1.00	31	0.129
153	A	7	4	1.00	29	0.138
154	A	2	2	1.00	22	0.091
155	N/A	0	0	1.00	31	0.000
156	N/A	0	0	1.00	31	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (d+ex^3)^5 (a+bx^3+cx^6)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 163

$$\begin{aligned}\int (d+ex^3)^5 (a+bx^3+cx^6)\, dx = {} & ad^5x+\frac{1}{4}d^4(bd+5ae)x^4+\frac{1}{7}d^3\left(cd^2+5e(bd+2ae)\right)x^7\\ &+\frac{1}{2}d^2e\left(cd^2+2e(bd+ae)\right)x^{10}\\ &+\frac{5}{13}de^2\left(2cd^2+e(2bd+ae)\right)x^{13}\\ &+\frac{1}{16}e^3\left(10cd^2+e(5bd+ae)\right)x^{16}\\ &+\frac{1}{19}e^4(5cd+be)x^{19}+\frac{1}{22}ce^5x^{22}\end{aligned}$$

```
[Out] a*d^5*x+1/4*d^4*(5*a*e+b*d)*x^4+1/7*d^3*(c*d^2+5*e*(2*a*e+b*d))*x^7+1/2*d^2
*e*(c*d^2+2*e*(a*e+b*d))*x^10+5/13*d*e^2*(2*c*d^2+e*(a*e+2*b*d))*x^13+1/16*
e^3*(10*c*d^2+e*(a*e+5*b*d))*x^16+1/19*e^4*(b*e+5*c*d)*x^19+1/22*c*e^5*x^22
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1421}

$$\begin{aligned}\int (d+ex^3)^5 (a+bx^3+cx^6)\, dx = {} & \frac{1}{16}e^3x^{16}\left(e(ae+5bd)+10cd^2\right)+\frac{5}{13}de^2x^{13}\left(e(ae+2bd)+2cd^2\right)\\ &+\frac{1}{2}d^2ex^{10}\left(2e(ae+bd)+cd^2\right)+\frac{1}{7}d^3x^7\left(5e(2ae+bd)+cd^2\right)\\ &+\frac{1}{4}d^4x^4(5ae+bd)+ad^5x+\frac{1}{19}e^4x^{19}(be+5cd)+\frac{1}{22}ce^5x^{22}\end{aligned}$$

```
[In] Int[(d + e*x^3)^5*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*e*(b*d + 2*a*e))*x^7)
/7 + (d^2*e*(c*d^2 + 2*e*(b*d + a*e))*x^10)/2 + (5*d*e^2*(2*c*d^2 + e*(2*b*
d + a*e))*x^13)/13 + (e^3*(10*c*d^2 + e*(5*b*d + a*e))*x^16)/16 + (e^4*(5*c
*d + b*e)*x^19)/19 + (c*e^5*x^22)/22
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ad^5 + d^4(bd+5ae)x^3 + d^3\left(cd^2+5e(bd+2ae)\right)x^6 + 5d^2e\left(cd^2+2e(bd+ae)\right)x^9 \right.\\
&\quad + 5de^2\left(2cd^2+e(2bd+ae)\right)x^{12} + e^3\left(10cd^2+e(5bd+ae)\right)x^{15} + e^4(5cd+be)x^{18} \\
&\quad \left. + ce^5x^{21}\right) dx \\
&= ad^5x + \frac{1}{4}d^4(bd+5ae)x^4 + \frac{1}{7}d^3\left(cd^2+5e(bd+2ae)\right)x^7 \\
&\quad + \frac{1}{2}d^2e\left(cd^2+2e(bd+ae)\right)x^{10} + \frac{5}{13}de^2\left(2cd^2+e(2bd+ae)\right)x^{13} \\
&\quad + \frac{1}{16}e^3\left(10cd^2+e(5bd+ae)\right)x^{16} + \frac{1}{19}e^4(5cd+be)x^{19} + \frac{1}{22}ce^5x^{22}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01

$$\begin{aligned}
\int \left(d+ex^3\right)^5\left(a+bx^3+cx^6\right)\,dx &= ad^5x + \frac{1}{4}d^4(bd+5ae)x^4 + \frac{1}{7}d^3\left(cd^2+5bde+10ae^2\right)x^7 \\
&\quad + \frac{1}{2}d^2e\left(cd^2+2bde+2ae^2\right)x^{10} \\
&\quad + \frac{5}{13}de^2\left(2cd^2+2bde+ae^2\right)x^{13} \\
&\quad + \frac{1}{16}e^3\left(10cd^2+5bde+ae^2\right)x^{16} \\
&\quad + \frac{1}{19}e^4(5cd+be)x^{19} + \frac{1}{22}ce^5x^{22}
\end{aligned}$$

```
[In] Integrate[(d + e*x^3)^5*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*b*d*e + 10*a*e^2)*x^7
)/7 + (d^2*e*(c*d^2 + 2*b*d*e + 2*a*e^2)*x^10)/2 + (5*d*e^2*(2*c*d^2 + 2*b*
d*e + a*e^2)*x^13)/13 + (e^3*(10*c*d^2 + 5*b*d*e + a*e^2)*x^16)/16 + (e^4*(
5*c*d + b*e)*x^19)/19 + (c*e^5*x^22)/22
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.01

method	result
norman	$a\,d^5x+\left(\frac{5}{4}d^4ea+\frac{1}{4}d^5b\right)x^4+\left(\frac{10}{7}a\,d^3e^2+\frac{5}{7}b\,d^4e+\frac{1}{7}d^5c\right)x^7+\left(a\,d^2e^3+b\,d^3e^2+\frac{1}{2}c\,d^4e\right)x^{10}+$
default	$\frac{c\,e^5x^{22}}{22}+\frac{(b\,e^5+5d\,e^4c)x^{19}}{19}+\frac{(a\,e^5+5bd\,e^4+10c\,d^2e^3)x^{16}}{16}+\frac{(5ad\,e^4+10b\,d^2e^3+10c\,d^3e^2)x^{13}}{13}+\frac{(10a\,d^2e^3+10b\,d^3e^2+5c}{10}$
gosper	$a\,d^5x+\frac{5}{4}x^4d^4ea+\frac{1}{4}x^4d^5b+\frac{10}{7}x^7a\,d^3e^2+\frac{5}{7}x^7b\,d^4e+\frac{1}{7}x^7d^5c+x^{10}a\,d^2e^3+x^{10}b\,d^3e^2+\frac{1}{2}x^{10}c$
risch	$a\,d^5x+\frac{5}{4}x^4d^4ea+\frac{1}{4}x^4d^5b+\frac{10}{7}x^7a\,d^3e^2+\frac{5}{7}x^7b\,d^4e+\frac{1}{7}x^7d^5c+x^{10}a\,d^2e^3+x^{10}b\,d^3e^2+\frac{1}{2}x^{10}c$
parallelrisch	$a\,d^5x+\frac{5}{4}x^4d^4ea+\frac{1}{4}x^4d^5b+\frac{10}{7}x^7a\,d^3e^2+\frac{5}{7}x^7b\,d^4e+\frac{1}{7}x^7d^5c+x^{10}a\,d^2e^3+x^{10}b\,d^3e^2+\frac{1}{2}x^{10}c$

```
[In] int((e*x^3+d)^5*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] a*d^5*x+(5/4*d^4*e*a+1/4*d^5*b)*x^4+(10/7*a*d^3*e^2+5/7*b*d^4*e+1/7*d^5*c)*
x^7+(a*d^2*e^3+b*d^3*e^2+1/2*c*d^4*e)*x^10+(5/13*a*d*e^4+10/13*b*d^2*e^3+10
/13*c*d^3*e^2)*x^13+(1/16*a*e^5+5/16*b*d*e^4+5/8*c*d^2*e^3)*x^16+(1/19*b*e^
5+5/19*d*e^4*c)*x^19+1/22*c*e^5*x^22
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \left(d+ex^3\right)^5\left(a+bx^3+cx^6\right)\,dx=&\ \frac{1}{22}\,ce^5x^{22}+\frac{1}{19}\left(5\,cde^4+be^5\right)x^{19}\\&+\frac{1}{16}\left(10\,cd^2e^3+5\,bde^4+ae^5\right)x^{16}\\&+\frac{5}{13}\left(2\,cd^3e^2+2\,bd^2e^3+ade^4\right)x^{13}\\&+\frac{1}{2}\left(cd^4e+2\,bd^3e^2+2\,ad^2e^3\right)x^{10}\\&+\frac{1}{7}\left(cd^5+5\,bd^4e+10\,ad^3e^2\right)x^7\\&+ad^5x+\frac{1}{4}\left(bd^5+5\,ad^4e\right)x^4\end{aligned}$$

```
[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/22*c*e^5*x^22 + 1/19*(5*c*d*e^4 + b*e^5)*x^19 + 1/16*(10*c*d^2*e^3 + 5*b*
d*e^4 + a*e^5)*x^16 + 5/13*(2*c*d^3*e^2 + 2*b*d^2*e^3 + a*d*e^4)*x^13 + 1/2
*(c*d^4*e + 2*b*d^3*e^2 + 2*a*d^2*e^3)*x^10 + 1/7*(c*d^5 + 5*b*d^4*e + 10*a
*d^3*e^2)*x^7 + a*d^5*x + 1/4*(b*d^5 + 5*a*d^4*e)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.15

$$\int \left(d+ex^3\right)^5\left(a+bx^3+cx^6\right)\,dx = ad^5x+\frac{ce^5x^{22}}{22}+x^{19}\left(\frac{be^5}{19}+\frac{5cde^4}{19}\right)+x^{16}\left(\frac{ae^5}{16}+\frac{5bde^4}{16}+\frac{5cd^2e^3}{8}\right)+x^{13}\cdot\left(\frac{5ade^4}{13}+\frac{10bd^2e^3}{13}+\frac{10cd^3e^2}{13}\right)+x^{10}\left(ad^2e^3+bd^3e^2+\frac{cd^4e}{2}\right)+x^7\cdot\left(\frac{10ad^3e^2}{7}+\frac{5bd^4e}{7}+\frac{cd^5}{7}\right)+x^4\cdot\left(\frac{5ad^4e}{4}+\frac{bd^5}{4}\right)$$

```
[In] integrate((e*x**3+d)**5*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**5*x + c*e**5*x**22/22 + x**19*(b*e**5/19 + 5*c*d*e**4/19) + x**16*(a*e
**5/16 + 5*b*d*e**4/16 + 5*c*d**2*e**3/8) + x**13*(5*a*d*e**4/13 + 10*b*d**
2*e**3/13 + 10*c*d**3*e**2/13) + x**10*(a*d**2*e**3 + b*d**3*e**2 + c*d**4*
e/2) + x**7*(10*a*d**3*e**2/7 + 5*b*d**4*e/7 + c*d**5/7) + x**4*(5*a*d**4*e
/4 + b*d**5/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02

$$\int \left(d+ex^3\right)^5\left(a+bx^3+cx^6\right)\,dx = \frac{1}{22}\,ce^5x^{22}+\frac{1}{19}\left(5\,cde^4+be^5\right)x^{19}+\frac{1}{16}\left(10\,cd^2e^3+5\,bde^4+ae^5\right)x^{16}+\frac{5}{13}\left(2\,cd^3e^2+2\,bd^2e^3+ade^4\right)x^{13}+\frac{1}{2}\left(cd^4e+2\,bd^3e^2+2\,ad^2e^3\right)x^{10}+\frac{1}{7}\left(cd^5+5\,bd^4e+10\,ad^3e^2\right)x^7+ad^5x+\frac{1}{4}\left(bd^5+5\,ad^4e\right)x^4$$

```
[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/22*c*e^5*x^22 + 1/19*(5*c*d*e^4 + b*e^5)*x^19 + 1/16*(10*c*d^2*e^3 + 5*b*
d*e^4 + a*e^5)*x^16 + 5/13*(2*c*d^3*e^2 + 2*b*d^2*e^3 + a*d*e^4)*x^13 + 1/2
*(c*d^4*e + 2*b*d^3*e^2 + 2*a*d^2*e^3)*x^10 + 1/7*(c*d^5 + 5*b*d^4*e + 10*a
*d^3*e^2)*x^7 + a*d^5*x + 1/4*(b*d^5 + 5*a*d^4*e)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12

$$\int \left(d+ex^3\right)^5 \left(a+bx^3+cx^6\right)\,dx = \frac{1}{22}ce^5x^{22}+\frac{5}{19}cde^4x^{19}+\frac{1}{19}be^5x^{19}+\frac{5}{8}cd^2e^3x^{16} + \frac{5}{16}bde^4x^{16}+\frac{1}{16}ae^5x^{16}+\frac{10}{13}cd^3e^2x^{13}+\frac{10}{13}bd^2e^3x^{13} + \frac{5}{13}ade^4x^{13}+\frac{1}{2}cd^4ex^{10}+bd^3e^2x^{10}+ad^2e^3x^{10}+\frac{1}{7}cd^5x^7 + \frac{5}{7}bd^4ex^7+\frac{10}{7}ad^3e^2x^7+\frac{1}{4}bd^5x^4+\frac{5}{4}ad^4ex^4+ad^5x$$

```
[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/22*c*e^5*x^22 + 5/19*c*d*e^4*x^19 + 1/19*b*e^5*x^19 + 5/8*c*d^2*e^3*x^16
+ 5/16*b*d*e^4*x^16 + 1/16*a*e^5*x^16 + 10/13*c*d^3*e^2*x^13 + 10/13*b*d^2*
e^3*x^13 + 5/13*a*d*e^4*x^13 + 1/2*c*d^4*e*x^10 + b*d^3*e^2*x^10 + a*d^2*e^
3*x^10 + 1/7*c*d^5*x^7 + 5/7*b*d^4*e*x^7 + 10/7*a*d^3*e^2*x^7 + 1/4*b*d^5*x
^4 + 5/4*a*d^4*e*x^4 + a*d^5*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.97

$$\int \left(d+ex^3\right)^5 \left(a+bx^3+cx^6\right)\,dx = x^4\left(\frac{b\,d^5}{4}+\frac{5\,a\,e\,d^4}{4}\right)+x^{19}\left(\frac{b\,e^5}{19}+\frac{5\,c\,d\,e^4}{19}\right) + x^7\left(\frac{c\,d^5}{7}+\frac{5\,b\,d^4\,e}{7}+\frac{10\,a\,d^3\,e^2}{7}\right) + x^{16}\left(\frac{5\,c\,d^2\,e^3}{8}+\frac{5\,b\,d\,e^4}{16}+\frac{a\,e^5}{16}\right)+\frac{c\,e^5\,x^{22}}{22} + a\,d^5\,x+\frac{d^2\,e\,x^{10}\,\left(c\,d^2+2\,b\,d\,e+2\,a\,e^2\right)}{2} + \frac{5\,d\,e^2\,x^{13}\,\left(2\,c\,d^2+2\,b\,d\,e+a\,e^2\right)}{13}$$

```
[In] int((d + e*x^3)^5*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^4*((b*d^5)/4 + (5*a*d^4*e)/4) + x^19*((b*e^5)/19 + (5*c*d*e^4)/19) + x^7*
((c*d^5)/7 + (10*a*d^3*e^2)/7 + (5*b*d^4*e)/7) + x^16*((a*e^5)/16 + (5*c*d^
2*e^3)/8 + (5*b*d*e^4)/16) + (c*e^5*x^22)/22 + a*d^5*x + (d^2*e*x^10*(2*a*e
^2 + c*d^2 + 2*b*d*e))/2 + (5*d*e^2*x^13*(a*e^2 + 2*c*d^2 + 2*b*d*e))/13
```

3.2 $\int (d+ex^3)^4 (a+bx^3+cx^6)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int (d+ex^3)^4 (a+bx^3+cx^6)\, dx = ad^4x + \frac{1}{4}d^3(bd+4ae)x^4 + \frac{1}{7}d^2(cd^2+4bde+6ae^2)\, x^7 + \frac{1}{5}de(2cd^2+e(3bd+2ae))\, x^{10} + \frac{1}{13}e^2(6cd^2+e(4bd+ae))\, x^{13} + \frac{1}{16}e^3(4cd+be)x^{16} + \frac{1}{19}ce^4x^{19}$$

```
[Out] a*d^4*x+1/4*d^3*(4*a*e+b*d)*x^4+1/7*d^2*(6*a*e^2+4*b*d*e+c*d^2)*x^7+1/5*d*e
*(2*c*d^2+e*(2*a*e+3*b*d))*x^10+1/13*e^2*(6*c*d^2+e*(a*e+4*b*d))*x^13+1/16*
e^3*(b*e+4*c*d)*x^16+1/19*c*e^4*x^19
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1421}

$$\int (d+ex^3)^4 (a+bx^3+cx^6)\, dx = \frac{1}{13}e^2x^{13}(e(ae+4bd)+6cd^2) + \frac{1}{7}d^2x^7(6ae^2+4bde+cd^2) + \frac{1}{5}dex^{10}(e(2ae+3bd)+2cd^2) + \frac{1}{4}d^3x^4(4ae+bd) + ad^4x + \frac{1}{16}e^3x^{16}(be+4cd) + \frac{1}{19}ce^4x^{19}$$

```
[In] Int[(d + e*x^3)^4*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^7)
/7 + (d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^10)/5 + (e^2*(6*c*d^2 + e*(4*b*d
+ a*e))*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/16 + (c*e^4*x^19)/19
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ad^4 + d^3(bd+4ae)x^3 + d^2\left(cd^2+4bde+6ae^2\right)x^6 + 2de\left(2cd^2+e(3bd+2ae)\right)x^9 \right.\\ &\qquad \left. + e^2\left(6cd^2+e(4bd+ae)\right)x^{12} + e^3(4cd+be)x^{15} + ce^4x^{18}\right)\,dx\\ &= ad^4x + \frac{1}{4}d^3(bd+4ae)x^4 + \frac{1}{7}d^2\left(cd^2+4bde+6ae^2\right)x^7 + \frac{1}{5}de\left(2cd^2+e(3bd+2ae)\right)x^{10}\\ &\quad + \frac{1}{13}e^2\left(6cd^2+e(4bd+ae)\right)x^{13} + \frac{1}{16}e^3(4cd+be)x^{16} + \frac{1}{19}ce^4x^{19}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(d+ex^3\right)^4\left(a+bx^3+cx^6\right)\,dx &= ad^4x + \frac{1}{4}d^3(bd+4ae)x^4 + \frac{1}{7}d^2\left(cd^2+4bde+6ae^2\right)x^7\\ &\quad + \frac{1}{5}de\left(2cd^2+3bde+2ae^2\right)x^{10}\\ &\quad + \frac{1}{13}e^2\left(6cd^2+4bde+ae^2\right)x^{13}\\ &\quad + \frac{1}{16}e^3(4cd+be)x^{16} + \frac{1}{19}ce^4x^{19}\end{aligned}$$

```
[In] Integrate[(d + e*x^3)^4*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^7)
/7 + (d*e*(2*c*d^2 + 3*b*d*e + 2*a*e^2)*x^10)/5 + (e^2*(6*c*d^2 + 4*b*d*e +
 a*e^2)*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/16 + (c*e^4*x^19)/19
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

method	result
norman	$a\,d^4x + \left(a\,d^3e + \frac{1}{4}d^4b\right)x^4 + \left(\frac{6}{7}e^2d^2a + \frac{4}{7}d^3eb + \frac{1}{7}d^4c\right)x^7 + \left(\frac{2}{5}d\,e^3a + \frac{3}{5}e^2d^2b + \frac{2}{5}d^3ec\right)x^{10} +$
default	$\frac{c\,e^4x^{19}}{19} + \frac{(b\,e^4+4d\,e^3c)x^{16}}{16} + \frac{(e^4a+4bd\,e^3+6e^2d^2c)x^{13}}{13} + \frac{(4d\,e^3a+6e^2d^2b+4d^3ec)x^{10}}{10} + \frac{(6e^2d^2a+4d^3eb+d^4c)x^7}{7} +$
gosper	$a\,d^4x + x^4a\,d^3e + \frac{1}{4}b\,x^4d^4 + \frac{6}{7}x^7e^2d^2a + \frac{4}{7}x^7d^3eb + \frac{1}{7}x^7d^4c + \frac{2}{5}x^{10}d\,e^3a + \frac{3}{5}x^{10}e^2d^2b + \frac{2}{5}x^{10}d$
risch	$a\,d^4x + x^4a\,d^3e + \frac{1}{4}b\,x^4d^4 + \frac{6}{7}x^7e^2d^2a + \frac{4}{7}x^7d^3eb + \frac{1}{7}x^7d^4c + \frac{2}{5}x^{10}d\,e^3a + \frac{3}{5}x^{10}e^2d^2b + \frac{2}{5}x^{10}d$
parallelrisch	$a\,d^4x + x^4a\,d^3e + \frac{1}{4}b\,x^4d^4 + \frac{6}{7}x^7e^2d^2a + \frac{4}{7}x^7d^3eb + \frac{1}{7}x^7d^4c + \frac{2}{5}x^{10}d\,e^3a + \frac{3}{5}x^{10}e^2d^2b + \frac{2}{5}x^{10}d$

```
[In] int((e*x^3+d)^4*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] a*d^4*x+(a*d^3*e+1/4*d^4*b)*x^4+(6/7*e^2*d^2*a+4/7*d^3*e*b+1/7*d^4*c)*x^7+(
2/5*d*e^3*a+3/5*e^2*d^2*b+2/5*d^3*e*c)*x^10+(1/13*e^4*a+4/13*b*d*e^3+6/13*e
^2*d^2*c)*x^13+(1/16*b*e^4+1/4*d*e^3*c)*x^16+1/19*c*e^4*x^19
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\int (d+ex^3)^4\left(a+bx^3+cx^6\right)\,dx = \frac{1}{19}\,ce^4x^{19} + \frac{1}{16}\,(4\,cde^3+be^4)x^{16} + \frac{1}{13}\,(6\,cd^2e^2+4\,bde^3+ae^4)x^{13} + \frac{1}{5}\,(2\,cd^3e+3\,bd^2e^2+2\,ade^3)x^{10} + \frac{1}{7}\,(cd^4+4\,bd^3e+6\,ad^2e^2)x^7 + ad^4x + \frac{1}{4}\,(bd^4+4\,ad^3e)x^4$$

```
[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/19*c*e^4*x^19 + 1/16*(4*c*d*e^3 + b*e^4)*x^16 + 1/13*(6*c*d^2*e^2 + 4*b*d
*e^3 + a*e^4)*x^13 + 1/5*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^10 + 1/7*(
c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^7 + a*d^4*x + 1/4*(b*d^4 + 4*a*d^3*e)*x^
4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.12

$$\int \left(d+ex^3\right)^4\left(a+bx^3+cx^6\right)\,dx = ad^4x+\frac{ce^4x^{19}}{19}+x^{16}\left(\frac{be^4}{16}+\frac{cde^3}{4}\right)+x^{13}\left(\frac{ae^4}{13}+\frac{4bde^3}{13}+\frac{6cd^2e^2}{13}\right)+x^{10}\cdot\left(\frac{2ade^3}{5}+\frac{3bd^2e^2}{5}+\frac{2cd^3e}{5}\right)+x^7\cdot\left(\frac{6ad^2e^2}{7}+\frac{4bd^3e}{7}+\frac{cd^4}{7}\right)+x^4\left(ad^3e+\frac{bd^4}{4}\right)$$

```
[In] integrate((e*x**3+d)**4*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**4*x + c*e**4*x**19/19 + x**16*(b*e**4/16 + c*d*e**3/4) + x**13*(a*e**4
/13 + 4*b*d*e**3/13 + 6*c*d**2*e**2/13) + x**10*(2*a*d*e**3/5 + 3*b*d**2*e*
*2/5 + 2*c*d**3*e/5) + x**7*(6*a*d**2*e**2/7 + 4*b*d**3*e/7 + c*d**4/7) + x
**4*(a*d**3*e + b*d**4/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\int \left(d+ex^3\right)^4\left(a+bx^3+cx^6\right)\,dx = \frac{1}{19}\,ce^4x^{19}+\frac{1}{16}\left(4\,cde^3+be^4\right)x^{16}+\frac{1}{13}\left(6\,cd^2e^2+4\,bde^3+ae^4\right)x^{13}+\frac{1}{5}\left(2\,cd^3e+3\,bd^2e^2+2\,ade^3\right)x^{10}+\frac{1}{7}\left(cd^4+4\,bd^3e+6\,ad^2e^2\right)x^7+ad^4x+\frac{1}{4}\left(bd^4+4\,ad^3e\right)x^4$$

```
[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/19*c*e^4*x^19 + 1/16*(4*c*d*e^3 + b*e^4)*x^16 + 1/13*(6*c*d^2*e^2 + 4*b*d
*e^3 + a*e^4)*x^13 + 1/5*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^10 + 1/7*(
c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^7 + a*d^4*x + 1/4*(b*d^4 + 4*a*d^3*e)*x^
4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.09

$$\int \left(d+ex^3\right)^4 \left(a+bx^3+cx^6\right) \, dx = \frac{1}{19} ce^4x^{19} + \frac{1}{4} cde^3x^{16} + \frac{1}{16} be^4x^{16} + \frac{6}{13} cd^2e^2x^{13} + \frac{4}{13} bde^3x^{13} + \frac{1}{13} ae^4x^{13} + \frac{2}{5} cd^3ex^{10} + \frac{3}{5} bd^2e^2x^{10} + \frac{2}{5} ade^3x^{10} + \frac{1}{7} cd^4x^7 + \frac{4}{7} bd^3ex^7 + \frac{6}{7} ad^2e^2x^7 + \frac{1}{4} bd^4x^4 + ad^3ex^4 + ad^4x$$

```
[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/19*c*e^4*x^19 + 1/4*c*d*e^3*x^16 + 1/16*b*e^4*x^16 + 6/13*c*d^2*e^2*x^13
+ 4/13*b*d*e^3*x^13 + 1/13*a*e^4*x^13 + 2/5*c*d^3*e*x^10 + 3/5*b*d^2*e^2*x^
10 + 2/5*a*d*e^3*x^10 + 1/7*c*d^4*x^7 + 4/7*b*d^3*e*x^7 + 6/7*a*d^2*e^2*x^7
 + 1/4*b*d^4*x^4 + a*d^3*e*x^4 + a*d^4*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

$$\int \left(d+ex^3\right)^4 \left(a+bx^3+cx^6\right) \, dx = x^4 \left(\frac{b\,d^4}{4}+a\,e\,d^3\right) + x^{16} \left(\frac{b\,e^4}{16}+\frac{c\,d\,e^3}{4}\right) + x^7 \left(\frac{c\,d^4}{7}+\frac{4\,b\,d^3\,e}{7}+\frac{6\,a\,d^2\,e^2}{7}\right) + x^{13} \left(\frac{6\,c\,d^2\,e^2}{13}+\frac{4\,b\,d\,e^3}{13}+\frac{a\,e^4}{13}\right) + \frac{c\,e^4\,x^{19}}{19} + a\,d^4\,x + \frac{d\,e\,x^{10}\,\left(2\,c\,d^2+3\,b\,d\,e+2\,a\,e^2\right)}{5}$$

```
[In] int((d + e*x^3)^4*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^4*((b*d^4)/4 + a*d^3*e) + x^16*((b*e^4)/16 + (c*d*e^3)/4) + x^7*((c*d^4)/
7 + (6*a*d^2*e^2)/7 + (4*b*d^3*e)/7) + x^13*((a*e^4)/13 + (6*c*d^2*e^2)/13
+ (4*b*d*e^3)/13) + (c*e^4*x^19)/19 + a*d^4*x + (d*e*x^10*(2*a*e^2 + 2*c*d^
2 + 3*b*d*e))/5
```

3.3 $\int (d+ex^3)^3 (a+bx^3+cx^6)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int (d+ex^3)^3 (a+bx^3+cx^6)\,dx = ad^3x+\frac{1}{4}d^2(bd+3ae)x^4+\frac{1}{7}d\left(cd^2+3e(bd+ae)\right)x^7+\frac{1}{10}e\left(3cd^2+e(3bd+ae)\right)x^{10}+\frac{1}{13}e^2(3cd+be)x^{13}+\frac{1}{16}ce^3x^{16}$$

```
[Out] a*d^3*x+1/4*d^2*(3*a*e+b*d)*x^4+1/7*d*(c*d^2+3*e*(a*e+b*d))*x^7+1/10*e*(3*c
*d^2+e*(a*e+3*b*d))*x^10+1/13*e^2*(b*e+3*c*d)*x^13+1/16*c*e^3*x^16
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1421}

$$\int (d+ex^3)^3 (a+bx^3+cx^6)\,dx = \frac{1}{10}ex^{10}\left(e(ae+3bd)+3cd^2\right)+\frac{1}{7}dx^7\left(3e(ae+bd)+cd^2\right)+\frac{1}{4}d^2x^4(3ae+bd)+ad^3x+\frac{1}{13}e^2x^{13}(be+3cd)+\frac{1}{16}ce^3x^{16}$$

```
[In] Int[(d + e*x^3)^3*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*e*(b*d + a*e))*x^7)/7 +
 (e*(3*c*d^2 + e*(3*b*d + a*e))*x^10)/10 + (e^2*(3*c*d + b*e)*x^13)/13 + (c
*e^3*x^16)/16
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(ad^3 + d^2(bd+3ae)x^3 + d\left(cd^2+3e(bd+ae)\right)x^6 + e\left(3cd^2+e(3bd+ae)\right)x^9 + e^2(3cd+be)x^{12} + ce^3x^{15}\right)\,dx$$

$$= ad^3x + \frac{1}{4}d^2(bd+3ae)x^4 + \frac{1}{7}d\left(cd^2+3e(bd+ae)\right)x^7 + \frac{1}{10}e\left(3cd^2+e(3bd+ae)\right)x^{10} + \frac{1}{13}e^2(3cd+be)x^{13} + \frac{1}{16}ce^3x^{16}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \left(d+ex^3\right)^3\left(a+bx^3+cx^6\right)\,dx = ad^3x + \frac{1}{4}d^2(bd+3ae)x^4 + \frac{1}{7}d\left(cd^2+3bde+3ae^2\right)x^7 + \frac{1}{10}e\left(3cd^2+3bde+ae^2\right)x^{10} + \frac{1}{13}e^2(3cd+be)x^{13} + \frac{1}{16}ce^3x^{16}$$

```
[In] Integrate[(d + e*x^3)^3*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*b*d*e + 3*a*e^2)*x^7)/7
 + (e*(3*c*d^2 + 3*b*d*e + a*e^2)*x^10)/10 + (e^2*(3*c*d + b*e)*x^13)/13 +
(c*e^3*x^16)/16
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

method	result
default	$\frac{c\,e^{3}x^{16}}{16}+\frac{\left(b\,e^{3}+3c d\,e^{2}\right)x^{13}}{13}+\frac{\left(a\,e^{3}+3d\,e^{2}b+3c\,d^{2}e\right)x^{10}}{10}+\frac{\left(3d\,e^{2}a+3b\,d^{2}e+d^{3}c\right)x^{7}}{7}+\frac{\left(3a\,d^{2}e+b\,d^{3}\right)x^{4}}{4}+a\,d^{3}x$
norman	$a\,d^{3}x+\left(\frac{3}{4}a\,d^{2}e+\frac{1}{4}b\,d^{3}\right)x^{4}+\left(\frac{3}{7}d\,e^{2}a+\frac{3}{7}b\,d^{2}e+\frac{1}{7}d^{3}c\right)x^{7}+\left(\frac{1}{10}a\,e^{3}+\frac{3}{10}d\,e^{2}b+\frac{3}{10}c\,d^{2}e\right)x^{10}$ -
gosper	$a\,d^{3}x+\frac{3}{4}x^{4}a\,d^{2}e+\frac{1}{4}x^{4}b\,d^{3}+\frac{3}{7}x^{7}d\,e^{2}a+\frac{3}{7}x^{7}b\,d^{2}e+\frac{1}{7}x^{7}d^{3}c+\frac{1}{10}x^{10}a\,e^{3}+\frac{3}{10}x^{10}d\,e^{2}b+\frac{3}{10}x^{10}$ c
risch	$a\,d^{3}x+\frac{3}{4}x^{4}a\,d^{2}e+\frac{1}{4}x^{4}b\,d^{3}+\frac{3}{7}x^{7}d\,e^{2}a+\frac{3}{7}x^{7}b\,d^{2}e+\frac{1}{7}x^{7}d^{3}c+\frac{1}{10}x^{10}a\,e^{3}+\frac{3}{10}x^{10}d\,e^{2}b+\frac{3}{10}x^{10}$ c
parallelrisch	$a\,d^{3}x+\frac{3}{4}x^{4}a\,d^{2}e+\frac{1}{4}x^{4}b\,d^{3}+\frac{3}{7}x^{7}d\,e^{2}a+\frac{3}{7}x^{7}b\,d^{2}e+\frac{1}{7}x^{7}d^{3}c+\frac{1}{10}x^{10}a\,e^{3}+\frac{3}{10}x^{10}d\,e^{2}b+\frac{3}{10}x^{10}$ c

```
[In] int((e*x^3+d)^3*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*c*e^3*x^16+1/13*(b*e^3+3*c*d*e^2)*x^13+1/10*(a*e^3+3*b*d*e^2+3*c*d^2*e
)*x^10+1/7*(3*a*d*e^2+3*b*d^2*e+c*d^3)*x^7+1/4*(3*a*d^2*e+b*d^3)*x^4+a*d^3*
x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int \left(d+ex^{3}\right)^{3}\left(a+bx^{3}+cx^{6}\right)\,dx=\frac{1}{16}\,ce^{3}x^{16}+\frac{1}{13}\,\left(3\,cde^{2}+be^{3}\right)x^{13}+\frac{1}{10}\,\left(3\,cd^{2}e+3\,bde^{2}+ae^{3}\right)x^{10}+\frac{1}{7}\,\left(cd^{3}+3\,bd^{2}e+3\,ade^{2}\right)x^{7}+ad^{3}x+\frac{1}{4}\,\left(bd^{3}+3\,ad^{2}e\right)x^{4}$$

```
[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/16*c*e^3*x^16 + 1/13*(3*c*d*e^2 + b*e^3)*x^13 + 1/10*(3*c*d^2*e + 3*b*d*e
^2 + a*e^3)*x^10 + 1/7*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^7 + a*d^3*x + 1/4*
(b*d^3 + 3*a*d^2*e)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.14

$$\int \left(d+ex^{3}\right)^{3}\left(a+bx^{3}+cx^{6}\right)\,dx=ad^{3}x+\frac{ce^{3}x^{16}}{16}+x^{13}\left(\frac{be^{3}}{13}+\frac{3cde^{2}}{13}\right)+x^{10}\left(\frac{ae^{3}}{10}+\frac{3bde^{2}}{10}+\frac{3cd^{2}e}{10}\right)+x^{7}\cdot\left(\frac{3ade^{2}}{7}+\frac{3bd^{2}e}{7}+\frac{cd^{3}}{7}\right)+x^{4}\cdot\left(\frac{3ad^{2}e}{4}+\frac{bd^{3}}{4}\right)$$

```
[In] integrate((e*x**3+d)**3*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**3*x + c*e**3*x**16/16 + x**13*(b*e**3/13 + 3*c*d*e**2/13) + x**10*(a*e
**3/10 + 3*b*d*e**2/10 + 3*c*d**2*e/10) + x**7*(3*a*d*e**2/7 + 3*b*d**2*e/7
 + c*d**3/7) + x**4*(3*a*d**2*e/4 + b*d**3/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int (d+ex^3)^3 (a+bx^3+cx^6)\, dx = \frac{1}{16}\, ce^3x^{16} + \frac{1}{13}\,(3\, cde^2 + be^3)x^{13} + \frac{1}{10}\,(3\, cd^2e + 3\, bde^2 + ae^3)x^{10} + \frac{1}{7}\,(cd^3 + 3\, bd^2e + 3\, ade^2)x^7 + ad^3x + \frac{1}{4}\,(bd^3 + 3\, ad^2e)x^4$$

```
[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/16*c*e^3*x^16 + 1/13*(3*c*d*e^2 + b*e^3)*x^13 + 1/10*(3*c*d^2*e + 3*b*d*e
^2 + a*e^3)*x^10 + 1/7*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^7 + a*d^3*x + 1/4*
(b*d^3 + 3*a*d^2*e)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09

$$\int (d+ex^3)^3 (a+bx^3+cx^6)\, dx = \frac{1}{16}\, ce^3x^{16} + \frac{3}{13}\, cde^2x^{13} + \frac{1}{13}\, be^3x^{13} + \frac{3}{10}\, cd^2ex^{10} + \frac{3}{10}\, bde^2x^{10} + \frac{1}{10}\, ae^3x^{10} + \frac{1}{7}\, cd^3x^7 + \frac{3}{7}\, bd^2ex^7 + \frac{3}{7}\, ade^2x^7 + \frac{1}{4}\, bd^3x^4 + \frac{3}{4}\, ad^2ex^4 + ad^3x$$

```
[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/16*c*e^3*x^16 + 3/13*c*d*e^2*x^13 + 1/13*b*e^3*x^13 + 3/10*c*d^2*e*x^10 +
 3/10*b*d*e^2*x^10 + 1/10*a*e^3*x^10 + 1/7*c*d^3*x^7 + 3/7*b*d^2*e*x^7 + 3/
7*a*d*e^2*x^7 + 1/4*b*d^3*x^4 + 3/4*a*d^2*e*x^4 + a*d^3*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int (d+ex^3)^3 (a+bx^3+cx^6) \, dx = x^4 \left(\frac{b\,d^3}{4}+\frac{3\,a\,e\,d^2}{4}\right)+x^{13}\left(\frac{b\,e^3}{13}+\frac{3\,c\,d\,e^2}{13}\right) + x^7\left(\frac{c\,d^3}{7}+\frac{3\,b\,d^2\,e}{7}+\frac{3\,a\,d\,e^2}{7}\right) + x^{10}\left(\frac{3\,c\,d^2\,e}{10}+\frac{3\,b\,d\,e^2}{10}+\frac{a\,e^3}{10}\right)+\frac{c\,e^3\,x^{16}}{16}+a\,d^3\,x$$

```
[In] int((d + e*x^3)^3*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^4*((b*d^3)/4 + (3*a*d^2*e)/4) + x^13*((b*e^3)/13 + (3*c*d*e^2)/13) + x^7*
((c*d^3)/7 + (3*a*d*e^2)/7 + (3*b*d^2*e)/7) + x^10*((a*e^3)/10 + (3*b*d*e^2
)/10 + (3*c*d^2*e)/10) + (c*e^3*x^16)/16 + a*d^3*x
```

3.4 $\int (d+ex^3)^2 (a+bx^3+cx^6)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int (d+ex^3)^2 (a+bx^3+cx^6)\, dx = ad^2x+\frac{1}{4}d(bd+2ae)x^4+\frac{1}{7}\left(cd^2+e(2bd+ae)\right)x^7 + \frac{1}{10}e(2cd+be)x^{10}+\frac{1}{13}ce^2x^{13}$$

```
[Out] a*d^2*x+1/4*d*(2*a*e+b*d)*x^4+1/7*(c*d^2+e*(a*e+2*b*d))*x^7+1/10*e*(b*e+2*c
*d)*x^10+1/13*c*e^2*x^13
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1421}

$$\int (d+ex^3)^2 (a+bx^3+cx^6)\, dx = \frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{4}dx^4(2ae+bd) + ad^2x+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{13}ce^2x^{13}$$

```
[In] Int[(d + e*x^3)^2*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + e*(2*b*d + a*e))*x^7)/7 + (e*
(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
```

```
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ad^2+d(bd+2ae)x^3+\left(cd^2+e(2bd+ae)\right)x^6+e(2cd+be)x^9+ce^2x^{12}\right)\,dx\\ &= ad^2x+\frac{1}{4}d(bd+2ae)x^4+\frac{1}{7}\left(cd^2+e(2bd+ae)\right)x^7+\frac{1}{10}e(2cd+be)x^{10}+\frac{1}{13}ce^2x^{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \left(d+ex^3\right)^2\left(a+bx^3+cx^6\right)\,dx=ad^2x+\frac{1}{4}d(bd+2ae)x^4+\frac{1}{7}\left(cd^2+2bde+ae^2\right)x^7+\frac{1}{10}e(2cd+be)x^{10}+\frac{1}{13}ce^2x^{13}$$

```
[In] Integrate[(d + e*x^3)^2*(a + b*x^3 + c*x^6),x]
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + 2*b*d*e + a*e^2)*x^7)/7 + (e*
(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result	s
default	$\frac{c\,e^2x^{13}}{13}+\frac{\left(b\,e^2+2dce\right)x^{10}}{10}+\frac{\left(a\,e^2+2bde+c\,d^2\right)x^7}{7}+\frac{\left(2eda+b\,d^2\right)x^4}{4}+a\,d^2x$	7
norman	$\frac{c\,e^2x^{13}}{13}+\left(\frac{1}{10}b\,e^2+\frac{1}{5}dce\right)x^{10}+\left(\frac{1}{7}a\,e^2+\frac{2}{7}bde+\frac{1}{7}c\,d^2\right)x^7+\left(\frac{1}{2}eda+\frac{1}{4}b\,d^2\right)x^4+a\,d^2x$	7
gosper	$\frac{1}{13}c\,e^2x^{13}+\frac{1}{10}x^{10}b\,e^2+\frac{1}{5}x^{10}dce+\frac{1}{7}x^7a\,e^2+\frac{2}{7}x^7bde+\frac{1}{7}x^7c\,d^2+\frac{1}{2}x^4eda+\frac{1}{4}b\,x^4d^2+a\,d^2x$	7
risch	$\frac{1}{13}c\,e^2x^{13}+\frac{1}{10}x^{10}b\,e^2+\frac{1}{5}x^{10}dce+\frac{1}{7}x^7a\,e^2+\frac{2}{7}x^7bde+\frac{1}{7}x^7c\,d^2+\frac{1}{2}x^4eda+\frac{1}{4}b\,x^4d^2+a\,d^2x$	7
parallelrisch	$\frac{1}{13}c\,e^2x^{13}+\frac{1}{10}x^{10}b\,e^2+\frac{1}{5}x^{10}dce+\frac{1}{7}x^7a\,e^2+\frac{2}{7}x^7bde+\frac{1}{7}x^7c\,d^2+\frac{1}{2}x^4eda+\frac{1}{4}b\,x^4d^2+a\,d^2x$	7

```
[In] int((e*x^3+d)^2*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/13*c*e^2*x^13+1/10*(b*e^2+2*c*d*e)*x^10+1/7*(a*e^2+2*b*d*e+c*d^2)*x^7+1/4
*(2*a*d*e+b*d^2)*x^4+a*d^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int (d+ex^3)^2 (a+bx^3+cx^6) \, dx = \frac{1}{13} ce^2x^{13} + \frac{1}{10} (2\,cde + be^2)x^{10} + \frac{1}{7} (cd^2 + 2\,bde + ae^2)x^7 + \frac{1}{4} (bd^2 + 2\,ade)x^4 + ad^2x$$

```
[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/13*c*e^2*x^13 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/7*(c*d^2 + 2*b*d*e + a*e^
2)*x^7 + 1/4*(b*d^2 + 2*a*d*e)*x^4 + a*d^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int (d+ex^3)^2 (a+bx^3+cx^6) \, dx = ad^2x + \frac{ce^2x^{13}}{13} + x^{10} \left(\frac{be^2}{10} + \frac{cde}{5} \right) + x^7 \left(\frac{ae^2}{7} + \frac{2bde}{7} + \frac{cd^2}{7} \right) + x^4 \left(\frac{ade}{2} + \frac{bd^2}{4} \right)$$

```
[In] integrate((e*x**3+d)**2*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**13/13 + x**10*(b*e**2/10 + c*d*e/5) + x**7*(a*e**2/7 +
 2*b*d*e/7 + c*d**2/7) + x**4*(a*d*e/2 + b*d**2/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int (d+ex^3)^2 (a+bx^3+cx^6) \, dx = \frac{1}{13} ce^2x^{13} + \frac{1}{10} (2\,cde + be^2)x^{10} + \frac{1}{7} (cd^2 + 2\,bde + ae^2)x^7 + \frac{1}{4} (bd^2 + 2\,ade)x^4 + ad^2x$$

```
[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/13*c*e^2*x^13 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/7*(c*d^2 + 2*b*d*e + a*e^
2)*x^7 + 1/4*(b*d^2 + 2*a*d*e)*x^4 + a*d^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int (d+ex^3)^2 (a+bx^3+cx^6) \, dx = \frac{1}{13} ce^2x^{13} + \frac{1}{5} cdex^{10} + \frac{1}{10} be^2x^{10} + \frac{1}{7} cd^2x^7 + \frac{2}{7} bdex^7 + \frac{1}{7} ae^2x^7 + \frac{1}{4} bd^2x^4 + \frac{1}{2} adex^4 + ad^2x$$

```
[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/13*c*e^2*x^13 + 1/5*c*d*e*x^10 + 1/10*b*e^2*x^10 + 1/7*c*d^2*x^7 + 2/7*b*
d*e*x^7 + 1/7*a*e^2*x^7 + 1/4*b*d^2*x^4 + 1/2*a*d*e*x^4 + a*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int (d+ex^3)^2 (a+bx^3+cx^6) \, dx = x^7 \left(\frac{c\,d^2}{7} + \frac{2\,b\,d\,e}{7} + \frac{a\,e^2}{7}\right) + x^4 \left(\frac{b\,d^2}{4} + \frac{a\,e\,d}{2}\right) + x^{10} \left(\frac{b\,e^2}{10} + \frac{c\,d\,e}{5}\right) + \frac{c\,e^2\,x^{13}}{13} + a\,d^2\,x$$

```
[In] int((d + e*x^3)^2*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^7*((a*e^2)/7 + (c*d^2)/7 + (2*b*d*e)/7) + x^4*((b*d^2)/4 + (a*d*e)/2) + x
^10*((b*e^2)/10 + (c*d*e)/5) + (c*e^2*x^13)/13 + a*d^2*x
```

3.5 $\int (d+ex^3)(a+bx^3+cx^6)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int (d+ex^3)\left(a+bx^3+cx^6\right)\,dx = adx+\frac{1}{4}(bd+ae)x^4+\frac{1}{7}(cd+be)x^7+\frac{1}{10}cex^{10}$$

```
[Out] a*d*x+1/4*(a*e+b*d)*x^4+1/7*(b*e+c*d)*x^7+1/10*c*e*x^10
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1421}

$$\int (d+ex^3)\left(a+bx^3+cx^6\right)\,dx = \frac{1}{4}x^4(ae+bd)+adx+\frac{1}{7}x^7(be+cd)+\frac{1}{10}cex^{10}$$

```
[In] Int[(d + e*x^3)*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ad+(bd+ae)x^3+(cd+be)x^6+cex^9\right)\,dx \\ &= adx+\frac{1}{4}(bd+ae)x^4+\frac{1}{7}(cd+be)x^7+\frac{1}{10}cex^{10}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (d+ex^3) (a+bx^3+cx^6) \, dx = adx + \frac{1}{4}(bd+ae)x^4 + \frac{1}{7}(cd+be)x^7 + \frac{1}{10}cex^{10}$$

[In] Integrate[(d + e*x^3)*(a + b*x^3 + c*x^6),x]

[Out] a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
default	$adx + \frac{(ae+bd)x^4}{4} + \frac{(be+cd)x^7}{7} + \frac{ce\,x^{10}}{10}$	37
norman	$\frac{ce\,x^{10}}{10} + \left(\frac{be}{7} + \frac{cd}{7}\right) x^7 + \left(\frac{ae}{4} + \frac{bd}{4}\right) x^4 + adx$	39
gosper	$\frac{1}{10}ce\,x^{10} + \frac{1}{7}x^7be + \frac{1}{7}x^7cd + \frac{1}{4}x^4ae + \frac{1}{4}x^4bd + adx$	41
risch	$\frac{1}{10}ce\,x^{10} + \frac{1}{7}x^7be + \frac{1}{7}x^7cd + \frac{1}{4}x^4ae + \frac{1}{4}x^4bd + adx$	41
parallelrisch	$\frac{1}{10}ce\,x^{10} + \frac{1}{7}x^7be + \frac{1}{7}x^7cd + \frac{1}{4}x^4ae + \frac{1}{4}x^4bd + adx$	41

[In] int((e*x^3+d)*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)

[Out] a*d*x+1/4*(a*e+b*d)*x^4+1/7*(b*e+c*d)*x^7+1/10*c*e*x^10

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int (d+ex^3) (a+bx^3+cx^6) \, dx = \frac{1}{10} cex^{10} + \frac{1}{7} (cd+be)x^7 + \frac{1}{4} (bd+ae)x^4 + adx$$

[In] integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] 1/10*c*e*x^10 + 1/7*(c*d + b*e)*x^7 + 1/4*(b*d + a*e)*x^4 + a*d*x

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int (d+ex^3)(a+bx^3+cx^6)\, dx = adx + \frac{cex^{10}}{10} + x^7\left(\frac{be}{7} + \frac{cd}{7}\right) + x^4\left(\frac{ae}{4} + \frac{bd}{4}\right)$$

```
[In] integrate((e*x**3+d)*(c*x**6+b*x**3+a),x)
[Out] a*d*x + c*e*x**10/10 + x**7*(b*e/7 + c*d/7) + x**4*(a*e/4 + b*d/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int (d+ex^3)(a+bx^3+cx^6)\, dx = \frac{1}{10}\, cex^{10} + \frac{1}{7}\,(cd+be)x^7 + \frac{1}{4}\,(bd+ae)x^4 + adx$$

```
[In] integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] 1/10*c*e*x^10 + 1/7*(c*d + b*e)*x^7 + 1/4*(b*d + a*e)*x^4 + a*d*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int (d+ex^3)(a+bx^3+cx^6)\, dx = \frac{1}{10}\, cex^{10} + \frac{1}{7}\, cdx^7 + \frac{1}{7}\, bex^7 + \frac{1}{4}\, bdx^4 + \frac{1}{4}\, aex^4 + adx$$

```
[In] integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] 1/10*c*e*x^10 + 1/7*c*d*x^7 + 1/7*b*e*x^7 + 1/4*b*d*x^4 + 1/4*a*e*x^4 + a*d
*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int (d+ex^3)(a+bx^3+cx^6)\, dx = \frac{c\,e\,x^{10}}{10} + \left(\frac{b\,e}{7} + \frac{c\,d}{7}\right)\,x^7 + \left(\frac{a\,e}{4} + \frac{b\,d}{4}\right)\,x^4 + a\,d\,x$$

```
[In] int((d + e*x^3)*(a + b*x^3 + c*x^6),x)
[Out] x^4*((a*e)/4 + (b*d)/4) + x^7*((b*e)/7 + (c*d)/7) + a*d*x + (c*e*x^10)/10
```

3.6 $\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 188

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx = -\frac{(cd-be)x}{e^2}+\frac{cx^4}{4e}-\frac{(cd^2-bde+ae^2)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{7/3}} + \frac{(cd^2-bde+ae^2)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{7/3}} - \frac{(cd^2-bde+ae^2)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{7/3}}$$

```
[Out] -(-b*e+c*d)*x/e^2+1/4*c*x^4/e+1/3*(a*e^2-b*d*e+c*d^2)*ln(d^(1/3)+e^(1/3)*x)
/d^(2/3)/e^(7/3)-1/6*(a*e^2-b*d*e+c*d^2)*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/
3)*x^2)/d^(2/3)/e^(7/3)-1/3*(a*e^2-b*d*e+c*d^2)*arctan(1/3*(d^(1/3)-2*e^(1/
3)*x)/d^(1/3)*3^(1/2))/d^(2/3)/e^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1425, 396, 206, 31, 648, 631, 210, 642}

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)(ae^2-bde+cd^2)}{\sqrt{3}d^{2/3}e^{7/3}} - \frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)(ae^2-bde+cd^2)}{6d^{2/3}e^{7/3}} + \frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)(ae^2-bde+cd^2)}{3d^{2/3}e^{7/3}} - \frac{x(cd-be)}{e^2}+\frac{cx^4}{4e}$$

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3),x]
```

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^4)/(4*e) - ((c*d^2 - b*d*e + a*e^2)*ArcTan[(d
^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(7/3)) + ((c*d
^2 - b*d*e + a*e^2)*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(7/3)) - ((c*d^2
 - b*d*e + a*e^2)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3
)*e^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1425

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{cx^4}{4e} + \frac{\int \frac{4ae-(4cd-4be)x^3}{d+ex^3}\,dx}{4e}\\
&= -\frac{(cd-be)x}{e^2} + \frac{cx^4}{4e} - \left(-a - \frac{d(cd-be)}{e^2}\right)\int \frac{1}{d+ex^3}\,dx\\
&= -\frac{(cd-be)x}{e^2} + \frac{cx^4}{4e} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{3d^{2/3}} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)\int \frac{2\sqrt[3]{d}-\sqrt[3]{e}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{3d^{2/3}}\\
&= -\frac{(cd-be)x}{e^2} + \frac{cx^4}{4e} + \frac{(cd^2-bde+ae^2)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{7/3}}\\
&\quad - \frac{(cd^2-bde+ae^2)\int \frac{-\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{6d^{2/3}e^{7/3}}\\
&\quad + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)\int \frac{1}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{2\sqrt[3]{d}}\\
&= -\frac{(cd-be)x}{e^2} + \frac{cx^4}{4e} + \frac{(cd^2-bde+ae^2)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{7/3}}\\
&\quad - \frac{(cd^2-bde+ae^2)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{7/3}}\\
&\quad + \frac{(cd^2-bde+ae^2)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{d^{2/3}e^{7/3}}
\end{aligned}
$$

$$= -\frac{(cd-be)x}{e^2} + \frac{cx^4}{4e} - \frac{(cd^2-bde+ae^2)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{7/3}}$$
$$+ \frac{(cd^2-bde+ae^2)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{7/3}}$$
$$- \frac{(cd^2-bde+ae^2)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{7/3}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.94

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx$$

$$= \frac{12\sqrt[3]{e}(-cd+be)x + 3ce^{4/3}x^4 - \frac{4\sqrt{3}(cd^2+e(-bd+ae))\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{2/3}} + \frac{4(cd^2+e(-bd+ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{d^{2/3}} - \frac{2(cd^2+}{}}{12e^{7/3}}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3),x]
```

```
[Out] (12*e^(1/3)*(-(c*d) + b*e)*x + 3*c*e^(4/3)*x^4 - (4*Sqrt[3]*(c*d^2 + e*(-(b
*d) + a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(2/3) + (4*(c*d^
2 + e*(-(b*d) + a*e))*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - (2*(c*d^2 + e*(-(
b*d) + a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2/3))/(12*e
^(7/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.71 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.36

method	result	size
risch	$\frac{c\,x^4}{4e}+\frac{bx}{e}-\frac{cdx}{e^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e\_Z^3+d\right)}\frac{\left(a\,e^2-bde+c\,d^2\right)\ln\left(x-\_R\right)}{\_R^2}}{3e^3}$	67
default	$\frac{\frac{1}{4}c\,x^4e+bex-cdx}{e^2}+\frac{\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)\left(a\,e^2-bde+c\,d^2\right)}{e^2}$	133

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c*x^4/e+1/e*b*x-c*d*x/e^2+1/3/e^3*sum((a*e^2-b*d*e+c*d^2)/_R^2*ln(x-_R)
,_R=RootOf(_Z^3*e+d))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.47

$$\int\frac{a+bx^3+cx^6}{d+ex^3}\,dx$$
$$=\left[\frac{3\,cd^2e^2x^4+6\sqrt{\frac{1}{3}}(cd^3e-bd^2e^2+ade^3)\sqrt{-\frac{(d^2e)^{\frac{1}{3}}}{e}}\log\left(\frac{2\,dex^3-3\,(d^2e)^{\frac{1}{3}}dx-d^2+3\sqrt{\frac{1}{3}}\left(2\,dex^2+(d^2e)^{\frac{2}{3}}x-(d^2e)^{\frac{1}{3}}d\right)\sqrt{-\frac{(\cdots}{}}}{ex^3+d}\right.}{}\right.$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] [1/12*(3*c*d^2*e^2*x^4 + 6*sqrt(1/3)*(c*d^3*e - b*d^2*e^2 + a*d*e^3)*sqrt(-
(d^2*e)^(1/3)/e)*log((2*d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)*(
2*d*e*x^2 + (d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(e*x
```

```
^3 + d)) - 2*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2
/3)*x + (d^2*e)^(1/3)*d) + 4*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*
x + (d^2*e)^(2/3)) - 12*(c*d^3*e - b*d^2*e^2)*x)/(d^2*e^3), 1/12*(3*c*d^2*e
^2*x^4 + 12*sqrt(1/3)*(c*d^3*e - b*d^2*e^2 + a*d*e^3)*sqrt((d^2*e)^(1/3)/e)
*arctan(sqrt(1/3)*(2*(d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt((d^2*e)^(1/3)/
e)/d^2) - 2*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/
3)*x + (d^2*e)^(1/3)*d) + 4*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x
 + (d^2*e)^(2/3)) - 12*(c*d^3*e - b*d^2*e^2)*x)/(d^2*e^3)]
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx = \frac{cx^4}{4e}+x\left(\frac{b}{e}-\frac{cd}{e^2}\right)$$
$$+\text{RootSum}\left(27t^3d^2e^7-a^3e^6+3a^2bde^5-3a^2cd^2e^4-3ab^2d^2e^4+6abcd^3e^3-3ac^2d^4e^2+b^3d^3e^3-3b^2cd^4e\right.$$

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d),x)
```

```
[Out] c*x**4/(4*e) + x*(b/e - c*d/e**2) + RootSum(27*_t**3*d**2*e**7 - a**3*e**6
+ 3*a**2*b*d*e**5 - 3*a**2*c*d**2*e**4 - 3*a*b**2*d**2*e**4 + 6*a*b*c*d**3*
e**3 - 3*a*c**2*d**4*e**2 + b**3*d**3*e**3 - 3*b**2*c*d**4*e**2 + 3*b*c**2*
d**5*e - c**3*d**6, Lambda(_t, _t*log(3*_t*d*e**2/(a*e**2 - b*d*e + c*d**2)
 + x)))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.02

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx = -\frac{\sqrt{3}(cd^2-bde+ae^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\,(-de^2)^{\frac{2}{3}}\,e} - \frac{(cd^2-bde+ae^2)\log\left(x^2+x\left(-\frac{d}{e}\right)^{\frac{1}{3}}+\left(-\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\,(-de^2)^{\frac{2}{3}}\,e} - \frac{(cd^2e^2-bde^3+ae^4)\left(-\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3\,de^4} + \frac{ce^3x^4-4\,cde^2x+4\,be^3x}{4\,e^4}$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(c*d^2 - b*d*e + a*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d/e)^(1/3)
)/(-d/e)^(1/3))/((-d*e^2)^(2/3)*e) - 1/6*(c*d^2 - b*d*e + a*e^2)*log(x^2 +
x*(-d/e)^(1/3) + (-d/e)^(2/3))/((-d*e^2)^(2/3)*e) - 1/3*(c*d^2*e^2 - b*d*e^
3 + a*e^4)*(-d/e)^(1/3)*log(abs(x - (-d/e)^(1/3)))/(d*e^4) + 1/4*(c*e^3*x^4
 - 4*c*d*e^2*x + 4*b*e^3*x)/e^4
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx = x\left(\frac{b}{e}-\frac{c\,d}{e^2}\right)+\frac{c\,x^4}{4\,e}+\frac{\ln\left(e^{1/3}\,x+d^{1/3}\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{3\,d^{2/3}\,e^{7/3}} + \frac{\ln\left(2\,e^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{3\,d^{2/3}\,e^{7/3}} - \frac{\ln\left(d^{1/3}-2\,e^{1/3}\,x+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{3\,d^{2/3}\,e^{7/3}}$$

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3),x)
```

```
[Out] x*(b/e - (c*d)/e^2) + (c*x^4)/(4*e) + (log(e^(1/3)*x + d^(1/3))*(a*e^2 + c*
d^2 - b*d*e))/(3*d^(2/3)*e^(7/3)) + (log(3^(1/2)*d^(1/3)*1i + 2*e^(1/3)*x -
 d^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*e^2 + c*d^2 - b*d*e))/(3*d^(2/3)*e^(7/3
)) - (log(3^(1/2)*d^(1/3)*1i - 2*e^(1/3)*x + d^(1/3))*((3^(1/2)*1i)/2 + 1/2
)*(a*e^2 + c*d^2 - b*d*e))/(3*d^(2/3)*e^(7/3))
```

3.7 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 213

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx=\frac{cx}{e^2}+\frac{(cd^2-bde+ae^2)\,x}{3de^2\,(d+ex^3)}+\frac{(4cd^2-e(bd+2ae))\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{3\sqrt{3}d^{5/3}e^{7/3}}-\frac{(4cd^2-e(bd+2ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{5/3}e^{7/3}}+\frac{(4cd^2-e(bd+2ae))\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{18d^{5/3}e^{7/3}}$$

```
[Out] c*x/e^2+1/3*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)-1/9*(4*c*d^2-e*(2*a*e+b*d
))*ln(d^(1/3)+e^(1/3)*x)/d^(5/3)/e^(7/3)+1/18*(4*c*d^2-e*(2*a*e+b*d))*ln(d^
(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(5/3)/e^(7/3)+1/9*(4*c*d^2-e*(2*a*e+
b*d))*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d^(5/3)/e^(7/3)*3^(
1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used

$= \{1423, 396, 206, 31, 648, 631, 210, 642\}$

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx = \frac{\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)(4cd^2-e(2ae+bd))}{3\sqrt{3}d^{5/3}e^{7/3}} + \frac{x(ae^2-bde+cd^2)}{3de^2(d+ex^3)} + \frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)(4cd^2-e(2ae+bd))}{18d^{5/3}e^{7/3}} - \frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)(4cd^2-e(2ae+bd))}{9d^{5/3}e^{7/3}} + \frac{cx}{e^2}$$

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*(d + e*x^3)) + ((4*c*d^2 -
 e*(b*d + 2*a*e))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(3*Sqr
t[3]*d^(5/3)*e^(7/3)) - ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(1/3) + e^(1/3)*
x])/(9*d^(5/3)*e^(7/3)) + ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(2/3) - d^(1/3
)*e^(1/3)*x + e^(2/3)*x^2])/(18*d^(5/3)*e^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Sim
p[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(cd^2-bde+ae^2)\,x}{3de^2\,(d+ex^3)}-\frac{\int \frac{cd^2-e(bd+2ae)-3cdex^3}{d+ex^3}\,dx}{3de^2}\\
&= \frac{cx}{e^2}+\frac{(cd^2-bde+ae^2)\,x}{3de^2\,(d+ex^3)}-\frac{(4cd^2-e(bd+2ae))\int \frac{1}{d+ex^3}\,dx}{3de^2}\\
&= \frac{cx}{e^2}+\frac{(cd^2-bde+ae^2)\,x}{3de^2\,(d+ex^3)}-\frac{(4cd^2-e(bd+2ae))\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{9d^{5/3}e^2}\\
&\quad -\frac{(4cd^2-e(bd+2ae))\int \frac{2\sqrt[3]{d}-\sqrt[3]{e}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{9d^{5/3}e^2}
\end{aligned}
$$

$$= \frac{cx}{e^2} + \frac{(cd^2 - bde + ae^2)x}{3de^2(d+ex^3)} - \frac{(4cd^2 - e(bd+2ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{5/3}e^{7/3}} + \frac{(4cd^2 - e(bd+2ae))\int \frac{-\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}dx}{18d^{5/3}e^{7/3}} - \frac{(4cd^2 - e(bd+2ae))\int \frac{1}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}dx}{6d^{4/3}e^2}$$

$$= \frac{cx}{e^2} + \frac{(cd^2 - bde + ae^2)x}{3de^2(d+ex^3)} - \frac{(4cd^2 - e(bd+2ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{5/3}e^{7/3}} + \frac{(4cd^2 - e(bd+2ae))\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{18d^{5/3}e^{7/3}} - \frac{(4cd^2 - e(bd+2ae))\operatorname{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{3d^{5/3}e^{7/3}}$$

$$= \frac{cx}{e^2} + \frac{(cd^2 - bde + ae^2)x}{3de^2(d+ex^3)} + \frac{(4cd^2 - e(bd+2ae))\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{3\sqrt{3}d^{5/3}e^{7/3}} - \frac{(4cd^2 - e(bd+2ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{5/3}e^{7/3}} + \frac{(4cd^2 - e(bd+2ae))\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{18d^{5/3}e^{7/3}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx$$

$$= \frac{18c\sqrt[3]{e}x + \frac{6\sqrt[3]{e}(cd^2+e(-bd+ae))x}{d(d+ex^3)} + \frac{2\sqrt{3}(4cd^2-e(bd+2ae))\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{5/3}} - \frac{2(4cd^2-e(bd+2ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{d^{5/3}} + \frac{(4cd^2-}{}}{18e^{7/3}}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^2,x]
```

```
[Out] (18*c*e^(1/3)*x + (6*e^(1/3)*(c*d^2 + e*(-(b*d) + a*e))*x)/(d*(d + e*x^3))
+ (2*Sqrt[3]*(4*c*d^2 - e*(b*d + 2*a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))
/Sqrt[3]])/d^(5/3) - (2*(4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(1/3) + e^(1/3)*x
])/d^(5/3) + ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x +
 e^(2/3)*x^2])/d^(5/3))/(18*e^(7/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.65 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.41

method	result	size
risch	$\frac{cx}{e^2}+\frac{(a\,e^2-bde+c\,d^2)x}{3d\,e^2(e\,x^3+d)}+\frac{\sum_{\_R=\mathrm{RootOf}(e\_Z^3+d)}\frac{(2a\,e^2+bde-4c\,d^2)\ln(x-\_R)}{\_R^2}}{9e^3d}$	88
default	$\frac{cx}{e^2}+\frac{\frac{(a\,e^2-bde+c\,d^2)x}{3d(e\,x^3+d)}+\frac{(2a\,e^2+bde-4c\,d^2)\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)}{3d}}{e^2}$	156

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*x/e^2+1/3*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)+1/9/e^3/d*sum((2*a*e^2+b*
d*e-4*c*d^2)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e+d))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 697, normalized size of antiderivative = 3.27

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^2}\,dx$$

$$= \left[\frac{18\,cd^3e^2x^4 - 3\,\sqrt{\frac{1}{3}}(4\,cd^4e - bd^3e^2 - 2\,ad^2e^3 + (4\,cd^3e^2 - bd^2e^3 - 2\,ade^4)x^3)\sqrt{-\frac{(d^2e)^{\frac{1}{3}}}{e}}\log\left(\frac{2\,dex^3 - 3\,(d^2e)^{\frac{1}{3}}\cdots}{\cdots}\right.}{\cdots}\right.$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(18*c*d^3*e^2*x^4 - 3*sqrt(1/3)*(4*c*d^4*e - b*d^3*e^2 - 2*a*d^2*e^3
+ (4*c*d^3*e^2 - b*d^2*e^3 - 2*a*d*e^4)*x^3)*sqrt(-(d^2*e)^(1/3)/e)*log((2*
```

```
d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)*(2*d*e*x^2 + (d^2*e)^(2/3
)*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(e*x^3 + d)) + (4*c*d^3 - b*
d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*log(
d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)*d) - 2*(4*c*d^3 - b*d^2*e - 2*a*d
*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x + (d^2*
e)^(2/3)) + 6*(4*c*d^4*e - b*d^3*e^2 + a*d^2*e^3)*x)/(d^3*e^4*x^3 + d^4*e^3
), 1/18*(18*c*d^3*e^2*x^4 - 6*sqrt(1/3)*(4*c*d^4*e - b*d^3*e^2 - 2*a*d^2*e^
3 + (4*c*d^3*e^2 - b*d^2*e^3 - 2*a*d*e^4)*x^3)*sqrt((d^2*e)^(1/3)/e)*arctan
(sqrt(1/3)*(2*(d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt((d^2*e)^(1/3)/e)/d^2)
 + (4*c*d^3 - b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d
^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)*d) - 2*(4*c*d^3 -
 b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*l
og(d*e*x + (d^2*e)^(2/3)) + 6*(4*c*d^4*e - b*d^3*e^2 + a*d^2*e^3)*x)/(d^3*e
^4*x^3 + d^4*e^3)]
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.97

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx = \frac{cx}{e^2} + \frac{x(ae^2-bde+cd^2)}{3d^2e^2+3de^3x^3}$$
$$+\mathrm{RootSum}\left(729t^3d^5e^7 - 8a^3e^6 - 12a^2bde^5 + 48a^2cd^2e^4 - 6ab^2d^2e^4 + 48abcd^3e^3 - 96ac^2d^4e^2 - b^3d^3e^3 + 1\right.$$

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(3*d**2*e**2 + 3*d*e**3*x**3) + Root
Sum(729*_t**3*d**5*e**7 - 8*a**3*e**6 - 12*a**2*b*d*e**5 + 48*a**2*c*d**2*e
**4 - 6*a*b**2*d**2*e**4 + 48*a*b*c*d**3*e**3 - 96*a*c**2*d**4*e**2 - b**3*
d**3*e**3 + 12*b**2*c*d**4*e**2 - 48*b*c**2*d**5*e + 64*c**3*d**6, Lambda(_
t, _t*log(9*_t*d**2*e**2/(2*a*e**2 + b*d*e - 4*c*d**2) + x)))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx = \frac{cx}{e^2} + \frac{\sqrt{3}(4\,cd^2-bde-2\,ae^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{9\,(-de^2)^{\frac{2}{3}}\,de} + \frac{(4\,cd^2-bde-2\,ae^2)\log\left(x^2+x\left(-\frac{d}{e}\right)^{\frac{1}{3}}+\left(-\frac{d}{e}\right)^{\frac{2}{3}}\right)}{18\,(-de^2)^{\frac{2}{3}}\,de} + \frac{(4\,cd^2-bde-2\,ae^2)\left(-\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{9\,d^2e^2} + \frac{cd^2x-bdex+ae^2x}{3\,(ex^3+d)de^2}$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="giac")
```

```
[Out] c*x/e^2 + 1/9*sqrt(3)*(4*c*d^2 - b*d*e - 2*a*e^2)*arctan(1/3*sqrt(3)*(2*x +
 (-d/e)^(1/3))/(-d/e)^(1/3))/((-d*e^2)^(2/3)*d*e) + 1/18*(4*c*d^2 - b*d*e -
 2*a*e^2)*log(x^2 + x*(-d/e)^(1/3) + (-d/e)^(2/3))/((-d*e^2)^(2/3)*d*e) + 1
/9*(4*c*d^2 - b*d*e - 2*a*e^2)*(-d/e)^(1/3)*log(abs(x - (-d/e)^(1/3)))/(d^2
*e^2) + 1/3*(c*d^2*x - b*d*e*x + a*e^2*x)/((e*x^3 + d)*d*e^2)
```

Mupad [B] (verification not implemented)

Time = 10.84 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx = \frac{c\,x}{e^2} + \frac{\ln\left(e^{1/3}\,x+d^{1/3}\right)\,\left(-4\,c\,d^2+b\,d\,e+2\,a\,e^2\right)}{9\,d^{5/3}\,e^{7/3}} + \frac{x\,(c\,d^2-b\,d\,e+a\,e^2)}{3\,d\,\left(e^3\,x^3+d\,e^2\right)} + \frac{\ln\left(2\,e^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-4\,c\,d^2+b\,d\,e+2\,a\,e^2\right)}{9\,d^{5/3}\,e^{7/3}} - \frac{\ln\left(d^{1/3}-2\,e^{1/3}\,x+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-4\,c\,d^2+b\,d\,e+2\,a\,e^2\right)}{9\,d^{5/3}\,e^{7/3}}$$

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^2,x)
```

```
[Out] (c*x)/e^2 + (log(e^(1/3)*x + d^(1/3))*(2*a*e^2 - 4*c*d^2 + b*d*e))/(9*d^(5/
3)*e^(7/3)) + (x*(a*e^2 + c*d^2 - b*d*e))/(3*d*(d*e^2 + e^3*x^3)) + (log(3^
```

```
(1/2)*d^(1/3)*1i + 2*e^(1/3)*x - d^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*a*e^2 -
 4*c*d^2 + b*d*e))/(9*d^(5/3)*e^(7/3)) - (log(3^(1/2)*d^(1/3)*1i - 2*e^(1/3
)*x + d^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*a*e^2 - 4*c*d^2 + b*d*e))/(9*d^(5/
3)*e^(7/3))
```

3.8 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 242

$$\begin{aligned}\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx = {} & \frac{(cd^2-bde+ae^2)\,x}{6de^2\,(d+ex^3)^2}-\frac{(7cd^2-e(bd+5ae))\,x}{18d^2e^2\,(d+ex^3)} \\ & -\frac{(2cd^2+e(bd+5ae))\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{9\sqrt{3}d^{8/3}e^{7/3}} \\ & +\frac{(2cd^2+e(bd+5ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{27d^{8/3}e^{7/3}} \\ & -\frac{(2cd^2+e(bd+5ae))\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{54d^{8/3}e^{7/3}}\end{aligned}$$

```
[Out] 1/6*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^2-1/18*(7*c*d^2-e*(5*a*e+b*d))*x/
d^2/e^2/(e*x^3+d)+1/27*(2*c*d^2+e*(5*a*e+b*d))*ln(d^(1/3)+e^(1/3)*x)/d^(8/3
)/e^(7/3)-1/54*(2*c*d^2+e*(5*a*e+b*d))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)
*x^2)/d^(8/3)/e^(7/3)-1/27*(2*c*d^2+e*(5*a*e+b*d))*arctan(1/3*(d^(1/3)-2*e^
(1/3)*x)/d^(1/3)*3^(1/2))/d^(8/3)/e^(7/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1423, 393, 206, 31, 648, 631, 210, 642}

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)(e(5ae+bd)+2cd^2)}{9\sqrt{3}d^{8/3}e^{7/3}} - \frac{x(7cd^2-e(5ae+bd))}{18d^2e^2(d+ex^3)} + \frac{x(ae^2-bde+cd^2)}{6de^2(d+ex^3)^2} - \frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)(e(5ae+bd)+2cd^2)}{54d^{8/3}e^{7/3}} + \frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)(e(5ae+bd)+2cd^2)}{27d^{8/3}e^{7/3}}$$

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^3,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(6*d*e^2*(d + e*x^3)^2) - ((7*c*d^2 - e*(b*d +
5*a*e))*x)/(18*d^2*e^2*(d + e*x^3)) - ((2*c*d^2 + e*(b*d + 5*a*e))*ArcTan[(
d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(9*Sqrt[3]*d^(8/3)*e^(7/3)) + ((
2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(1/3) + e^(1/3)*x])/(27*d^(8/3)*e^(7/3)) -
 ((2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2
])/(54*d^(8/3)*e^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Sim
p[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{(cd^2 - bde + ae^2)\,x}{6de^2\,(d+ex^3)^2} - \frac{\int \frac{cd^2 - e(bd+5ae) - 6cdex^3}{(d+ex^3)^2}\,dx}{6de^2}$$

$$= \frac{(cd^2 - bde + ae^2)\,x}{6de^2\,(d+ex^3)^2} - \frac{(7cd^2 - e(bd+5ae))\,x}{18d^2e^2\,(d+ex^3)} + \frac{(2cd^2 + e(bd+5ae))\int \frac{1}{d+ex^3}\,dx}{9d^2e^2}$$

$$= \frac{(cd^2 - bde + ae^2)\,x}{6de^2\,(d+ex^3)^2} - \frac{(7cd^2 - e(bd+5ae))\,x}{18d^2e^2\,(d+ex^3)} + \frac{(2cd^2 + e(bd+5ae)) \int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{27d^{8/3}e^2}$$

$$+ \frac{(2cd^2 + e(bd+5ae)) \int \frac{2\sqrt[3]{d}-\sqrt[3]{e}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{27d^{8/3}e^2}$$

$$= \frac{(cd^2 - bde + ae^2)\,x}{6de^2\,(d+ex^3)^2} - \frac{(7cd^2 - e(bd+5ae))\,x}{18d^2e^2\,(d+ex^3)}$$

$$+ \frac{(2cd^2 + e(bd+5ae)) \log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{27d^{8/3}e^{7/3}}$$

$$- \frac{(2cd^2 + e(bd+5ae)) \int \frac{-\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{54d^{8/3}e^{7/3}}$$

$$+ \frac{(2cd^2 + e(bd+5ae)) \int \frac{1}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{18d^{7/3}e^2}$$

$$= \frac{(cd^2 - bde + ae^2)\,x}{6de^2\,(d+ex^3)^2} - \frac{(7cd^2 - e(bd+5ae))\,x}{18d^2e^2\,(d+ex^3)}$$

$$+ \frac{(2cd^2 + e(bd+5ae)) \log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{27d^{8/3}e^{7/3}}$$

$$- \frac{(2cd^2 + e(bd+5ae)) \log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{54d^{8/3}e^{7/3}}$$

$$+ \frac{(2cd^2 + e(bd+5ae))\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{9d^{8/3}e^{7/3}}$$

$$= \frac{(cd^2 - bde + ae^2)\,x}{6de^2\,(d+ex^3)^2} - \frac{(7cd^2 - e(bd+5ae))\,x}{18d^2e^2\,(d+ex^3)}$$

$$- \frac{(2cd^2 + e(bd+5ae)) \tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{9\sqrt{3}d^{8/3}e^{7/3}}$$

$$+ \frac{(2cd^2 + e(bd+5ae)) \log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{27d^{8/3}e^{7/3}}$$

$$- \frac{(2cd^2 + e(bd+5ae)) \log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{54d^{8/3}e^{7/3}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx$$

$$= \frac{-\frac{3d^{2/3}\sqrt[3]{e}x(cd^2(4d+7ex^3)-e(bd(-2d+ex^3)+ae(8d+5ex^3)))}{(d+ex^3)^2} - 2\sqrt{3}(2cd^2+e(bd+5ae))\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right) + 2(2cd^2}{54d^{8/3}e^{7/3}}$$

[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^3,x]

[Out] ((-3*d^(2/3)*e^(1/3)*x*(c*d^2*(4*d + 7*e*x^3) - e*(b*d*(-2*d + e*x^3) + a*e *(8*d + 5*e*x^3))))/(d + e*x^3)^2 - 2*Sqrt[3]*(2*c*d^2 + e*(b*d + 5*a*e))*A rcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] + 2*(2*c*d^2 + e*(b*d + 5*a*e))* Log[d^(1/3) + e^(1/3)*x] - (2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(54*d^(8/3)*e^(7/3))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.47

method	result
risch	$\frac{\frac{(5a e^2+bde-7c d^2)x^4}{18d^2e}+\frac{(4a e^2-bde-2c d^2)x}{9d e^2}}{(e x^3+d)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(e\_Z^3+d)}\frac{(5a e^2+bde+2c d^2)\ln(x-\_R)}{\_R^2}}{27e^3d^2}$
default	$\frac{\frac{(5a e^2+bde-7c d^2)x^4}{18d^2e}+\frac{(4a e^2-bde-2c d^2)x}{9d e^2}}{(e x^3+d)^2}+\frac{(5a e^2+bde+2c d^2)\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right.}{3}\right.}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right.}{9e^2d^2}$

[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^3,x,method=_RETURNVERBOSE)

[Out] (1/18*(5*a*e^2+b*d*e-7*c*d^2)/d^2/e*x^4+1/9*(4*a*e^2-b*d*e-2*c*d^2)/d/e^2*x)/(e*x^3+d)^2+1/27/e^3/d^2*sum((5*a*e^2+b*d*e+2*c*d^2)/_R^2*ln(x-_R),_R=Roo tOf(_Z^3*e+d))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(201) = 402.

Time = 0.32 (sec) , antiderivative size = 941, normalized size of antiderivative = 3.89

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx$$

$$= \left[-\frac{3\,(7\,cd^4e^2 - bd^3e^3 - 5\,ad^2e^4)x^4 - 3\,\sqrt{\tfrac{1}{3}}(2\,cd^5e + bd^4e^2 + 5\,ad^3e^3 + (2\,cd^3e^3 + bd^2e^4 + 5\,ade^5)x^6 + 2\,(2\,c\cdots}{\cdots}\right.$$

$$-\frac{3\,(7\,cd^4e^2 - bd^3e^3 - 5\,ad^2e^4)x^4 - 6\,\sqrt{\tfrac{1}{3}}(2\,cd^5e + bd^4e^2 + 5\,ad^3e^3 + (2\,cd^3e^3 + bd^2e^4 + 5\,ade^5)x^6 + 2\,(2\,c\cdots}{\cdots}$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(3*(7*c*d^4*e^2 - b*d^3*e^3 - 5*a*d^2*e^4)*x^4 - 3*sqrt(1/3)*(2*c*d^
5*e + b*d^4*e^2 + 5*a*d^3*e^3 + (2*c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^6 +
 2*(2*c*d^4*e^2 + b*d^3*e^3 + 5*a*d^2*e^4)*x^3)*sqrt(-(d^2*e)^(1/3)/e)*log(
(2*d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)*(2*d*e*x^2 + (d^2*e)^(
2/3)*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(e*x^3 + d)) + ((2*c*d^2*
e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3
*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*
x + (d^2*e)^(1/3)*d) - 2*((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 +
 b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)
^(2/3)*log(d*e*x + (d^2*e)^(2/3)) + 6*(2*c*d^5*e + b*d^4*e^2 - 4*a*d^3*e^3)
*x)/(d^4*e^5*x^6 + 2*d^5*e^4*x^3 + d^6*e^3), -1/54*(3*(7*c*d^4*e^2 - b*d^3*
e^3 - 5*a*d^2*e^4)*x^4 - 6*sqrt(1/3)*(2*c*d^5*e + b*d^4*e^2 + 5*a*d^3*e^3 +
 (2*c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^6 + 2*(2*c*d^4*e^2 + b*d^3*e^3 + 5
*a*d^2*e^4)*x^3)*sqrt((d^2*e)^(1/3)/e)*arctan(sqrt(1/3)*(2*(d^2*e)^(2/3)*x
- (d^2*e)^(1/3)*d)*sqrt((d^2*e)^(1/3)/e)/d^2) + ((2*c*d^2*e^2 + b*d*e^3 + 5
*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 +
5*a*d*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)
*d) - 2*((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^
```

```
2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x
+ (d^2*e)^(2/3)) + 6*(2*c*d^5*e + b*d^4*e^2 - 4*a*d^3*e^3)*x)/(d^4*e^5*x^6
+ 2*d^5*e^4*x^3 + d^6*e^3)]
```

Sympy [A] (verification not implemented)

Time = 11.31 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.02

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx = \frac{x^4\cdot(5ae^3+bde^2-7cd^2e)+x(8ade^2-2bd^2e-4cd^3)}{18d^4e^2+36d^3e^3x^3+18d^2e^4x^6}$$
$$+\text{RootSum}\left(19683t^3d^8e^7-125a^3e^6-75a^2bde^5-150a^2cd^2e^4-15ab^2d^2e^4-60abcd^3e^3-60ac^2d^4e^2-b^3\right.$$

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**3,x)
```

```
[Out] (x**4*(5*a*e**3 + b*d*e**2 - 7*c*d**2*e) + x*(8*a*d*e**2 - 2*b*d**2*e - 4*c
*d**3))/(18*d**4*e**2 + 36*d**3*e**3*x**3 + 18*d**2*e**4*x**6) + RootSum(19
683*_t**3*d**8*e**7 - 125*a**3*e**6 - 75*a**2*b*d*e**5 - 150*a**2*c*d**2*e*
*4 - 15*a*b**2*d**2*e**4 - 60*a*b*c*d**3*e**3 - 60*a*c**2*d**4*e**2 - b**3*
d**3*e**3 - 6*b**2*c*d**4*e**2 - 12*b*c**2*d**5*e - 8*c**3*d**6, Lambda(_t,
 _t*log(27*_t*d**3*e**2/(5*a*e**2 + b*d*e + 2*c*d**2) + x)))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx = -\frac{\sqrt{3}(2\,cd^2+bde+5\,ae^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{27\,\left(-de^2\right)^{\frac{2}{3}}\,d^2e} - \frac{(2\,cd^2+bde+5\,ae^2)\log\left(x^2+x\left(-\frac{d}{e}\right)^{\frac{1}{3}}+\left(-\frac{d}{e}\right)^{\frac{2}{3}}\right)}{54\,\left(-de^2\right)^{\frac{2}{3}}\,d^2e} - \frac{(2\,cd^2+bde+5\,ae^2)\left(-\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{27\,d^3e^2} - \frac{7\,cd^2ex^4-bde^2x^4-5\,ae^3x^4+4\,cd^3x+2\,bd^2ex-8\,ade^2x}{18\,(ex^3+d)^2d^2e^2}$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(2*c*d^2 + b*d*e + 5*a*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d/e)^
(1/3))/(-d/e)^(1/3))/((-d*e^2)^(2/3)*d^2*e) - 1/54*(2*c*d^2 + b*d*e + 5*a*e
^2)*log(x^2 + x*(-d/e)^(1/3) + (-d/e)^(2/3))/((-d*e^2)^(2/3)*d^2*e) - 1/27*
(2*c*d^2 + b*d*e + 5*a*e^2)*(-d/e)^(1/3)*log(abs(x - (-d/e)^(1/3)))/(d^3*e^
2) - 1/18*(7*c*d^2*e*x^4 - b*d*e^2*x^4 - 5*a*e^3*x^4 + 4*c*d^3*x + 2*b*d^2*
e*x - 8*a*d*e^2*x)/((e*x^3 + d)^2*d^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx = \frac{\ln\left(e^{1/3}\,x+d^{1/3}\right)\,\left(2\,c\,d^2+b\,d\,e+5\,a\,e^2\right)}{27\,d^{8/3}\,e^{7/3}} - \frac{\frac{x\,\left(2\,c\,d^2+b\,d\,e-4\,a\,e^2\right)}{9\,d\,e^2}-\frac{x^4\,\left(-7\,c\,d^2+b\,d\,e+5\,a\,e^2\right)}{18\,d^2\,e}}{d^2+2\,d\,e\,x^3+e^2\,x^6} + \frac{\ln\left(2\,e^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,c\,d^2+b\,d\,e+5\,a\,e^2\right)}{27\,d^{8/3}\,e^{7/3}} - \frac{\ln\left(d^{1/3}-2\,e^{1/3}\,x+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2\,c\,d^2+b\,d\,e+5\,a\,e^2\right)}{27\,d^{8/3}\,e^{7/3}}$$

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^3,x)
```

```
[Out] (log(e^(1/3)*x + d^(1/3))*(5*a*e^2 + 2*c*d^2 + b*d*e))/(27*d^(8/3)*e^(7/3))
 - ((x*(2*c*d^2 - 4*a*e^2 + b*d*e))/(9*d*e^2) - (x^4*(5*a*e^2 - 7*c*d^2 + b
*d*e))/(18*d^2*e))/(d^2 + e^2*x^6 + 2*d*e*x^3) + (log(3^(1/2)*d^(1/3)*1i +
2*e^(1/3)*x - d^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*a*e^2 + 2*c*d^2 + b*d*e))/
(27*d^(8/3)*e^(7/3)) - (log(3^(1/2)*d^(1/3)*1i - 2*e^(1/3)*x + d^(1/3))*((3
^(1/2)*1i)/2 + 1/2)*(5*a*e^2 + 2*c*d^2 + b*d*e))/(27*d^(8/3)*e^(7/3))
```

3.9 $\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6} dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6} dx = \frac{(cd-be)x^3}{3c^2} + \frac{ex^6}{6c} - \frac{(b^2cd-2ac^2d-b^3e+3abce)\,\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}} - \frac{(bcd-b^2e+ace)\log(a+bx^3+cx^6)}{6c^3}$$

```
[Out] 1/3*(-b*e+c*d)*x^3/c^2+1/6*e*x^6/c-1/6*(a*c*e-b^2*e+b*c*d)*ln(c*x^6+b*x^3+a
)/c^3-1/3*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)*arctanh((2*c*x^3+b)/(-4*a*c+b
^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1488, 814, 648, 632, 212, 642}

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6} dx = -\frac{\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)(3abce-2ac^2d+b^3(-e)+b^2cd)}{3c^3\sqrt{b^2-4ac}} - \frac{(ace+b^2(-e)+bcd)\log(a+bx^3+cx^6)}{6c^3} + \frac{x^3(cd-be)}{3c^2} + \frac{ex^6}{6c}$$

```
[In] Int[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((c*d - b*e)*x^3)/(3*c^2) + (e*x^6)/(6*c) - ((b^2*c*d - 2*a*c^2*d - b^3*e +
 3*a*b*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c^3*Sqrt[b^2 - 4*a
*c]) - ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^3 + c*x^6])/(6*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(d+ex)}{a+bx+cx^2}\,dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{cd-be}{c^2}+\frac{ex}{c}-\frac{a(cd-be)+(bcd-b^2e+ace)\,x}{c^2\,(a+bx+cx^2)}\right)\,dx, x, x^3\right)
\end{aligned}$$

$$= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{\text{Subst}\left(\int \frac{a(cd-be)+(bcd-b^2e+ace)x}{a+bx+cx^2}dx,x,x^3\right)}{3c^2}$$

$$= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{(bcd-b^2e+ace)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}dx,x,x^3\right)}{6c^3}+\frac{(b^2cd-2ac^2d-b^3e+3abce)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}dx,x,x^3\right)}{6c^3}$$

$$= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{(bcd-b^2e+ace)\log\left(a+bx^3+cx^6\right)}{6c^3}-\frac{(b^2cd-2ac^2d-b^3e+3abce)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}dx,x,b+2cx^3\right)}{3c^3}$$

$$= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{(b^2cd-2ac^2d-b^3e+3abce)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}}-\frac{(bcd-b^2e+ace)\log\left(a+bx^3+cx^6\right)}{6c^3}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{2c(cd-be)x^3+c^2ex^6+\frac{2(b^2cd-2ac^2d-b^3e+3abce)\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(-bcd+b^2e-ace)\log\left(a+bx^3+cx^6\right)}{6c^3}$$

```
[In] Integrate[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*(c*d - b*e)*x^3 + c^2*e*x^6 + (2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*
c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*c*
d) + b^2*e - a*c*e)*Log[a + b*x^3 + c*x^6])/(6*c^3)
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{-\frac{1}{2}ce\,x^6+e\,x^3b-cd\,x^3}{3c^2}+\frac{\frac{\left(-ace+b^2e-bcd\right)\ln\left(c\,x^6+b\,x^3+a\right)}{2c}+\frac{2\left(abe-acd-\frac{\left(-ace+b^2e-bcd\right)b}{2c}\right)\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{3c^2}$	136
risch	Expression too large to display	2131

```
[In] int(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/c^2*(-1/2*c*e*x^6+e*x^3*b-c*d*x^3)+1/3/c^2*(1/2*(-a*c*e+b^2*e-b*c*d)/c
*ln(c*x^6+b*x^3+a)+2*(a*b*e-a*c*d-1/2*(-a*c*e+b^2*e-b*c*d)*b/c)/(4*a*c-b^2)
^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.26

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx$$
$$= \left[\frac{(b^2c^2-4\,ac^3)ex^6+2\,((b^2c^2-4\,ac^3)d-(b^3c-4\,abc^2)e)x^3+\sqrt{b^2-4\,ac}((b^2c-2\,ac^2)d-(b^3-3\,abc)e)\,\mathrm{l}}{6\,(b^2c^3\,\cdots}\right.$$

```
[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*((b^2*c^2 - 4*a*c^3)*e*x^6 + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b
*c^2)*e)*x^3 + sqrt(b^2 - 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*
log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c - (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))
/(c*x^6 + b*x^3 + a)) - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c
^2)*e)*log(c*x^6 + b*x^3 + a))/(b^2*c^3 - 4*a*c^4), 1/6*((b^2*c^2 - 4*a*c^3
)*e*x^6 + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*x^3 - 2*sqrt(-b
^2 + 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*arctan(-(2*c*x^3 + b)
*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^
2*c + 4*a^2*c^2)*e)*log(c*x^6 + b*x^3 + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

```
[In] integrate(x**8*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{cex^6+2\,cdx^3-2\,bex^3}{6\,c^2} - \frac{(bcd-b^2e+ace)\log\left(cx^6+bx^3+a\right)}{6\,c^3} + \frac{(b^2cd-2\,ac^2d-b^3e+3\,abce)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c^3}$$

```
[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*(c*e*x^6 + 2*c*d*x^3 - 2*b*e*x^3)/c^2 - 1/6*(b*c*d - b^2*e + a*c*e)*log
(c*x^6 + b*x^3 + a)/c^3 + 1/3*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*arc
tan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 11.12 (sec) , antiderivative size = 3586, normalized size of antiderivative = 27.17

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] int((x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] x^3*(d/(3*c) - (b*e)/(3*c^2)) + (e*x^6)/(6*c) - (log(a + b*x^3 + c*x^6)*(3*
b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c
^4 - 9*b^2*c^3)) - (atan((4*c^6*(4*a*c - b^2)^(3/2)*(x^3*((b*((b^5*c^3*d^3
- b^8*e^3 - 2*a*b^3*c^4*d^3 + a^2*b*c^5*d^3 + a^3*c^5*d^2*e - 3*b^6*c^2*d^2
*e - 8*a^2*b^4*c^2*e^3 + 4*a^3*b^2*c^3*e^3 + 5*a*b^6*c*e^3 + 3*b^7*c*d*e^2
```

```
+ 9*a*b^4*c^3*d^2*e - 12*a*b^5*c^2*d*e^2 - 4*a^3*b*c^4*d*e^2 - 7*a^2*b^2*c^
4*d^2*e + 14*a^2*b^3*c^3*d*e^2)/c^6 - (((6*a^2*c^7*d^2 + 12*b^4*c^5*d^2 + 1
2*b^6*c^3*e^2 - 18*a*b^2*c^6*d^2 - 42*a*b^4*c^4*e^2 + 36*a^2*b^2*c^5*e^2 -
24*b^5*c^4*d*e + 60*a*b^3*c^5*d*e - 30*a^2*b*c^6*d*e)/c^6 - (((45*b^3*c^7*d
 - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3*b^4*e
 + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b
^2*c^3))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e)
)/(2*(36*a*c^4 - 9*b^2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*
c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((((45*b^3*c^7*d - 45*
b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3*b^4*e + 12*
a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3
))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) -
 (9*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c^2*e -
 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^4
 - 9*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c -
b^2)^(1/2)) + (3*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)^2*(3*b^4*e +
 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(4*c^3*(4*a*c - b
^2)*(36*a*c^4 - 9*b^2*c^3))))/(4*a^2*c) - ((2*a*c - b^2)*((((((45*b^3*c^7*d
 - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3*b^4*e
 + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b
^2*c^3))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1
/2)) - (9*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c
^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(4*a*c - b^2)^(1/2)*(36
*a*c^4 - 9*b^2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d -
15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((6*a^2*c^7*d^2 + 12*b^4*c^5*d
^2 + 12*b^6*c^3*e^2 - 18*a*b^2*c^6*d^2 - 42*a*b^4*c^4*e^2 + 36*a^2*b^2*c^5*
e^2 - 24*b^5*c^4*d*e + 60*a*b^3*c^5*d*e - 30*a^2*b*c^6*d*e)/c^6 - (((45*b^3
*c^7*d - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3
*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4
 - 9*b^2*c^3))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^
2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*
e))/(6*c^3*(4*a*c - b^2)^(1/2)) + (b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b
*c*e)^3)/(4*c^6*(4*a*c - b^2)^(3/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2))) - (b*
((a*b^7*e^3 - a*b^4*c^3*d^3 - 4*a^2*b^5*c*e^3 - 2*a^4*b*c^3*e^3 + a^4*c^4*d
*e^2 + a^2*b^2*c^4*d^3 + 5*a^3*b^3*c^2*e^3 - 3*a*b^6*c*d*e^2 + 3*a*b^5*c^2*
d^2*e + 2*a^3*b*c^4*d^2*e - 6*a^2*b^3*c^3*d^2*e + 9*a^2*b^4*c^2*d*e^2 - 7*a
^3*b^2*c^3*d*e^2)/c^6 + (((15*a*b^3*c^5*d^2 - 12*a^2*b*c^6*d^2 + 15*a*b^5*c
^3*e^2 + 27*a^3*b*c^5*e^2 - 42*a^2*b^3*c^4*e^2 - 12*a^3*c^6*d*e - 30*a*b^4*
c^4*d*e + 54*a^2*b^2*c^5*d*e)/c^6 + (((36*a^2*c^8*d - 72*a*b^2*c^7*d + 72*a
*b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c^2*e - 3
*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(3*b^4*e +
 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*
b^2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*
e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((((36*a^2*c^8*d - 72*a*b^2*c^7*d + 72*a*
b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c^2*e - 3*
```

```
b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(b^3*e + 2*
a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) + (9*a*b*(b^3*e
 + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 1
2*a*b*c^2*d - 15*a*b^2*c*e))/((4*a*c - b^2)^(1/2)*(36*a*c^4 - 9*b^2*c^3)))*
(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) - (3
*a*b*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)^2*(3*b^4*e + 12*a^2*c^2*e -
3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*c^3*(4*a*c - b^2)*(36*a*c^4 -
9*b^2*c^3))))/(4*a^2*c) + ((2*a*c - b^2)*((((((36*a^2*c^8*d - 72*a*b^2*c^7*
d + 72*a*b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c
^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(b
^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) + (9*a
*b*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c^2*e - 3*b^
3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/((4*a*c - b^2)^(1/2)*(36*a*c^4 - 9*b^
2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e)
)/(2*(36*a*c^4 - 9*b^2*c^3)) + (((15*a*b^3*c^5*d^2 - 12*a^2*b*c^6*d^2 + 15*
a*b^5*c^3*e^2 + 27*a^3*b*c^5*e^2 - 42*a^2*b^3*c^4*e^2 - 12*a^3*c^6*d*e - 30
*a*b^4*c^4*d*e + 54*a^2*b^2*c^5*d*e)/c^6 + (((36*a^2*c^8*d - 72*a*b^2*c^7*d
 + 72*a*b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c^
2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(3*
b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c
^4 - 9*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c
- b^2)^(1/2)) - (a*b*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)^3)/(2*c^6*(4
*a*c - b^2)^(3/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2))))/(b^9*e^3 + 8*a^3*c^6*d
^3 - b^6*c^3*d^3 + 6*a*b^4*c^4*d^3 + 3*b^7*c^2*d^2*e - 12*a^2*b^2*c^5*d^3 +
 27*a^2*b^5*c^2*e^3 - 27*a^3*b^3*c^3*e^3 - 9*a*b^7*c*e^3 - 3*b^8*c*d*e^2 -
21*a*b^5*c^3*d^2*e + 24*a*b^6*c^2*d*e^2 - 36*a^3*b*c^5*d^2*e + 48*a^2*b^3*c
^4*d^2*e - 63*a^2*b^4*c^3*d*e^2 + 54*a^3*b^2*c^4*d*e^2))*(b^3*e + 2*a*c^2*d
 - b^2*c*d - 3*a*b*c*e))/(3*c^3*(4*a*c - b^2)^(1/2))
```

3.10 $\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{ex^3}{3c}+\frac{(bcd-b^2e+2ace)\,\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}$$

```
[Out] 1/3*e*x^3/c+1/6*(-b*e+c*d)*ln(c*x^6+b*x^3+a)/c^2+1/3*(2*a*c*e-b^2*e+b*c*d)*
arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1488, 787, 648, 632, 212, 642}

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)(2ace+b^2(-e)+bcd)}{3c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{ex^3}{3c}$$

```
[In] Int[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x^3)/(3*c) + ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 -
 4*a*c]])/(3*c^2*Sqrt[b^2 - 4*a*c]) + ((c*d - b*e)*Log[a + b*x^3 + c*x^6])/
(6*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x(d+ex)}{a+bx+cx^2}\,dx, x, x^3\right)$$

$$= \frac{ex^3}{3c} + \frac{\text{Subst}\left(\int \frac{-ae+(cd-be)x}{a+bx+cx^2}\,dx, x, x^3\right)}{3c}$$

$$= \frac{ex^3}{3c} + \frac{(cd-be)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6c^2} - \frac{(bcd-b^2e+2ace)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6c^2}$$

$$= \frac{ex^3}{3c} + \frac{(cd-be)\log(a+bx^3+cx^6)}{6c^2} + \frac{(bcd-b^2e+2ace)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3c^2}$$

$$= \frac{ex^3}{3c} + \frac{(bcd-b^2e+2ace)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}} + \frac{(cd-be)\log(a+bx^3+cx^6)}{6c^2}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{2cex^3 + \frac{2(-bcd+b^2e-2ace)\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + (cd-be)\log(a+bx^3+cx^6)}{6c^2}$$

```
[In] Integrate[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*e*x^3 + (2*(-(b*c*d) + b^2*e - 2*a*c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2
 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*d - b*e)*Log[a + b*x^3 + c*x^6])/(6*c^2
)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{e\,x^3}{3c} + \frac{\frac{(-be+cd)\ln\left(c\,x^6+b\,x^3+a\right)}{2c} + \frac{2\left(-ae-\frac{(-be+cd)b}{2c}\right)\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{3c}$	98
risch	Expression too large to display	1400

```
[In] int(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*e*x^3/c+1/3/c*(1/2*(-b*e+c*d)/c*ln(c*x^6+b*x^3+a)+2*(-a*e-1/2*(-b*e+c*d
)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.14

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx = \left[\frac{2\,(b^2c-4\,ac^2)ex^3+(bcd-(b^2-2\,ac)e)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)+((b^2c-4\,ac}{6\,(b^2c^2-4\,ac^3)}\right.$$

```
[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c - 4*a*c^2)*e*x^3 + (b*c*d - (b^2 - 2*a*c)*e)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*
c))/(c*x^6 + b*x^3 + a)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log(c*
x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3), 1/6*(2*(b^2*c - 4*a*c^2)*e*x^3 + 2*(
b*c*d - (b^2 - 2*a*c)*e)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^3 + b)*sqrt(-b^2
 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log(c*
x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(94) = 188$.

Time = 101.95 (sec) , antiderivative size = 434, normalized size of antiderivative = 4.47

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx = \left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}\right.$$
$$\left.-\frac{be-cd}{6c^2}\right)\log\left(x^3+\frac{-abe-12ac^2\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}-\frac{be-cd}{6c^2}\right)+2acd+3b^2c\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+}{6c^2\cdot(4ac-b^2)}\right.}{2ace-b^2e+bcd}\right.$$
$$+\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}\right.$$
$$\left.-\frac{be-cd}{6c^2}\right)\log\left(x^3+\frac{-abe-12ac^2\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}-\frac{be-cd}{6c^2}\right)+2acd+3b^2c\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}\right.}{2ace-b^2e+bcd}\right.$$
$$+\frac{ex^3}{3c}$$

```
[In] integrate(x**5*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) -
(b*e - c*d)/(6*c**2))*log(x**3 + (-a*b*e - 12*a*c**2*(-sqrt(-4*a*c + b**2)*
```

```
(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2))
+ 2*a*c*d + 3*b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c*
*2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d)) + (
sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b
*e - c*d)/(6*c**2))*log(x**3 + (-a*b*e - 12*a*c**2*(sqrt(-4*a*c + b**2)*(2*
a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)) + 2
*a*c*d + 3*b**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(
4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d)) + e*x**
3/(3*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{ex^3}{3\,c} + \frac{(cd-be)\log\left(cx^6+bx^3+a\right)}{6\,c^2} - \frac{(bcd-b^2e+2\,ace)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c^2}$$

```
[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*e*x^3/c + 1/6*(c*d - b*e)*log(c*x^6 + b*x^3 + a)/c^2 - 1/3*(b*c*d - b^2
*e + 2*a*c*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*
c^2)
```

Mupad [B] (verification not implemented)

Time = 11.31 (sec) , antiderivative size = 2624, normalized size of antiderivative = 27.05

$$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] int((x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] (e*x^3)/(3*c) + (log(a + b*x^3 + c*x^6)*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d -
 12*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) + (atan((4*c^3*(4*a*c - b^2)^(3/2)
*(x^3*((b*((b^2*c^3*d^3 - b^5*e^3 - a^2*b*c^2*e^3 + a^2*c^3*d*e^2 - 3*b^3*c
^2*d^2*e + 2*a*b^3*c*e^3 + 3*b^4*c*d*e^2 + 2*a*b*c^3*d^2*e - 4*a*b^2*c^2*d*
e^2)/c^3 - (((6*a^2*c^4*e^2 + 12*b^2*c^4*d^2 + 12*b^4*c^2*e^2 - 18*a*b^2*c^
3*e^2 - 24*b^3*c^3*d*e + 18*a*b*c^4*d*e)/c^3 - (((45*b^2*c^5*d - 45*b^3*c^4
*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12
*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a
*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12
*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) - (((((45*b^2*c^5*d - 45*b^3*c^4*e +
36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*
c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^
2)^(1/2)) - (9*b^2*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e + 12*a*c^2*d - 3*b^
2*c*d - 12*a*b*c*e))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c
*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^2)^(1/2)) + (3*b^2*(2*a*c*e - b^2*e
+ b*c*d)^2*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(4*c*(4*a*c - b
^2)*(36*a*c^3 - 9*b^2*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*((((((45*b^2*c^5*d
 - 45*b^3*c^4*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3
*b^2*c*d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/
(6*c^2*(4*a*c - b^2)^(1/2)) - (9*b^2*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e +
 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9
*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(36*a*c^3 -
 9*b^2*c^2)) + (b^2*(2*a*c*e - b^2*e + b*c*d)^3)/(4*c^3*(4*a*c - b^2)^(3/2)
) - (((6*a^2*c^4*e^2 + 12*b^2*c^4*d^2 + 12*b^4*c^2*e^2 - 18*a*b^2*c^3*e^2 -
 24*b^3*c^3*d*e + 18*a*b*c^4*d*e)/c^3 - (((45*b^2*c^5*d - 45*b^3*c^4*e + 36
*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*
e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e)
)/(2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^
2)^(1/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2))) + (b*((a^2*b^2*c*e^3 - a*b^4*e^3
 + a^2*c^3*d^2*e + a*b*c^3*d^3 + 3*a*b^3*c*d*e^2 - 3*a*b^2*c^2*d^2*e - 2*a^
2*b*c^2*d*e^2)/c^3 - (((15*a*b^3*c^2*e^2 - 12*a^2*b*c^3*e^2 + 15*a*b*c^4*d^
2 + 12*a^2*c^4*d*e - 30*a*b^2*c^3*d*e)/c^3 - (((36*a^2*c^5*e + 72*a*b*c^5*d
 - 72*a*b^2*c^4*e)/c^3 - (54*a*b*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12
*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a
*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12
*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) - (((((36*a^2*c^5*e + 72*a*b*c^5*d -
```

```
72*a*b^2*c^4*e)/c^3 - (54*a*b*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*
b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c -
b^2)^(1/2)) - (9*a*b*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e + 12*a*c^2*d - 3*
b^2*c*d - 12*a*b*c*e))/((4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c
*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^2)^(1/2)) + (3*a*b*(2*a*c*e - b^2*e
+ b*c*d)^2*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*c*(4*a*c - b
^2)*(36*a*c^3 - 9*b^2*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*((((((36*a^2*c^5*e
 + 72*a*b*c^5*d - 72*a*b^2*c^4*e)/c^3 - (54*a*b*c^3*(3*b^3*e + 12*a*c^2*d -
 3*b^2*c*d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d)
)/(6*c^2*(4*a*c - b^2)^(1/2)) - (9*a*b*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e
 + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/((4*a*c - b^2)^(1/2)*(36*a*c^3 - 9
*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(36*a*c^3 -
 9*b^2*c^2)) - (((15*a*b^3*c^2*e^2 - 12*a^2*b*c^3*e^2 + 15*a*b*c^4*d^2 + 12
*a^2*c^4*d*e - 30*a*b^2*c^3*d*e)/c^3 - (((36*a^2*c^5*e + 72*a*b*c^5*d - 72*
a*b^2*c^4*e)/c^3 - (54*a*b*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c
*e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e
))/(2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b
^2)^(1/2)) + (a*b*(2*a*c*e - b^2*e + b*c*d)^3)/(2*c^3*(4*a*c - b^2)^(3/2)))
)/(4*a^2*c*(4*a*c - b^2)^(1/2))))/(8*a^3*c^3*e^3 - b^6*e^3 + b^3*c^3*d^3 -
3*b^4*c^2*d^2*e - 12*a^2*b^2*c^2*e^3 + 6*a*b^4*c*e^3 + 3*b^5*c*d*e^2 + 6*a*
b^2*c^3*d^2*e - 12*a*b^3*c^2*d*e^2 + 12*a^2*b*c^3*d*e^2))*(2*a*c*e - b^2*e
+ b*c*d))/(3*c^2*(4*a*c - b^2)^(1/2))
```

3.11 $\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = -\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}} + \frac{e\log\left(a+bx^3+cx^6\right)}{6c}$$

```
[Out] 1/6*e*ln(c*x^6+b*x^3+a)/c-1/3*(-b*e+2*c*d)*arctanh((2*c*x^3+b)/(-4*a*c+b^2)
^(1/2))/c/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1482, 648, 632, 212, 642}

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{e\log\left(a+bx^3+cx^6\right)}{6c} - \frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}$$

```
[In] Int[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] -1/3*((2*c*d - b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 -
 4*a*c]) + (e*Log[a + b*x^3 + c*x^6])/(6*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{d+ex}{a+bx+cx^2}\,dx, x, x^3\right)\\
&= \frac{e\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6c} + \frac{(2cd-be)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6c}\\
&= \frac{e\log\left(a+bx^3+cx^6\right)}{6c} - \frac{(2cd-be)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3c}\\
&= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}} + \frac{e\log\left(a+bx^3+cx^6\right)}{6c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{-\frac{2(-2cd+be)\arctan\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+e\log\left(a+bx^3+cx^6\right)}{6c}$$

```
[In] Integrate[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] + e*Log[a + b*x^3 + c*x^6])/(6*c)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

method	result
default	$\frac{e\ln(c\,x^6+b\,x^3+a)}{6c}+\frac{2\left(d-\frac{be}{2c}\right)\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}}$
risch	$\frac{2\ln\left(\left(-4abce+8a\,c^2d+b^3e-2b^2cd+\sqrt{-(be-2cd)^2(4ac-b^2)}\,b\right)x^3+2\sqrt{-(be-2cd)^2(4ac-b^2)}\,a\right)ae}{3(4ac-b^2)}-\frac{\ln\left(\left(-4abce+8a\,c^2d+b^3e-2b^2\right.\right.}{}$

```
[In] int(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*e*ln(c*x^6+b*x^3+a)/c+2/3*(d-1/2/c*b*e)/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^3+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.00

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \left[\frac{(b^2-4\,ac)e\log\left(cx^6+bx^3+a\right)-\sqrt{b^2-4\,ac}(2\,cd-be)\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)}{6\,(b^2c-4\,ac^2)},\ \frac{(b^2-4\,\cdots}{}\right.$$

```
[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*((b^2 - 4*a*c)*e*log(c*x^6 + b*x^3 + a) - sqrt(b^2 - 4*a*c)*(2*c*d - b
*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a
*c))/(c*x^6 + b*x^3 + a)))/(b^2*c - 4*a*c^2), 1/6*((b^2 - 4*a*c)*e*log(c*x^
```

```
6 + b*x^3 + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^3 + b)*s
qrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(65) = 130$.

Time = 10.30 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.99

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \left(\frac{e}{6c} - \frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)\log\left(x^3 + \frac{-12ac\left(\frac{e}{6c}-\frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)+2ae+3b^2\left(\frac{e}{6c}-\frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)\cdots}{be-2cd}\right.$$
$$+\left(\frac{e}{6c} + \frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)\log\left(x^3 + \frac{-12ac\left(\frac{e}{6c}+\frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)+2ae+3b^2\left(\frac{e}{6c}+\frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)\cdots}{be-2cd}\right.$$

```
[In] integrate(x**2*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] (e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2)))*log(x**3
 + (-12*a*c*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2
))) + 2*a*e + 3*b**2*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a
*c - b**2))) - b*d)/(b*e - 2*c*d)) + (e/(6*c) + sqrt(-4*a*c + b**2)*(b*e -
2*c*d)/(6*c*(4*a*c - b**2)))*log(x**3 + (-12*a*c*(e/(6*c) + sqrt(-4*a*c + b
**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) + 2*a*e + 3*b**2*(e/(6*c) + sqrt(-
4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{e\log\left(cx^6+bx^3+a\right)}{6\,c} + \frac{(2\,cd-be)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c}$$

```
[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*e*log(c*x^6 + b*x^3 + a)/c + 1/3*(2*c*d - b*e)*arctan((2*c*x^3 + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

Mupad [B] (verification not implemented)

Time = 11.27 (sec) , antiderivative size = 1632, normalized size of antiderivative = 22.67

$$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] int((x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] - (log(a + b*x^3 + c*x^6)*(3*b^2*e - 12*a*c*e))/(2*(36*a*c^2 - 9*b^2*c)) -
(atan((b*(4*a*c - b^2)^(3/2)*(a*c*d*e^2 - a*b*e^3 - ((3*b^2*e - 12*a*c*e)*(
((3*b^2*e - 12*a*c*e)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 1
2*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 15*a*b*c*e^2 -
12*a*c^2*d*e))/(2*(36*a*c^2 - 9*b^2*c)) + ((((b*e - 2*c*d)*(72*a*b*c^2*e -
36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(6*c*
(4*a*c - b^2)^(1/2)) + (9*a*b*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d))/((36*
a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(6*c*(4*a*c - b^2)^(1
/2)) + (3*a*b*c*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d)^2)/(2*(36*a*c^2 - 9*b^2*
c)*(4*a*c - b^2))))/(a^2*c*(b^3*e^3 - 8*c^3*d^3 + 12*b*c^2*d^2*e - 6*b^2*c*
d*e^2)) - (4*x^3*((b*(b^2*e^3 + c^2*d^2*e + ((3*b^2*e - 12*a*c*e)*(6*c^3*d^
2 + ((3*b^2*e - 12*a*c*e)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e
 - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 12*b^2*c*e^
2 - 18*b*c^2*d*e))/(2*(36*a*c^2 - 9*b^2*c)) - 2*b*c*d*e^2 - (((b*e - 2*c*d
)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2
- 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) + (9*b^2*c^2*(3*b^2*e - 12*a*c*e)*(b
*e - 2*c*d))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(
6*c*(4*a*c - b^2)^(1/2)) - (3*b^2*c*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d)^2)/(
4*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2))))/(4*a^2*c) - ((2*a*c - b^2)*(((3*b^2
*e - 12*a*c*e)*(((b*e - 2*c*d)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*
b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) + (9*b^
2*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c -
```

```
b^2)^(1/2))))/(2*(36*a*c^2 - 9*b^2*c)) - (b^2*(b*e - 2*c*d)^3)/(4*(4*a*c -
b^2)^(3/2)) + ((b*e - 2*c*d)*(6*c^3*d^2 + ((3*b^2*e - 12*a*c*e)*(45*b^2*c^2
*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/
(2*(36*a*c^2 - 9*b^2*c)) + 12*b^2*c*e^2 - 18*b*c^2*d*e))/(6*c*(4*a*c - b^2)
^(1/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^(3/2))/(b^3*e^3 - 8*
c^3*d^3 + 12*b*c^2*d^2*e - 6*b^2*c*d*e^2) + ((2*a*c - b^2)*(4*a*c - b^2)*((
(3*b^2*e - 12*a*c*e)*(((b*e - 2*c*d)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c
^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) +
 (9*a*b*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d))/((36*a*c^2 - 9*b^2*c)*(4*a*
c - b^2)^(1/2))))/(2*(36*a*c^2 - 9*b^2*c)) + ((b*e - 2*c*d)*(((3*b^2*e - 12
*a*c*e)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 12*a*c*e))/(36*
a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 15*a*b*c*e^2 - 12*a*c^2*d*e))
/(6*c*(4*a*c - b^2)^(1/2)) - (a*b*(b*e - 2*c*d)^3)/(2*(4*a*c - b^2)^(3/2)))
)/(a^2*c*(b^3*e^3 - 8*c^3*d^3 + 12*b*c^2*d^2*e - 6*b^2*c*d*e^2)))*(b*e - 2*
c*d))/(3*c*(4*a*c - b^2)^(1/2))
```

3.12 $\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx = \frac{(bd-2ae)\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}} + \frac{d\log(x)}{a} - \frac{d\log(a+bx^3+cx^6)}{6a}$$

```
[Out] d*ln(x)/a-1/6*d*ln(c*x^6+b*x^3+a)/a+1/3*(-2*a*e+b*d)*arctanh((2*c*x^3+b)/(-
4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1488, 814, 648, 632, 212, 642}

$$\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx = \frac{(bd-2ae)\operatorname{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}} - \frac{d\log(a+bx^3+cx^6)}{6a} + \frac{d\log(x)}{a}$$

```
[In] Int[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a*Sqrt[b^2 - 4*
a*c]) + (d*Log[x])/a - (d*Log[a + b*x^3 + c*x^6])/(6*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{d+ex}{x\,(a+bx+cx^2)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{d}{ax}+\frac{-bd+ae-cdx}{a\,(a+bx+cx^2)}\right)\,dx,x,x^3\right)\\
&= \frac{d\log(x)}{a}+\frac{\text{Subst}\left(\int \frac{-bd+ae-cdx}{a+bx+cx^2}\,dx,x,x^3\right)}{3a}\\
&= \frac{d\log(x)}{a}-\frac{d\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6a}+\frac{(-bd+2ae)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^3\right)}{6a}\\
&= \frac{d\log(x)}{a}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}-\frac{(-bd+2ae)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^3\right)}{3a}
\end{aligned}
$$

$$= \frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}} + \frac{d\log(x)}{a} - \frac{d\log\left(a+bx^3+cx^6\right)}{6a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

$$\int \frac{d+ex^3}{x\left(a+bx^3+cx^6\right)}\,dx$$
$$= \frac{d\log(x)}{a} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&, \frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^3}{b+2c\#1^3}\&\right]}{3a}$$

```
[In] Integrate[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
[Out] (d*Log[x])/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log[x
 - #1] + c*d*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

method	result
default	$\frac{d\ln(x)}{a}+\frac{-\frac{d\ln\left(c\,x^6+b\,x^3+a\right)}{2}+\frac{2\left(ae-\frac{bd}{2}\right)\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{3a}$
risch	$\frac{d\ln(x)}{a}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4c\,a^2-b^2a\right)\_Z^2+\left(4acd-b^2d\right)\_Z+a\,e^2-bde+c\,d^2\right)}\_R\ln\left(\left(\left(-14ac+4b^2\right)\_R^2+\left(be-7cd\right)\_R-3e^2\right)x^3+b\_\right.\right.}{3}$

```
[In] int((e*x^3+d)/x/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] d*ln(x)/a+1/3/a*(-1/2*d*ln(c*x^6+b*x^3+a)+2*(a*e-1/2*b*d)/(4*a*c-b^2)^(1/2)
*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.08

$$\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx = \left[-\frac{(b^2-4ac)d\log(cx^6+bx^3+a)-6(b^2-4ac)d\log(x)+\sqrt{b^2-4ac}(bd-2ae)\log\left(\frac{2c^2x^6+2bcx^3+b^2-2ac-(2cx^3+b)\sqrt{b^2-4ac}}{cx^6+bx^3+a}\right)}{6(ab^2-4a^2c)}, -\frac{(b^2-4ac)d\log(cx^6+bx^3+a)-6(b^2-4ac)d\log(x)-2\sqrt{-b^2+4ac}(bd-2ae)\arctan\left(-\frac{(2cx^3+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{6(ab^2-4a^2c)}\right]$$

```
[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*((b^2 - 4*a*c)*d*log(c*x^6 + b*x^3 + a) - 6*(b^2 - 4*a*c)*d*log(x) +
sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c -
(2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)))/(a*b^2 - 4*a^2*c), -
1/6*((b^2 - 4*a*c)*d*log(c*x^6 + b*x^3 + a) - 6*(b^2 - 4*a*c)*d*log(x) - 2*
sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**3+d)/x/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$\int \frac{d+ex^3}{x\left(a+bx^3+cx^6\right)}\,dx = -\frac{d\log\left(cx^6+bx^3+a\right)}{6\,a}+\frac{d\log\left(|x|\right)}{a}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a}$$

```
[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/6*d*log(c*x^6 + b*x^3 + a)/a + d*log(abs(x))/a - 1/3*(b*d - 2*a*e)*arcta
n((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

Mupad [B] (verification not implemented)

Time = 13.61 (sec) , antiderivative size = 4149, normalized size of antiderivative = 53.19

$$\int \frac{d+ex^3}{x\left(a+bx^3+cx^6\right)}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x)
```

```
[Out] (d*log(x))/a - (log(a + b*x^3 + c*x^6)*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 -
36*a^2*c)) - (atan((48*a^4*x^3*(4*a*c - b^2)^2*(((((((3*b^2*d - 12*a*c*d)*(
((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*
a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a*(4*
a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*
a*e - b*d))/(12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(9*a*b^2 -
 36*a^2*c)) - ((2*a*e - b*d)*(42*a*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12
*a*c*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 -
 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 3
6*a^2*c)) + 42*b*c^4*d*e))/(6*a*(4*a*c - b^2)^(1/2)))*(3*b^2*d - 12*a*c*d))
/(2*(9*a*b^2 - 36*a^2*c)) + ((2*a*e - b*d)*(5*b*c^3*e^3 - ((3*b^2*d - 12*a*
c*d)*(42*a*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*
a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4
*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e
))/(2*(9*a*b^2 - 36*a^2*c)) + 7*c^4*d*e^2))/(6*a*(4*a*c - b^2)^(1/2)) - (((
(2*a*e - b*d)*(((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b
^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c
^4*e))/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378
*a*b^2*c^4)*(2*a*e - b*d))/(12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2)))
)/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^
```

```
2*c^4)*(2*a*e - b*d)^2)/(72*a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)))*(2*a*e
 - b*d))/(6*a*(4*a*c - b^2)^(1/2)) - ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 3
78*a*b^2*c^4)*(2*a*e - b*d)^3)/(432*a^3*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^
(3/2)))*(4*b^4*d + 7*a^2*c^2*d - a*b^3*e - 15*a*b^2*c*d + 2*a^2*b*c*e))/(16
*a^4*c^3*(a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e)) - ((c^3*e^4 - ((3*b
^2*d - 12*a*c*d)*(5*b*c^3*e^3 - ((3*b^2*d - 12*a*c*d)*(42*a*c^4*e^2 - 9*b^2
*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*
a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*
b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e))/(2*(9*a*b^2 - 36*a^2*c)
) + 7*c^4*d*e^2))/(2*(9*a*b^2 - 36*a^2*c)) + ((((2*a*e - b*d)*(((2*a*e - b*
d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a
^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a*(4*a*c - b^2)^(
1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d))/
(12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(6*a*(4*a*c - b^2)^(1/2))
 + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d)^2)/(72
*a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)))*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2
 - 36*a^2*c)) + ((((3*b^2*d - 12*a*c*d)*(((2*a*e - b*d)*(((3*b^2*d - 12*a*c
*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d
 - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12
*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d))/(12*a*(9*a*b^2 - 36*a^
2*c)*(4*a*c - b^2)^(1/2))))/(2*(9*a*b^2 - 36*a^2*c)) - ((2*a*e - b*d)*(42*a
*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(10
8*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^
3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e))/(6*a*(4
*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(6*a*(4*a*c - b^2)^(1/2)) - ((108*b^4*c^
3 - 378*a*b^2*c^4)*(2*a*e - b*d)^4)/(1296*a^4*(4*a*c - b^2)^2))*(4*b^5*d -
2*a^3*c^2*e - a*b^4*e - 23*a*b^3*c*d + 29*a^2*b*c^2*d + 4*a^2*b^2*c*e))/(16
*a^4*c^3*(4*a*c - b^2)^(1/2)*(a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e))
))/(8*a^3*c^3*e^3 - b^3*c^3*d^3 + 6*a*b^2*c^3*d^2*e - 12*a^2*b*c^3*d*e^2) -
 (3*(4*a*c - b^2)^(3/2)*(c^3*d*e^3 + ((3*b^2*d - 12*a*c*d)*(((2*a*e - b*d)*
(((2*a*e - b*d)*(27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 1
2*a*c*d))/(2*(9*a*b^2 - 36*a^2*c))))/(6*a*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3
*(3*b^2*d - 12*a*c*d)*(2*a*e - b*d))/(4*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^
(1/2))))/(6*a*(4*a*c - b^2)^(1/2)) + (3*b^3*c^3*(3*b^2*d - 12*a*c*d)*(2*a*e
 - b*d)^2)/(8*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2))))/(2*(9*a*b^2 - 36*a^2*
c)) - ((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(
27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a
*b^2 - 36*a^2*c))))/(2*(9*a*b^2 - 36*a^2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d
*e))/(2*(9*a*b^2 - 36*a^2*c)) - a*c^3*e^3 + 9*b*c^3*d*e^2))/(2*(9*a*b^2 - 3
6*a^2*c)) + ((((3*b^2*d - 12*a*c*d)*(((2*a*e - b*d)*(27*b^3*c^3*d - 27*a*b^
2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*c))))/(6
*a*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3*(3*b^2*d - 12*a*c*d)*(2*a*e - b*d))/(4
*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(9*a*b^2 - 36*a^2*c)) + ((2
*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b
^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*c))))/(2*(9*a*b^2 - 36*a^
```

```
2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d*e))/(6*a*(4*a*c - b^2)^(1/2)))*(2*a*e
- b*d))/(6*a*(4*a*c - b^2)^(1/2)) - (b^3*c^3*(2*a*e - b*d)^4)/(48*a^3*(4*a*
c - b^2)^2))*(4*b^5*d - 2*a^3*c^2*e - a*b^4*e - 23*a*b^3*c*d + 29*a^2*b*c^2
*d + 4*a^2*b^2*c*e))/(c^3*(8*a^3*c^3*e^3 - b^3*c^3*d^3 + 6*a*b^2*c^3*d^2*e
- 12*a^2*b*c^3*d*e^2)*(a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e)) + (3*(
4*a*c - b^2)^2*(((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(((2*a*e - b*d
)*(27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(
9*a*b^2 - 36*a^2*c))))/(6*a*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3*(3*b^2*d - 12
*a*c*d)*(2*a*e - b*d))/(4*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(9
*a*b^2 - 36*a^2*c)) + ((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(27*b^3*c^3*d -
 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*
c))))/(2*(9*a*b^2 - 36*a^2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d*e))/(6*a*(4*a
*c - b^2)^(1/2))))/(2*(9*a*b^2 - 36*a^2*c)) - ((2*a*e - b*d)*(((2*a*e - b*d
)*(((2*a*e - b*d)*(27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d -
 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*c))))/(6*a*(4*a*c - b^2)^(1/2)) + (9*b^3*c
^3*(3*b^2*d - 12*a*c*d)*(2*a*e - b*d))/(4*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2
)^(1/2))))/(6*a*(4*a*c - b^2)^(1/2)) + (3*b^3*c^3*(3*b^2*d - 12*a*c*d)*(2*a
*e - b*d)^2)/(8*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2))))/(6*a*(4*a*c - b^2)^
(1/2)) + ((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(27*b
^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2
 - 36*a^2*c))))/(2*(9*a*b^2 - 36*a^2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d*e))
/(2*(9*a*b^2 - 36*a^2*c)) - a*c^3*e^3 + 9*b*c^3*d*e^2))/(6*a*(4*a*c - b^2)^
(1/2)) - (b^3*c^3*(3*b^2*d - 12*a*c*d)*(2*a*e - b*d)^3)/(16*a^2*(9*a*b^2 -
36*a^2*c)*(4*a*c - b^2)^(3/2)))*(4*b^4*d + 7*a^2*c^2*d - a*b^3*e - 15*a*b^2
*c*d + 2*a^2*b*c*e))/(c^3*(8*a^3*c^3*e^3 - b^3*c^3*d^3 + 6*a*b^2*c^3*d^2*e
- 12*a^2*b*c^3*d*e^2)*(a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e)))*(2*a*
e - b*d))/(3*a*(4*a*c - b^2)^(1/2))
```

3.13 $\int \frac{d+ex^3}{x^4(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{d+ex^3}{x^4(a+bx^3+cx^6)}\,dx = -\frac{d}{3ax^3} - \frac{(b^2d-2acd-abe)\,\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}} - \frac{(bd-ae)\log(x)}{a^2} + \frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}$$

```
[Out] -1/3*d/a/x^3-(-a*e+b*d)*ln(x)/a^2+1/6*(-a*e+b*d)*ln(c*x^6+b*x^3+a)/a^2-1/3*
(-a*b*e-2*a*c*d+b^2*d)*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+
b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1488, 814, 648, 632, 212, 642}

$$\int \frac{d+ex^3}{x^4(a+bx^3+cx^6)}\,dx = -\frac{\text{arctanh}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)(-abe-2acd+b^2d)}{3a^2\sqrt{b^2-4ac}} + \frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2} - \frac{\log(x)(bd-ae)}{a^2} - \frac{d}{3ax^3}$$

```
[In] Int[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/3*d/(a*x^3) - ((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2
- 4*a*c]])/(3*a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x])/a^2 + ((b*d - a
*e)*Log[a + b*x^3 + c*x^6])/(6*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{d+ex}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{d}{ax^2}+\frac{-bd+ae}{a^2x}+\frac{b^2d-acd-abe+c(bd-ae)x}{a^2\left(a+bx+cx^2\right)}\right)\,dx, x, x^3\right)
\end{aligned}$$

$$= -\frac{d}{3ax^3} - \frac{(bd-ae)\log(x)}{a^2} + \frac{\text{Subst}\left(\int \frac{b^2d-acd-abe+c(bd-ae)x}{a+bx+cx^2}\,dx, x, x^3\right)}{3a^2}$$

$$= -\frac{d}{3ax^3} - \frac{(bd-ae)\log(x)}{a^2} + \frac{(bd-ae)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6a^2} + \frac{(b^2d-2acd-abe)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6a^2}$$

$$= -\frac{d}{3ax^3} - \frac{(bd-ae)\log(x)}{a^2} + \frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2} - \frac{(b^2d-2acd-abe)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3a^2}$$

$$= -\frac{d}{3ax^3} - \frac{(b^2d-2acd-abe)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}} - \frac{(bd-ae)\log(x)}{a^2} + \frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.16

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx = -\frac{d}{3ax^3} + \frac{(-bd+ae)\log(x)}{a^2} + \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&, \frac{b^2d\log(x-\#1)-acd\log(x-\#1)-abe\log(x-\#1)+bcd\log(x-\#1)\#1^3-ace\log(x-\#1)\#1^3}{b+2c\#1^3}\right.}{3a^2}$$

```
[In] Integrate[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/3*d/(a*x^3) + ((-(b*d) + a*e)*Log[x])/a^2 + RootSum[a + b*#1^3 + c*#1^6
& , (b^2*d*Log[x - #1] - a*c*d*Log[x - #1] - a*b*e*Log[x - #1] + b*c*d*Log[
x - #1]*#1^3 - a*c*e*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a^2)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

method	result
default	$-\frac{d}{3a\,x^{3}}+\frac{(ae-bd)\ln(x)}{a^{2}}-\frac{\frac{(ace-bcd)\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{2c}+\frac{2\left(abe+acd-b^{2}d-\frac{(ace-bcd)b}{2c}\right)\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{3a^{2}}$
risch	$-\frac{d}{3a\,x^{3}}+\frac{\ln(x)e}{a}-\frac{\ln(x)bd}{a^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(4a^{2}ce-a\,b^{2}e-4abcd+b^{3}d\right)\_Z+e^{2}ac-bcde+c^{2}d^{2}\right)}\_R\ln\left(\left(\left(-14a^{3}c\right.\right.\right.\right.}{}$

[In] int((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)

[Out] -1/3*d/a/x^3+(a*e-b*d)/a^2*ln(x)-1/3/a^2*(1/2*(a*c*e-b*c*d)/c*ln(c*x^6+b*x^3+a)+2*(a*b*e+a*c*d-b^2*d-1/2*(a*c*e-b*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.68 (sec) , antiderivative size = 385, normalized size of antiderivative = 3.44

$$\int \frac{d+ex^{3}}{x^{4}\left(a+bx^{3}+cx^{6}\right)}\,dx = \left[\frac{(abe-(b^{2}-2\,ac)d)\sqrt{b^{2}-4\,ac}x^{3}\log\left(\frac{2\,c^{2}x^{6}+2\,bcx^{3}+b^{2}-2\,ac+(2\,cx^{3}+b)\sqrt{b^{2}-4\,ac}}{cx^{6}+bx^{3}+a}\right)+((b^{3}-4\,abc)d-(ab^{2}-4\,a^{2}c}{6\,(a^{2}b^{2}-4\,a^{3}c)x^{3}}\right.$$

```
[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] [1/6*((a*b*e - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^6 + 2*b*
c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a))
 + ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(c*x^6 + b*x^3 + a) - 6
*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(x) - 2*(a*b^2 - 4*a^2*c)
*d)/((a^2*b^2 - 4*a^3*c)*x^3), 1/6*(2*(a*b*e - (b^2 - 2*a*c)*d)*sqrt(-b^2 +
 4*a*c)*x^3*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^3
 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(c*x^6 + b*x^3 + a) - 6*((b^3 -
 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(x) - 2*(a*b^2 - 4*a^2*c)*d)/((a^
2*b^2 - 4*a^3*c)*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**3+d)/x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.11

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx = \frac{(bd-ae)\log\left(cx^6+bx^3+a\right)}{6\,a^2} - \frac{(bd-ae)\log\left(|x|\right)}{a^2} + \frac{\left(b^2d-2\,acd-abe\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a^2} + \frac{bdx^3-aex^3-ad}{3\,a^2x^3}$$

```
[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*(b*d - a*e)*log(c*x^6 + b*x^3 + a)/a^2 - (b*d - a*e)*log(abs(x))/a^2 +
1/3*(b^2*d - 2*a*c*d - a*b*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqr
t(-b^2 + 4*a*c)*a^2) + 1/3*(b*d*x^3 - a*e*x^3 - a*d)/(a^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 15.50 (sec) , antiderivative size = 7282, normalized size of antiderivative = 65.02

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x)
```

```
[Out] (log(x)*(a*e - b*d))/a^2 - (log(((((((((a*e - b*d + a^2*(-(a*b*e - b^2*d + 2
*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((27*b^2*c^3*(a*b*e - b^2*d + a*c*d))
/a + (9*b*c^4*x^3*(2*b^2*d + 7*a*b*e - 28*a*c*d))/a + (9*b^2*c^3*(a*b + 4*b
^2*x^3 - 14*a*c*x^3)*(a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4
*a*c - b^2)))^(1/2)))/(2*a^2)))/(6*a^2) - (3*c^5*d*x^3*(11*b^2*d - 14*a*b*e
 + 14*a*c*d))/a^2 + (9*b*c^4*d*(3*a*b*e - 3*b^2*d + a*c*d))/a^2)*(a*e - b*d
 + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(6*a^2) +
 (c^5*d^2*(9*a*b*e - 9*b^2*d + a*c*d))/a^3 + (c^6*d^2*x^3*(7*a*e - 12*b*d))
/a^3)*(a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(
1/2)))/(6*a^2) + (c^6*d^3*(a*e - b*d))/a^4 - (c^7*d^4*x^3)/a^4)*(((((((b*d
- a*e + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((27*
b^2*c^3*(a*b*e - b^2*d + a*c*d))/a + (9*b*c^4*x^3*(2*b^2*d + 7*a*b*e - 28*a
*c*d))/a - (9*b^2*c^3*(a*b + 4*b^2*x^3 - 14*a*c*x^3)*(b*d - a*e + a^2*(-(a*
b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(2*a^2)))/(6*a^2) + (
3*c^5*d*x^3*(11*b^2*d - 14*a*b*e + 14*a*c*d))/a^2 - (9*b*c^4*d*(3*a*b*e - 3
*b^2*d + a*c*d))/a^2)*(b*d - a*e + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(
4*a*c - b^2)))^(1/2)))/(6*a^2) + (c^5*d^2*(9*a*b*e - 9*b^2*d + a*c*d))/a^3
+ (c^6*d^2*x^3*(7*a*e - 12*b*d))/a^3)*(b*d - a*e + a^2*(-(a*b*e - b^2*d + 2
*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(6*a^2) - (c^6*d^3*(a*e - b*d))/a^4
+ (c^7*d^4*x^3)/a^4))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(
36*a^3*c - 9*a^2*b^2)) - d/(3*a*x^3) - (atan((48*a^8*x^3*((((((((((18*a^3*b
^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 + ((108*a^4*b^4*c^3 - 37
8*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^4*(36*
a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2))
 + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)*(3*b^3*d
- 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3
*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a
^3*c - 9*a^2*b^2)) - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5*d^2 - 42*a^3*b*c^5*
d*e)/a^4 - (((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 +
((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12
*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*
e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*
a^2*(4*a*c - b^2)^(1/2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/
(2*(36*a^3*c - 9*a^2*b^2)) - (((((((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 2
52*a^4*b*c^5*d)/a^4 + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b
^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b
```

```
^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*
b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a
*b*c*d))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*
d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*b^2
*c^4)*(a*b*e - b^2*d + 2*a*c*d)^2*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*
b*c*d))/(72*a^8*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a
*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + (((7*a^2*c^6*d^2*e - 12*a*b*c^6*d^3)/a
^4 - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5*d^2 - 42*a^3*b*c^5*d*e)/a^4 - (((18
*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 + ((108*a^4*b^4*c^
3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^
4*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))
/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d
))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b
^2)^(1/2)) - ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)
^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(432*a^10*(4*a*c - b^2)
^(3/2)*(36*a^3*c - 9*a^2*b^2)))*(4*b^5*d - 7*a^3*c^2*e - 4*a*b^4*e - 16*a*b
^3*c*d + 9*a^2*b*c^2*d + 15*a^2*b^2*c*e))/(16*a^4*c^3*(49*a^3*c*e^2 - 12*b^
4*d^2 - 12*a^2*b^2*e^2 + a^2*c^2*d^2 + 24*a*b^3*d*e + 48*a*b^2*c*d^2 - 97*a
^2*b*c*d*e)) - ((((((((((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5
*d)/a^4 + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^
2*c*e - 12*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c
*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*
b*e - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(12
*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d)
)/(6*a^2*(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e
 - b^2*d + 2*a*c*d)^2*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(72*
a^8*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*
e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) - (((7*a^2*c^6*d^2*e - 12*a*b*c
^6*d^3)/a^4 - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5*d^2 - 42*a^3*b*c^5*d*e)/a^
4 - (((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 + ((108*a
^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*
d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*
a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 1
2*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e -
 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) + (c^7*d^4)/a^4 - ((108*a^4*b^4*c^
3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)^4)/(1296*a^12*(4*a*c - b^2)^
2) + (((((((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 + ((
108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a
*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*
(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d
+ 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(12*a^6*(4*a*c
- b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 1
2*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5
*d^2 - 42*a^3*b*c^5*d*e)/a^4 - (((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252
*a^4*b*c^5*d)/a^4 + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2
```

```
*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3
*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b
^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)))*(a*b*e - b^2*d + 2*a*c*d))/(6
*a^2*(4*a*c - b^2)^(1/2)))*(8*a^3*c^3*d - 16*b^6*d + 16*a*b^5*e - 132*a^2*b
^2*c^2*d + 96*a*b^4*c*d - 92*a^2*b^3*c*e + 116*a^3*b*c^2*e))/(64*a^4*c^3*(4
*a*c - b^2)^(1/2)*(49*a^3*c*e^2 - 12*b^4*d^2 - 12*a^2*b^2*e^2 + a^2*c^2*d^2
 + 24*a*b^3*d*e + 48*a*b^2*c*d^2 - 97*a^2*b*c*d*e)))*(4*a*c - b^2)^2)/(8*a^
3*c^6*d^3 - b^6*c^3*d^3 + 6*a*b^4*c^4*d^3 - 12*a^2*b^2*c^5*d^3 + a^3*b^3*c^
3*e^3 + 3*a*b^5*c^3*d^2*e + 12*a^3*b*c^5*d^2*e - 12*a^2*b^3*c^4*d^2*e - 3*a
^2*b^4*c^3*d*e^2 + 6*a^3*b^2*c^4*d*e^2) + (3*a^4*(4*a*c - b^2)^2*(((((((27
*a^4*b^2*c^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(
3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))
*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3*(a*b*e
 - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(4*a*(
4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c
*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) + (((9*a^3*b*c^5*d^2 - 27*a^2*
b^3*c^4*d^2 + 27*a^3*b^2*c^4*d*e)/a^4 + (((27*a^4*b^2*c^4*d - 27*a^3*b^4*c^
3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c
*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2
*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/
(6*a^2*(4*a*c - b^2)^(1/2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d
))/(2*(36*a^3*c - 9*a^2*b^2)) + (((a^2*c^6*d^3 - 9*a*b^2*c^5*d^3 + 9*a^2*b*
c^5*d^2*e)/a^4 + (((9*a^3*b*c^5*d^2 - 27*a^2*b^3*c^4*d^2 + 27*a^3*b^2*c^4*d
*e)/a^4 + (((27*a^4*b^2*c^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 +
(27*a*b^3*c^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c
 - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3
*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a
^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2))
- (((((((27*a^4*b^2*c^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*
a*b^3*c^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9
*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + (9*b^3
*c^3*(a*b*e - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c
*d))/(4*a*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a
*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + (3*b^3*c^3*(a*b*e - b^2*d + 2*a*c*d)^2
*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(8*a^3*(4*a*c - b^2)*(36*
a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2))
 - (b^3*c^3*(a*b*e - b^2*d + 2*a*c*d)^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e -
 12*a*b*c*d))/(16*a^5*(4*a*c - b^2)^(3/2)*(36*a^3*c - 9*a^2*b^2)))*(4*b^5*d
 - 7*a^3*c^2*e - 4*a*b^4*e - 16*a*b^3*c*d + 9*a^2*b*c^2*d + 15*a^2*b^2*c*e)
)/(c^3*(49*a^3*c*e^2 - 12*b^4*d^2 - 12*a^2*b^2*e^2 + a^2*c^2*d^2 + 24*a*b^3
*d*e + 48*a*b^2*c*d^2 - 97*a^2*b*c*d*e)*(8*a^3*c^6*d^3 - b^6*c^3*d^3 + 6*a*
b^4*c^4*d^3 - 12*a^2*b^2*c^5*d^3 + a^3*b^3*c^3*e^3 + 3*a*b^5*c^3*d^2*e + 12
*a^3*b*c^5*d^2*e - 12*a^2*b^3*c^4*d^2*e - 3*a^2*b^4*c^3*d*e^2 + 6*a^3*b^2*c
^4*d*e^2)) - (3*a^4*(4*a*c - b^2)^(3/2)*((b*c^6*d^4 - a*c^6*d^3*e)/a^4 - ((
(a^2*c^6*d^3 - 9*a*b^2*c^5*d^3 + 9*a^2*b*c^5*d^2*e)/a^4 + (((9*a^3*b*c^5*d^
```

```
2 - 27*a^2*b^3*c^4*d^2 + 27*a^3*b^2*c^4*d*e)/a^4 + (((27*a^4*b^2*c^4*d - 27
*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(3*b^3*d - 3*a*b^2*e
 + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2
*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b
^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a
*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) + (((((((27*a
^4*b^2*c^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(3*
b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(
a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3*(a*b*e -
 b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(4*a*(4*
a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2
*(4*a*c - b^2)^(1/2)) + (3*b^3*c^3*(a*b*e - b^2*d + 2*a*c*d)^2*(3*b^3*d - 3
*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(8*a^3*(4*a*c - b^2)*(36*a^3*c - 9*a^2
*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a
^2*b^2)) + (((((((27*a^4*b^2*c^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a
^4 + (27*a*b^3*c^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*
a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2))
 + (9*b^3*c^3*(a*b*e - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e -
 12*a*b*c*d))/(4*a*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d -
3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) + (((9*a^3
*b*c^5*d^2 - 27*a^2*b^3*c^4*d^2 + 27*a^3*b^2*c^4*d*e)/a^4 + (((27*a^4*b^2*c
^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(3*b^3*d -
3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d
- 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e
- b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)))*(a*b*e - b^2*d + 2*a*c*d))
/(6*a^2*(4*a*c - b^2)^(1/2)) - (b^3*c^3*(a*b*e - b^2*d + 2*a*c*d)^4)/(48*a^
7*(4*a*c - b^2)^2))*(8*a^3*c^3*d - 16*b^6*d + 16*a*b^5*e - 132*a^2*b^2*c^2*
d + 96*a*b^4*c*d - 92*a^2*b^3*c*e + 116*a^3*b*c^2*e))/(4*c^3*(49*a^3*c*e^2
- 12*b^4*d^2 - 12*a^2*b^2*e^2 + a^2*c^2*d^2 + 24*a*b^3*d*e + 48*a*b^2*c*d^2
 - 97*a^2*b*c*d*e)*(8*a^3*c^6*d^3 - b^6*c^3*d^3 + 6*a*b^4*c^4*d^3 - 12*a^2*
b^2*c^5*d^3 + a^3*b^3*c^3*e^3 + 3*a*b^5*c^3*d^2*e + 12*a^3*b*c^5*d^2*e - 12
*a^2*b^3*c^4*d^2*e - 3*a^2*b^4*c^3*d*e^2 + 6*a^3*b^2*c^4*d*e^2)))*(a*b*e -
b^2*d + 2*a*c*d))/(3*a^2*(4*a*c - b^2)^(1/2))
```

3.14 $\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 723

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*e*x^2/c-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(c*d-b*e
+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b-(-4*a*c+b^2)
^(1/2))^(1/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)
^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)
/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/6*arcta
n(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(c*d-b*
e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)*3^(1/2)/(b-(-4
*a*c+b^2)^(1/2))^(1/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3
))*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b+(-
4*a*c+b^2)^(1/2))^(1/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-
```

```
4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(c*d-b*e+(2*a*c*e-b^2
*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b+(-4*a*c+b^2)^(1/2))^(1/3)-
1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2)
)*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)*3^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1516, 1524, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$
$$+\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$
$$-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{ex^2}{2c}$$

```
[In] Int[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x^2)/(2*c) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*
ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/
(2^(2/3)*Sqrt[3]*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e + (b*
c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)
/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)*(b + Sqr
t[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 -
 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)
*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e + (b*c*d - b^2*e + 2*
a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/
3)*x])/(3*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((c*d - b*e - (b
*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3
) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])
/(6*2^(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((c*d - b*e + (b*c*d -
 b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^
(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

<u>Rule 298</u>

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

<u>Rule 631</u>

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

<u>Rule 642</u>

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{ex^2}{2c} - \frac{\int \frac{x\left(2ae-2(cd-be)x^3\right)}{a+bx^3+cx^6}\,dx}{2c} \\
&= \frac{ex^2}{2c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} \\
&\quad + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}
\end{aligned}
$$

$$= \frac{ex^2}{2c} - \frac{\left(cd - be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\ 2^{2/3}c^{4/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(cd - be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\ 2^{2/3}c^{4/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\left(cd - be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\ 2^{2/3}c^{4/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(cd - be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\ 2^{2/3}c^{4/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$=\frac{ex^2}{2c}-\frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$-\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+\frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{4c^{4/3}}$$

$$+\frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int\frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{4c^{4/3}}$$

$$+\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int\frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$= \frac{ex^2}{2c} - \frac{\left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b - \sqrt{b^2-4ac}}}$$

$$- \frac{\left(cd - be + \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b + \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b + \sqrt{b^2-4ac}}}$$

$$+ \frac{\left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right) \log\left(\left(b - \sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b - \sqrt{b^2-4ac}}x + 2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b - \sqrt{b^2-4ac}}}$$

$$+ \frac{\left(cd - be + \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right) \log\left(\left(b + \sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b + \sqrt{b^2-4ac}}x + 2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b + \sqrt{b^2-4ac}}}$$

$$+ \frac{\left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right) \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b - \sqrt{b^2-4ac}}}\right)}{2^{2/3}c^{5/3}\sqrt[3]{b - \sqrt{b^2-4ac}}}$$

$$+ \frac{\left(cd - be + \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right) \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b + \sqrt{b^2-4ac}}}\right)}{2^{2/3}c^{5/3}\sqrt[3]{b + \sqrt{b^2-4ac}}}$$

$$
= \frac{ex^2}{2c} - \frac{\left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}
- \frac{\left(cd - be + \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}
- \frac{\left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}
- \frac{\left(cd - be + \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}
+ \frac{\left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x + 2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}
+ \frac{\left(cd - be + \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x + 2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.12

$$
\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{3ex^2 - 2\mathrm{RootSum}\left[a + b\#1^3 + c\#1^6\&, \frac{ae\log(x-\#1) - cd\log(x-\#1)\#1^3 + be\log(x-\#1)\#1^3}{b\#1 + 2c\#1^4}\&\right]}{6c}
$$

```
[In] Integrate[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
[Out] (3*e*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*e*Log[x - #1] - c*d*Log[x -
 #1]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ])/(6*c)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.10

method	result	size
default	$\frac{e\,x^{2}}{2c}-\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\_R^{4}(be-cd)+ae\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	70
risch	$\frac{e\,x^{2}}{2c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left((-be+cd)\_R^{4}-ae\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	71

```
[In] int(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/2/c*e*x^2-1/3/c*sum((_R^4*(b*e-c*d)+a*e*_R)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R
=RootOf(_Z^6*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13535 vs. 2(583) = 1166.

Time = 52.23 (sec) , antiderivative size = 13535, normalized size of antiderivative = 18.72

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*(e*x**3+d)/(c*x**6+b*x**3+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \int \frac{(ex^3+d)x^4}{cx^6+bx^3+a}\,dx$$

[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] 1/2*e*x^2/c - integrate(-((c*d - b*e)*x^4 - a*e*x)/(c*x^6 + b*x^3 + a), x)/c

Giac [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] Timed out

Mupad [B] (verification not implemented)

Time = 38.70 (sec) , antiderivative size = 13112, normalized size of antiderivative = 18.14

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

[In] int((x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x)

[Out] log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a*c*e^2 - 2*b*c*d*e) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3

```
*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3
*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(
4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2
*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*
c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/6 - (9*a*(4*a*c - b^2)*(b*
e - c*d)*(b^4*e^2 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d*e -
 4*a*b^2*c*e^2 + 5*a*b*c^2*d*e))/c^2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3
*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^
3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2
*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*
b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^
2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
)/(c^5*(4*a*c - b^2)^3))^(2/3))/18 - (a^2*x*(a*e^2 + c*d^2 - b*d*e)^2*(a*c*
e - b^2*e + b*c*d))/c^2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d
^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b
^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*
a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2
*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^
(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3
*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + log((2^(1/3)*((2^
(2/3)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a*c*e^2 - 2*b*c*d*
e) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*
c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5
*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*
d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2
)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 +
 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2
*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1
/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3
*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^
3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2
*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^
```

```
3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*
b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^
2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
)/(c^5*(4*a*c - b^2)^3))^(1/3))/6 - (9*a*(4*a*c - b^2)*(b*e - c*d)*(b^4*e^2
 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d*e - 4*a*b^2*c*e^2 +
5*a*b*c^2*d*e))/c^2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c
^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b
^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^
4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2
*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2
) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b
^2)^3))^(2/3))/18 - (a^2*x*(a*e^2 + c*d^2 - b*d*e)^2*(a*c*e - b^2*e + b*c*d
))/c^2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)
^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a
^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*
b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e +
 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e -
96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/
2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 +
12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + (e*x^2)/(2*c) + log(- (2^(1/3)*((2
^(2/3)*(3^(1/2)*1i - 1)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*
a*c*e^2 - 2*b*c*d*e) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2
*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1
/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*
c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d
*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b
^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*
b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*
a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-
(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2)
+ 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2)
 - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3
```

*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^
2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*
c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5
*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3
*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b
*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 - (9*
a*(4*a*c - b^2)*(b*e - c*d)*(b^4*e^2 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*
d^2 - 2*b^3*c*d*e - 4*a*b^2*c*e^2 + 5*a*b*c^2*d*e))/c^2)*(3^(1/2)*1i + 1)*(
-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2
) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^
3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e
^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5
*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) +
5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^
3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*
b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/36 - (a
^2*x*(a*e^2 + c*d^2 - b*d*e)^2*(a*c*e - b^2*e + b*c*d))/c^2)*((3^(1/2)*1i)/
2 - 1/2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2
)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*
a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3
*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e
+ 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3
)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e -
 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^
2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 +
 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + log(- (2^(1/3)*((2^(2/3)*(3^(1/2)
1i - 1)(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a*c*e^2 - 2*b*c
*d*e) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b^8*e^3 + 1
6*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^
4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^
5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c
^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*
c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 +
96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e
^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 +
 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)
^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(
-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8*e^3 + 16*a
^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d
^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d

^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*
d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*
a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*
a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4
*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 - (9*a*(4*a*c - b^2)
*(b*e - c*d)*(b^4*e^2 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d
*e - 4*a*b^2*c*e^2 + 5*a*b*c^2*d*e))/c^2)*(3^(1/2)*1i + 1)*(-(b^8*e^3 + 16*
a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*
d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*
d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3
d^3(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96
*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3
*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6
*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(
4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/36 - (a^2*x*(a*e^2 + c
*d^2 - b*d*e)^2*(a*c*e - b^2*e + b*c*d))/c^2)*((3^(1/2)*1i)/2 - 1/2)*(-(b^8
*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*
a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 4
8*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3
 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 +
5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*
d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2
*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3
*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3
*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 -
 48*a^2*b^2*c^7)))^(1/3) - log(- (2^(1/3)*((2^(2/3)*(3^(1/2)*1i + 1)*(27*a^
2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a*c*e^2 - 2*b*c*d*e) - (27*2^(
1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^8*e^3 + 16*a^4*c^4*e^3 -
 b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*
b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6
*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c
 - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c
- b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*
e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*
a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^
3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^
5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c

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^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^
2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c -
b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2
 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)
^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b
^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^
(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 + (9*a*(4*a*c - b^2)*(b*e - c*d)*(b
^4*e^2 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d*e - 4*a*b^2*c*
e^2 + 5*a*b*c^2*d*e))/c^2)*(3^(1/2)*1i - 1)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b
^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*
c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c
^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c -
b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^
2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2
)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*
b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)
^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/36 - (a^2*x*(a*e^2 + c*d^2 - b*d*e)^2
*(a*c*e - b^2*e + b*c*d))/c^2)*((3^(1/2)*1i)/2 + 1/2)*(-(b^8*e^3 + 16*a^4*c
^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 -
 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e
 + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*
(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(
-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*
b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4
*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*
c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7
)))^(1/3) - log(- (2^(1/3)*((2^(2/3)*(3^(1/2)*1i + 1)*(27*a^2*c*x*(4*a*c -
b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a*c*e^2 - 2*b*c*d*e) - (27*2^(1/3)*a*b*c^3*(3
^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 -
b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a
*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41
*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2
) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) +
72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2
*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*
(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5
```

```
*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^
4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^
2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*
a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*
a*c - b^2)^3))^(1/3))/12 + (9*a*(4*a*c - b^2)*(b*e - c*d)*(b^4*e^2 - a*c^3*
d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d*e - 4*a*b^2*c*e^2 + 5*a*b*c^2
*d*e))/c^2)*(3^(1/2)*1i - 1)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^
5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c
^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a
^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2)
- 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72
*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^2*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4
*a*c - b^2)^3))^(2/3))/36 - (a^2*x*(a*e^2 + c*d^2 - b*d*e)^2*(a*c*e - b^2*e
 + b*c*d))/c^2)*((3^(1/2)*1i)/2 + 1/2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^
3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d
^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^
2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3
*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^
(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c
^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2
))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3)
```

3.15 $\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 718

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{ex}{c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

```
[Out] e*x/c+1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(c*d-b*e+(-2*a
*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2)
)^(2/3)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2)
)^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a
*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*arctan(1/3*
(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(c*d-b*e+(-2*
a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b-(-4*a*c+b
^2)^(1/2))^(2/3)+1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(c*
d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+
b^2)^(1/2))^(2/3)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+
b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(c*d-b*e+(2*a*c*e-b^2*e+b*c
*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)-1/6*ar
```

```
ctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(c*d
-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1516, 1436, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$
$$-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$
$$+\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{ex}{c}$$

```
[In] Int[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x)/c - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/
3)*Sqrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e + (b*c*d -
b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b +
```

```
Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2
- 4*a*c])^(2/3)) + ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c
])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/
3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)
/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e - (b*c*d -
 b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e + (b*c*d - b^2*e
 + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*
c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*
c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{ex}{c} - \frac{\int \frac{ae-(cd-be)x^3}{a+bx^3+cx^6}\,dx}{c}\\ &= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}\\ &\quad + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}\end{aligned}$$

$$= \frac{ex}{c} + \frac{\left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}c\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}} - \sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2}\,dx}{3\sqrt[3]{2}c\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(cd - be + \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}c\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(cd - be + \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}} - \sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2}\,dx}{3\sqrt[3]{2}c\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= \frac{ex}{c} + \frac{\left(cd - be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(cd - be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(cd - be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(cd - be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{2\ 2^{2/3}c\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\left(cd - be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(cd - be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{2\ 2^{2/3}c\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$= \frac{ex}{c} + \frac{\left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}}$$
$$+ \frac{\left(cd - be + \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b + \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{\left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b - \sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b - \sqrt{b^2-4ac}}x + 2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{\left(cd - be + \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b + \sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b + \sqrt{b^2-4ac}}x + 2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}}$$
$$+ \frac{\left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b - \sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}c^{4/3}\left(b - \sqrt{b^2-4ac}\right)^{2/3}}$$
$$+ \frac{\left(cd - be + \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b + \sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}c^{4/3}\left(b + \sqrt{b^2-4ac}\right)^{2/3}}$$

$$= \frac{ex}{c} - \frac{\left(cd - be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(cd - be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(cd - be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(cd - be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(cd - be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(cd - be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.12

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{ex}{c} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&, \frac{ae\log(x-\#1)-cd\log(x-\#1)\#1^3+be\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3c}$$

```
[In] Integrate[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x)/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*e*Log[x - #1] - c*d*Log[x - #1
]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.09

method	result	size
default	$\frac{ex}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(-be+cd\right)\_R^{3}-ae\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	67
risch	$\frac{ex}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(-be+cd\right)\_R^{3}-ae\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	67

```
[In] int(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] e*x/c+1/3/c*sum(((-b*e+c*d)*_R^3-a*e)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(
_Z^6*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8705 vs. 2(580) = 1160.

Time = 4.25 (sec) , antiderivative size = 8705, normalized size of antiderivative = 12.12

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6} \, dx = \int \frac{(ex^3+d)x^3}{cx^6+bx^3+a} \, dx$$

```
[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] e*x/c - integrate(-((c*d - b*e)*x^3 - a*e)/(c*x^6 + b*x^3 + a), x)/c
```

Giac [F]

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6} \, dx = \int \frac{(ex^3+d)x^3}{cx^6+bx^3+a} \, dx$$

```
[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] integrate((e*x^3 + d)*x^3/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 29.75 (sec) , antiderivative size = 11453, normalized size of antiderivative = 15.95

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6} \, dx = \text{Too large to display}$$

```
[In] int((x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
[Out] log((3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3 + 2*a^3*c^2*e^4 + b^2*c^3*d
^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d^3*e
+ 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/3)*
((2^(1/3)*(81*a*c^3*d*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c - b^2)
^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1
/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3
*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^
2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4
*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^
3))^(1/3))/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c -
 b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c -
b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2
*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*
```

```
a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*
e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c
*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*
a*c - b^2)^3))^(2/3))/18 + (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2*c^
2*e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2 + 6
*a*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4
*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*
a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e
^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^
2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*
c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3
*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^
2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c
^4*(4*a*c - b^2)^3))^(1/3))/6)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b
^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^
3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^
2*b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3
*b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e
+ 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3
*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3
)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3)
 + log((3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3 + 2*a^3*c^2*e^4 + b^2*c^
3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d^3
*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/
3)*((2^(1/3)*(81*a*c^3*d*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c - b
^2)^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a^2*c^2
*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*b^2*c*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48
*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^
2)^3))^(1/3))/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*
c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c
 - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3
- 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 +
 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2
*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^
3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*
(4*a*c - b^2)^3))^(2/3))/18 + (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2
*c^2*e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2
```

```
+ 6*a*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(
-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-
(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^
2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d
*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b
^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
+ 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))
/(c^4*(4*a*c - b^2)^3))^(1/3))/6)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3
- b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b
*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32
*a^2*b^3*c^2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3
- 3*b^6*c*d*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2
*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2
- 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2
)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1
/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((2^(1/3)*(3^(1/2)*1i + 1)*(81*a*c^3*d*
x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((
b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) +
 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) +
48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(
4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4
*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c
^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(
1/3))/4)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)
^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*
c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2
*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 +
 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*
c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c -
 b^2)^3))^(2/3))/36 - (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2*c^2*e^3
 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2 + 6*a*b*
c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c
- b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c -
 b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 +
2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4
*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d
*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*
c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4
```

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*a*c - b^2)^3))^(1/3))/12 + (3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3 + 2
*a^3*c^2*e^4 + b^2*c^3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c*d^
2*e^2 + 8*a*b*c^3*d^3*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*c^2
*d^2*e^2))/c)*((3^(1/2)*1i)/2 - 1/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d
^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3
- b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e +
 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e
^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*
d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e
^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c -
b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))
^(1/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((2^(1/3)*(3^(1/2)*1i + 1)*(81*a*c^3
*d*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2
*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2
) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2)
 + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a^2*c^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*b^2*c*e^3*(
-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*
b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)
^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3)
)^(1/3))/4)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b
^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^
2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a
^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*
b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^
2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*
c - b^2)^3))^(2/3))/36 - (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2*c^2*
e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2 + 6*a
*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3
 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2
+ 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^
2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b
^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4
*(4*a*c - b^2)^3))^(1/3))/12 + (3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3
+ 2*a^3*c^2*e^4 + b^2*c^3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c
*d^2*e^2 + 8*a*b*c^3*d^3*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*
c^2*d^2*e^2))/c)*((3^(1/2)*1i)/2 - 1/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^
3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e
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^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*
e + 32*a^2*b^3*c^2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*
c*e^3 - 3*b^6*c*d*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c
^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*
d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6
)))^(1/3) - log(- (2^(2/3)*(3^(1/2)*1i + 1)*((2^(1/3)*(3^(1/2)*1i - 1)*(81*
a*c^3*d*x*(4*a*c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b
^2)^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2
*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48
*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c -
 b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^
2)^3))^(1/3))/4)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*
c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c
 - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3
+ 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 -
 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2
*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^
3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*
(4*a*c - b^2)^3))^(2/3))/36 + (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2
*c^2*e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2
+ 6*a*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(
-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-
(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^
2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d
*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b
^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
- 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))
/(c^4*(4*a*c - b^2)^3))^(1/3))/12 - (3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d
*e^3 + 2*a^3*c^2*e^4 + b^2*c^3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*
b^4*c*d^2*e^2 + 8*a*b*c^3*d^3*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a
*b^2*c^2*d^2*e^2))/c)*((3^(1/2)*1i)/2 + 1/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b
^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*
c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2
*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a
*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*
b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2
```

```
*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(
4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^
2*c^6)))^(1/3) - log(- (2^(2/3)*(3^(1/2)*1i + 1)*((2^(1/3)*(3^(1/2)*1i - 1)
*(81*a*c^3*d*x*(4*a*c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*
c - b^2)^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^
2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2
)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a^
2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*b
^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2
 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*
e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c
 - b^2)^3))^(1/3))/4)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(
4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2
*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*
e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^
4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/
(c^4*(4*a*c - b^2)^3))^(2/3))/36 + (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3
+ a^2*c^2*e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d
*e^2 + 6*a*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*
e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d
^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b
^3*c^2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^
6*c*d*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 2
7*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(
1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^
2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(
1/2))/(c^4*(4*a*c - b^2)^3))^(1/3))/12 - (3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 -
b^5*d*e^3 + 2*a^3*c^2*e^4 + b^2*c^3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e
 + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d^3*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3
- 9*a*b^2*c^2*d^2*e^2))/c)*((3^(1/2)*1i)/2 + 1/2)*((b^7*e^3 - 16*a^2*c^5*d^
3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a
^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^
5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^
2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^
2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a
^2*b^2*c^6)))^(1/3) + (e*x)/c
```

3.16 $\int \frac{x(d+ex^3)}{a+bx^3+cx^6} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 634

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}} + \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(e+(-b*e+2*c*d)/(-4
*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/12*ln(2^(2/
3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^
2)^(1/2))^(2/3))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3)/(b-(-4
*a*c+b^2)^(1/2))^(1/3)-1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2
)^(1/2))^(1/3))*3^(1/2))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3
)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+
b^2)^(1/2))^(1/3))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3)/(b+(-
```

```
4*a*c+b^2)^(1/2))^(1/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-
4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(e+(b*e-2*c*d)/(-4*a*
c+b^2)^(1/2))*2^(1/3)/c^(2/3)/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/6*arctan(1/3*(
1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(e+(b*e-2*c*d)
/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1524, 298, 31, 648, 631, 210, 642}

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx$$

$$= -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$

$$+ \frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$

$$- \frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$

```
[In] Int[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] -(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/
(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3))) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sq
rt[3]*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - ((e + (2*c*d - b*e)/Sqrt[b^2
 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/
3)*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((e - (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*
c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a
*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4
*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx \\
&\quad+\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx \\
&= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\ 2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}} \\
&\quad+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\ 2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}} \\
&\quad-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\ 2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}} \\
&\quad+\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\ 2^{2/3}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}
\end{aligned}$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{4\sqrt[3]{c}}$$

$$+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int\frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{4\sqrt[3]{c}}$$

$$+\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int\frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.09

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]$$

```
[In] Integrate[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1^3)/(b*#1
+ 2*c*#1^4) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.08

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\_R^{4}e+\_Rd\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}\right)}{3}$	49
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\_R^{4}e+\_Rd\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}\right)}{3}$	49

```
[In] int(x*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3*sum((_R^4*e+_R*d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8268 vs. 2(496) = 992.

Time = 14.50 (sec) , antiderivative size = 8268, normalized size of antiderivative = 13.04

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

```
[In] integrate(x*(e*x**3+d)/(c*x**6+b*x**3+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \int \frac{(ex^3+d)x}{cx^6+bx^3+a}\,dx$$

```
[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \int \frac{(ex^3+d)x}{cx^6+bx^3+a}\,dx$$

```
[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 26.70 (sec) , antiderivative size = 7457, normalized size of antiderivative = 11.76

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] int((x*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(1/3)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 +
 a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 -
b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2
) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(
4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - (2^(2/3)*(27*c^3*x*(4*a*c - b^2)*(
2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e) - (27*2^(1/3)*a*b*c^3*(4*a*c -
 b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^
2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2
*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a
^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 2
4*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c
 - b^2)^3))^(2/3))/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*
c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c
^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2
```

```
)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))
/(a*c^2*(4*a*c - b^2)^3))^(1/3))/6 - 108*a^2*c^4*d^2*e - 45*a^2*b^2*c^2*e^3
 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*
d*e^2))/18 + c*x*(b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2)*((a*b^5*e^3 + 16*a^
2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1
/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*
e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b
*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4
*c^3 - 48*a^3*b^2*c^4)))^(1/3) + log((2^(1/3)*((a*b^5*e^3 + 16*a^2*c^4*d^3
+ b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^
2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2
*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*
c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - (
2^(2/3)*(27*c^3*x*(4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*
e) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4
*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3
*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a
*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3))/2)*((a*b^5*e^3 + 16*a^2
*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e
^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*
c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(1/3))/6 - 108*a
^2*c^4*d^2*e - 45*a^2*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27
*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*d*e^2))/18 + c*x*(b*e - c*d)*(a*e^2 + c*d^
2 - b*d*e)^2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3
- a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 +
 b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
- 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64
*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*c^4)))^(1/3) - log(c*x*(
b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2 + (2^(1/3)*(3^(1/2)*1i - 1)*((a*b^5*e^
3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a
*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^
2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3)*
(36*a^3*c^3*e^3 - 108*a^2*c^4*d^2*e + (2^(2/3)*(3^(1/2)*1i + 1)*(27*c^3*x*(
4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e) - (27*2^(1/3)*a*
b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c
^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c
```

*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2
e(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3))/4)*((a*b^5*e^3 + 16*a^2*c
^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2)
- 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2
 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*
d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(1/3))/12 - 45*a^2
*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27*a*b^3*c^2*d*e^2 + 10
8*a^2*b*c^3*d*e^2))/36)*((3^(1/2)*1i)/2 + 1/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3
 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a
^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^
2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a
*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*
(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4*c^3 - 48
*a^3*b^2*c^4)))^(1/3) - log(c*x*(b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2 + (2^
(1/3)*(3^(1/2)*1i - 1)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2
*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*
c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
)/(a*c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - 108*a^2*c^4*d^2*e + (2^(
2/3)*(3^(1/2)*1i + 1)*(27*c^3*x*(4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*
d^2 - 2*a*b*d*e) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((a
*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*
a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a
*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^
2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c
^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))
^(2/3))/4)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a
*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*
c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 4
8*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
+ 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*
a*c - b^2)^3))^(1/3))/12 - 45*a^2*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^
3*d^2*e - 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*d*e^2))/36)*((3^(1/2)*1i)/2 +
1/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^
3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*
c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a
^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^4*c^5
- a*b^6*c^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*c^4)))^(1/3) + log(c*x*(b*e - c*d
)*(a*e^2 + c*d^2 - b*d*e)^2 - (2^(1/3)*(3^(1/2)*1i + 1)*((a*b^5*e^3 + 16*a^
2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1

```
/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*
e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b
*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c
^3*e^3 - 108*a^2*c^4*d^2*e - (2^(2/3)*(3^(1/2)*1i - 1)*(27*c^3*x*(4*a*c - b
^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e) + (27*2^(1/3)*a*b*c^3*(3^
(1/2)*1i + 1)*(4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 -
8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16
*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c
- b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a
*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3
)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3))/4)*((a*b^5*e^3 + 16*a^2*c^4*d^3 +
b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*
b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c
*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^
2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(1/3))/12 - 45*a^2*b^2*c^2*
e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c
^3*d*e^2))/36)*((3^(1/2)*1i)/2 - 1/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^
2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*
e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(
-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c
- b^2)^3)^(1/2))/(54*(64*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*
c^4)))^(1/3) + log(c*x*(b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2 - (2^(1/3)*(3^
(1/2)*1i + 1)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3
- a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 +
 b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
- 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*
(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - 108*a^2*c^4*d^2*e - (2^(2/3)*(3^(
1/2)*1i - 1)*(27*c^3*x*(4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a
*b*d*e) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*((a*b^5*e^3
+ 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2
)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)
^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b
^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2
+ 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3))/4
)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(
-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3
*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*
b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2
)^3))^(1/3))/12 - 45*a^2*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e -
 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*d*e^2))/36)*((3^(1/2)*1i)/2 - 1/2)*((a*
```

```
b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2
 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^
2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^4*c^5 - a*b^6*c
^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*c^4)))^(1/3)
```

3.17 $\int \frac{d+ex^3}{a+bx^3+cx^6} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 634

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$$
$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+ \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$- \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

```
[Out] 1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(e+(-b*e+2*c*d)/(-4*
a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/12*ln(2^(2/3
)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2
)^(1/2))^(2/3))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)/(b-(-4*
a*c+b^2)^(1/2))^(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)
^(1/2))^(1/3))*3^(1/2))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)
*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b
^2)^(1/2))^(1/3))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)/(b+(-4
*a*c+b^2)^(1/2))^(2/3)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4
*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(e+(b*e-2*c*d)/(-4*a*c
```

```
+b^2)^(1/2))*2^(2/3)/c^(1/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)-1/6*arctan(1/3*(1
-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(e+(b*e-2*c*d)/
(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1436, 206, 31, 648, 631, 210, 642}

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$$
$$= -\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}} - \frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}} + \frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

```
[In] Int[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```

```
[Out] -(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/
(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*(b - Sqrt
```

```
[b^2 - 4*a*c])^(2/3))) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sq
rt[3]*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((e + (2*c*d - b*e)/Sqrt[b^2
 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((e - (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*
c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a
*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(2/3)) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4
*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx \\
&\quad+\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx \\
&= \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} \\
&\quad+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int\frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} \\
&\quad+\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int\frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} \\
&\quad+\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int\frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}
\end{aligned}$$

$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{2\ 2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{2\ 2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.10

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]$$

```
[In] Integrate[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1^3)/(b*#1^
2 + 2*c*#1^5) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\_R^{3}e+d\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}\right)}{3}$	47
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\_R^{3}e+d\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}\right)}{3}$	47

```
[In] int((e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3*sum((_R^3*e+d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6748 vs. 2(496) = 992.

Time = 1.87 (sec) , antiderivative size = 6748, normalized size of antiderivative = 10.64

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**3+d)/(c*x**6+b*x**3+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx=\int \frac{ex^3+d}{cx^6+bx^3+a}\,dx$$

```
[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a), x)
```

Giac [F]

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx=\int \frac{ex^3+d}{cx^6+bx^3+a}\,dx$$

```
[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 21.64 (sec) , antiderivative size = 7469, normalized size of antiderivative = 11.78

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx=\text{Too large to display}$$

```
[In] int((d + e*x^3)/(a + b*x^3 + c*x^6),x)
[Out] log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d^2*
e^2 - 4*b*c^2*d^3*e) - (2^(2/3)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3
 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2
) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2
)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*((2^(1/3)*(81*c^3*x*(4*a*c - b^2
)^2*(a*e - b*d) - (81*2^(2/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^5*c*d^3 + a^2*b^
4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(
-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*
e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2
*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*
c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3))/2)*(-(b^5
*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3
- 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e
```

```
- 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2
*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(
2/3))/18 - 36*a*c^5*d^3 + 9*b^2*c^4*d^3 + 9*a*b^3*c^2*e^3 - 36*a^2*b*c^3*e^
3 + 108*a^2*c^4*d*e^2 - 27*a*b^2*c^3*d*e^2))/6)*(-(b^5*c*d^3 + a^2*b^4*e^3
+ 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*
c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 +
b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6
*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*
e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 -
48*a^4*b^2*c^3)))^(1/3) + log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e^4
- b^3*d*e^3 + 3*b^2*c*d^2*e^2 - 4*b*c^2*d^3*e) - (2^(2/3)*(-(b^5*c*d^3 + a^
2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d
^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^
2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c
*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*
a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*((2^(1
/3)*(81*c^3*x*(4*a*c - b^2)^2*(a*e - b*d) - (81*2^(2/3)*a*b*c^3*(4*a*c - b^
2)^2*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2
*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^
2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3
*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*
a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c -
 b^2)^3))^(1/3))/2)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c
^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)
^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/
(a^2*c*(4*a*c - b^2)^3))^(2/3))/18 - 36*a*c^5*d^3 + 9*b^2*c^4*d^3 + 9*a*b^3
*c^2*e^3 - 36*a^2*b*c^3*e^3 + 108*a^2*c^4*d*e^2 - 27*a*b^2*c^3*d*e^2))/6)*(
-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3
*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^
(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d
^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^
2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^5*c^4 - a^2
*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)))^(1/3) + log(3*c^2*x*(2*c^3*d^4
+ a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d^2*e^2 - 4*b*c^2*d^3*e) +
(2^(2/3)*(3^(1/2)*1i - 1)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a
*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^
2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b
^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(
1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*(36*a*c^5*d^3 - 9*b^2*c^4*d^3 - 9*a*b^
3*c^2*e^3 + 36*a^2*b*c^3*e^3 - 108*a^2*c^4*d*e^2 + (2^(1/3)*(3^(1/2)*1i + 1
)*(81*c^3*x*(4*a*c - b^2)^2*(a*e - b*d) - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i -
 1)*(4*a*c - b^2)^2*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c
```

```
^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)
^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/
(a^2*c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*
e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(
1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(
4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c -
b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(2/3))/36 + 27*a*b^2*c^3*d*e^2))/12
)*((3^(1/2)*1i)/2 - 1/2)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*
b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2
*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1
/2))/(54*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)))^(1/3)
 + log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d
^2*e^2 - 4*b*c^2*d^3*e) + (2^(2/3)*(3^(1/2)*1i - 1)*(-(b^5*c*d^3 + a^2*b^4*
e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(
4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^
3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e
 - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*
d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*(36*a*c^5*d^
3 - 9*b^2*c^4*d^3 - 9*a*b^3*c^2*e^3 + 36*a^2*b*c^3*e^3 - 108*a^2*c^4*d*e^2
+ (2^(1/3)*(3^(1/2)*1i + 1)*(81*c^3*x*(4*a*c - b^2)^2*(a*e - b*d) - (81*2^(
2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^5*c*d^3 + a^2*b^4*e^3 +
16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c
- b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^
2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a
^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*
(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5*c*d^3 +
 a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^
2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3
*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^
4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e +
 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(2/3))/36
 + 27*a*b^2*c^3*d*e^2))/12)*((3^(1/2)*1i)/2 - 1/2)*(-(b^5*c*d^3 + a^2*b^4*e
^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4
*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3
 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e
- 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d
^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2
 - 48*a^4*b^2*c^3)))^(1/3) - log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e
^4 - b^3*d*e^3 + 3*b^2*c*d^2*e^2 - 4*b*c^2*d^3*e) + (2^(2/3)*(3^(1/2)*1i +
1)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b
```

```
*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c
^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^
2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b
^2)^3))^(1/3)*(9*b^2*c^4*d^3 - 36*a*c^5*d^3 + 9*a*b^3*c^2*e^3 - 36*a^2*b*c^
3*e^3 + 108*a^2*c^4*d*e^2 + (2^(1/3)*(3^(1/2)*1i - 1)*(81*c^3*x*(4*a*c - b^
2)^2*(a*e - b*d) + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(
b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d
^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1
/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2
*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*
c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3)
)^(1/3))/4)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 +
 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2
) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(
4*a*c - b^2)^3))^(2/3))/36 - 27*a*b^2*c^3*d*e^2))/12)*((3^(1/2)*1i)/2 + 1/2
)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*
c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^
3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^
3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2
*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^5*c^4 -
a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)))^(1/3) - log(3*c^2*x*(2*c^3*d
^4 + a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d^2*e^2 - 4*b*c^2*d^3*e)
 + (2^(2/3)*(3^(1/2)*1i + 1)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 -
8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) +
 a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c
- b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3
)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*(9*b^2*c^4*d^3 - 36*a*c^5*d^3 + 9*a
*b^3*c^2*e^3 - 36*a^2*b*c^3*e^3 + 108*a^2*c^4*d*e^2 + (2^(1/3)*(3^(1/2)*1i
- 1)*(81*c^3*x*(4*a*c - b^2)^2*(a*e - b*d) + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1
i + 1)*(4*a*c - b^2)^2*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^
3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b
*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2
))/(a^2*c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c
^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3
)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*
(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e
^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(2/3))/36 - 27*a*b^2*c^3*d*e^2))
/12)*((3^(1/2)*1i)/2 + 1/2)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8
```

```
*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) +
a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*
c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)
^(1/2))/(54*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)))^(1
/3)
```

3.18 $\int \frac{d+ex^3}{x^2(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 653

$$\int \frac{d+ex^3}{x^2(a+bx^3+cx^6)}\,dx$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$
$$+ \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$+ \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$
$$- \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$- \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

```
[Out] -d/a/x+1/6*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(d+(-
2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/12*
c^(1/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/
3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(1/3
)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3
)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/
2))*2^(1/3)/a*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*ln(2^(1/3)*c
^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2
```

```
^(1/3)/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/12*c^(1/3)*ln(2^(2/3)*c^(2/3)*x^2-2
^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))
*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)+
1/6*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))
*3^(1/2))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/a*3^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/3)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1518, 1524, 298, 31, 648, 631, 210, 642}

$$
\begin{aligned}
&\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx \\
&= \frac{\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} \\
&+ \frac{\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{\sqrt{b^2-4ac}+b}} \\
&- \frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} \\
&- \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}} \\
&+ \frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} \\
&+ \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}} - \frac{d}{ax}
\end{aligned}
$$

```
[In] Int[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(d/(a*x)) + (c^(1/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*
2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3
]*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 -
 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/S
qrt[3]])/(2^(2/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d +
(b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3
)*c^(1/3)*x])/(3*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d - (
b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)
*c^(1/3)*x])/(3*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(d + (b
*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*
c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*
a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4
*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 -
4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c]
)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\frac{d}{ax} - \frac{\int \frac{x(bd-ae+cdx^3)}{a+bx^3+cx^6}\,dx}{a}$$

$$= -\frac{d}{ax} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}$$

$$= -\frac{d}{ax} + \frac{\left(c^{2/3}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} \, dx}{3\ 2^{2/3} a \sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\left(c^{2/3}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right) \int \frac{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2} \, dx}{3\ 2^{2/3} a \sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(c^{2/3}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} \, dx}{3\ 2^{2/3} a \sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\left(c^{2/3}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right) \int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}} + c^{2/3}x^2} \, dx}{3\ 2^{2/3} a \sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$-\frac{\left(c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{4a}$$

$$-\frac{\left(\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int\frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$-\frac{\left(c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{4a}$$

$$-\frac{\left(\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int\frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\left(\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\left(\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\sqrt[3]{c}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+ \frac{\sqrt[3]{c}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$+ \frac{\sqrt[3]{c}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$- \frac{\sqrt[3]{c}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x + 2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$- \frac{\sqrt[3]{c}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x + 2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.13

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx$$

$$= -\frac{d}{ax} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]}{3a}$$

```
[In] Integrate[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
[Out] -(d/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log[x -
 #1] + c*d*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ]/(3*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.11

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(-cd\_R^{4}+(ae-bd)\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3a}-\frac{d}{ax}$
risch	$-\frac{d}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^{7}c^{3}-48b^{2}c^{2}a^{6}+12a^{5}b^{4}c-b^{6}a^{4}\right)\_Z^{6}+\left(-16a^{5}c^{2}e^{3}+8a^{4}b^{2}c\,e^{3}+48a^{4}b\,c^{2}d\,e^{2}+48a^{4}c^{3}d^{2}e-a^{3}b^{4}e^{3}-24a^{3}b^{3}cd\,e^{2}-72\right.\right.}\right.}{}$

```
[In] int((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a*sum((-c*d*_R^4+(a*e-b*d)*_R)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^
6*c+_Z^3*b+a))-d/a/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11285 vs. 2(517) = 1034.

Time = 37.04 (sec) , antiderivative size = 11285, normalized size of antiderivative = 17.28

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx=\text{Timed out}$$

```
[In] integrate((e*x**3+d)/x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx=\int \frac{ex^3+d}{(cx^6+bx^3+a)x^2}\,dx$$

```
[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] -integrate((c*d*x^4 + (b*d - a*e)*x)/(c*x^6 + b*x^3 + a), x)/a - d/(a*x)
```

Giac [F]

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx=\int \frac{ex^3+d}{(cx^6+bx^3+a)x^2}\,dx$$

```
[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 36.13 (sec) , antiderivative size = 11174, normalized size of antiderivative = 17.11

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx=\text{Too large to display}$$

```
[In] int((d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x)
[Out] log((2^(1/3)*(-(b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) -
16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*
a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2
*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*
a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c
*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*
a*c - b^2)^3))^(2/3)*((2^(2/3)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a^3
*c*e^2 + a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*
b*c*d*e) - (27*2^(1/3)*a^10*b*c^3*(4*a*c - b^2)^2*(-(b^7*d^3 - a^3*b^4*e^3
+ b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^
3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a
^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2)
- 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c
```

*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(1/3))/6 + 36*a^9*c^6*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5*d^3 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^4*d*e^2))/18 + a^7*c^4*e*x*(a*e^2 + c*d^2 - b*d*e)^2)*((b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) + log((2^(1/3)*(-(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3)*((2^(2/3)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a^3*c*e^2 + a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*b*c*d*e) - (27*2^(1/3)*a^10*b*c^3*(4*a*c - b^2)^2*(-(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(

```
-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b
^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*
c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3)
)^(1/3))/6 + 36*a^9*c^6*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a
^8*b^2*c^5*d^3 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^
4*d*e^2))/18 + a^7*c^4*e*x*(a*e^2 + c*d^2 - b*d*e)^2)*((b^7*d^3 - a^3*b^4*e
^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 +
 a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 4
8*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^
3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3
*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a^2*b*c*
d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c +
48*a^6*b^2*c^2)))^(1/3) - log((2^(1/3)*(3^(1/2)*1i - 1)*(-(b^7*d^3 - a^3*b^
4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^
3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2
+ 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^
(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(
1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3
*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b
*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3)*(36*a^9*c^6
*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5*d^3 - (2^(2/
3)*(3^(1/2)*1i + 1)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a^3*c*e^2 + a^
2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*b*c*d*e) -
(27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^7*d^3 - a^3*b^
4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^
3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2
+ 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^
(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(
1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3
*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b
*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^7*
d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*
a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^
2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*
c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c
 - b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^
2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2
*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(1/3)
)/12 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^4*d*e^2))/
```

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36 + a^7*c^4*e*x*(a*e^2 + c*d^2 - b*d*e)^2)*((3^(1/2)*1i)/2 + 1/2)*((b^7*d^
3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^
3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*
b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c
- b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*
e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e
 + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12
*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) - log((2^(1/3)*(3^(1/2)*1i - 1)*(-(b^7
*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32
*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a
^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d
^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^
2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3
)*(36*a^9*c^6*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5
*d^3 - (2^(2/3)*(3^(1/2)*1i + 1)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a
^3*c*e^2 + a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^
2*b*c*d*e) - (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^7
*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32
*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a
^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d
^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^
2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3
))/4)*(-(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*
c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2
*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^
2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c
*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27
*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^
3*b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b
^2)^3))^(1/3))/12 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2
*c^4*d*e^2))/36 + a^7*c^4*e*x*(a*e^2 + c*d^2 - b*d*e)^2)*((3^(1/2)*1i)/2 +
1/2)*((b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^
2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c
*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*
d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d
^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a
```

```
^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*
b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64
*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) + log(a^7*c^4*e*x*(a*e^2
+ c*d^2 - b*d*e)^2 - (2^(1/3)*(3^(1/2)*1i + 1)*(-(b^7*d^3 - a^3*b^4*e^3 + b
^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b
*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*
c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 1
0*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*
a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*
e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b
^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*
(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3)*(36*a^9*c^6*d^3 - 10
8*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5*d^3 + (2^(2/3)*(3^(1/
2)*1i - 1)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a^3*c*e^2 + a^2*b^2*e^2
 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*b*c*d*e) + (27*2^(1/
3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b^7*d^3 - a^3*b^4*e^3 + b
^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b
*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*
c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 1
0*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*
a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*
e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b
^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*
(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^7*d^3 - a^3
*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3
*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e
^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3
)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*
a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^
2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(1/3))/12 + 10
8*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^4*d*e^2))/36)*((3^(
1/2)*1i)/2 - 1/2)*((b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2
) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2)
+ 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3
 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e
- 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a
^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*
(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) + log(a^7*c^
4*e*x*(a*e^2 + c*d^2 - b*d*e)^2 - (2^(1/3)*(3^(1/2)*1i + 1)*(-(b^7*d^3 - a^
3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^
```

```
3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*
e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24
*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a
^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3)*(36*a^9
*c^6*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5*d^3 + (2
^(2/3)*(3^(1/2)*1i - 1)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a^3*c*e^2
+ a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*b*c*d*e
) + (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b^7*d^3 - a^
3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^
3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*
e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24
*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a
^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(
b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 -
 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 +
3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(
4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4
*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*
c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2
*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(
1/3))/12 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^4*d*e^
2))/36)*((3^(1/2)*1i)/2 - 1/2)*((b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b
^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^
2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3
*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^
2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3
)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3)
 - d/(a*x)
```

3.19 $\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 655

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx$$

$$= -\frac{d}{2ax^2}+\frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

```
[Out] -1/2*d/a/x^2-1/6*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))
*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b-(-4*a*c+b^2)^(1/2))^(2/3)
+1/12*c^(2/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2
))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*
2^(2/3)/a/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/6*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*
c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(d+(-2*a*e+b*d)/(-4*a*c+b^
2)^(1/2))*2^(2/3)/a*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*c^(2/3)*ln(2^(
1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1
/2))*2^(2/3)/a/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/12*c^(2/3)*ln(2^(2/3)*c^(2/3)
*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^
```

```
(2/3))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b+(-4*a*c+b^2)^(1/2))^
(2/3)+1/6*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^
(1/3))*3^(1/2))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a*3^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1518, 1436, 206, 31, 648, 631, 210, 642}

$$\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx$$

$$= \frac{c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}} + \frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}} - \frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}} - \frac{d}{2ax^2}$$

```
[In] Int[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/2*d/(a*x^2) + (c^(2/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sq
rt[3]*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d - (b*d - 2*a*e)/Sqrt[b
^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3
))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(
d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^
(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(d
 - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(
1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d
+ (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1
/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1
/3)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d - (b*d - 2*a*e)/Sqrt[b^2
 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^
2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b + Sqrt[b^2 - 4*
a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{d}{2ax^2} - \frac{\int \frac{2(bd-ae)+2cdx^3}{a+bx^3+cx^6}\,dx}{2a} \\
&= -\frac{d}{2ax^2} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}
\end{aligned}$$

$$= -\frac{d}{2ax^2} - \frac{\left(c\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(c\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(c\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(c\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= -\frac{d}{2ax^2} - \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{\left(c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int\frac{-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{2\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$+ \frac{\left(c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int\frac{-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+2c^{2/3}x}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int\frac{1}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}dx}{2\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$= -\frac{d}{2ax^2} - \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{\left(c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}\right)}{\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= -\frac{d}{2ax^2} + \frac{c^{2/3}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{c^{2/3}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{c^{2/3}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$- \frac{c^{2/3}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{c^{2/3}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b-\sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-4ac}}x + 2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$+ \frac{c^{2/3}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\left(b+\sqrt{b^2-4ac}\right)^{2/3} - \sqrt[3]{2}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x + 2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.14

$$\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx$$

$$= -\frac{d}{2ax^2} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&, \frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3a}$$

```
[In] Integrate[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/2*d/(a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log
[x - #1] + c*d*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.10

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6} c+\_Z^{3} b+a\right)}\frac{\left(-\_R^{3} c d+a e-b d\right) \ln\left(x-\_R\right)}{2\_R^{5} c+\_R^{2} b}}{3 a}-\frac{d}{2 a \,x^{2}}$
risch	$-\frac{d}{2 a \,x^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64 c^{3} a^{8}-48 b^{2} c^{2} a^{7}+12 b^{4} c \,a^{6}-b^{6} a^{5}\right) \_Z^{6}+\left(16 a^{5} b \,c^{2} e^{3}+48 a^{5} c^{3} d \,e^{2}-8 a^{4} b^{3} c \,e^{3}-72 a^{4} b^{2} c^{2} d \,e^{2}-96 a^{4} b \,c^{3} d^{2} e-16 a^{4} c^{4}\right.\right.}$

```
[In] int((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a*sum((-_R^3*c*d+a*e-b*d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_
Z^3*b+a))-1/2*d/a/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11459 vs. 2(517) = 1034.

Time = 18.76 (sec) , antiderivative size = 11459, normalized size of antiderivative = 17.49

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx=\text{Timed out}$$

```
[In] integrate((e*x**3+d)/x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx=\int \frac{ex^3+d}{(cx^6+bx^3+a)x^3}\,dx$$

```
[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*d*x^3 + b*d - a*e)/(c*x^6 + b*x^3 + a), x)/a - 1/2*d/(a*x^2)
```

Giac [F]

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx=\int \frac{ex^3+d}{(cx^6+bx^3+a)x^3}\,dx$$

```
[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 35.79 (sec) , antiderivative size = 13466, normalized size of antiderivative = 20.56

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx=\text{Too large to display}$$

```
[In] int((d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x)
```

```
[Out] log(- (2^(2/3)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c
- b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3
- 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^
3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4
*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a
^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))
^(1/3)*((2^(1/3)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b*e - b^2*d + a*c*d) + (8
1*2^(2/3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d
^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3
+ 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2
 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)
^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^
```

(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^
3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a
^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/
(a^5*(4*a*c - b^2)^3))^(1/3))/2)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 +
b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*
a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 4
1*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2
) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^
4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) +
3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b
^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5
*(4*a*c - b^2)^3))^(2/3))/18 + 36*a^10*c^5*e^3 + 72*a^8*b*c^6*d^3 - 108*a^9
*c^6*d^2*e + 9*a^6*b^5*c^4*d^3 - 54*a^7*b^3*c^5*d^3 - 9*a^9*b^2*c^4*e^3 - 1
08*a^9*b*c^5*d*e^2 - 27*a^7*b^4*c^4*d^2*e + 135*a^8*b^2*c^5*d^2*e + 27*a^8*
b^3*c^4*d*e^2))/6 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*
d^3*e + 3*a*b^2*d^2*e^2 - 4*a^2*b*d*e^3))*(-(b^8*d^3 - a^3*b^5*e^3 + 16*a^4
*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^
2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3
*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e -
27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e
+ 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(
1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3) +
log(- (2^(2/3)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c
- b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c
- b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3
- 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^
3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4
*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a
^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))
^(1/3)*((2^(1/3)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b*e - b^2*d + a*c*d) + (8
1*2^(2/3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d
^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3
- 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2
+ 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)

^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))/2)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/18 + 36*a^10*c^5*e^3 + 72*a^8*b*c^6*d^3 - 108*a^9*c^6*d^2*e + 9*a^6*b^5*c^4*d^3 - 54*a^7*b^3*c^5*d^3 - 9*a^9*b^2*c^4*e^3 - 108*a^9*b*c^5*d*e^2 - 27*a^7*b^4*c^4*d^2*e + 135*a^8*b^2*c^5*d^2*e + 27*a^8*b^3*c^4*d*e^2))/6 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*d^3*e + 3*a*b^2*d^2*e^2 - 4*a^2*b*d*e^3))*(-(b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3) - d/(2*a*x^2) + log((2^(2/3)*(3^(1/2)*1i - 1)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(108*a^9*c^6*d^2*e - 72*a^8*b*c^6*d^3 - 36*a^10*c^5*e^3 + (2^(1/3)*(3^(1/2)*1i + 1)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b*e - b^2*d + a*c*d) + (81*2^(2/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3

```
)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3
*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*
b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c
 - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d
^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
 - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))/
4)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1
/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2
*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*
d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e
 + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9
*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/36 -
 9*a^6*b^5*c^4*d^3 + 54*a^7*b^3*c^5*d^3 + 9*a^9*b^2*c^4*e^3 + 108*a^9*b*c^5
*d*e^2 + 27*a^7*b^4*c^4*d^2*e - 135*a^8*b^2*c^5*d^2*e - 27*a^8*b^3*c^4*d*e^
2))/12 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*d^3*e + 3*a
*b^2*d^2*e^2 - 4*a^2*b*d*e^3))*((3^(1/2)*1i)/2 - 1/2)*(-(b^8*d^3 - a^3*b^5*
e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 -
 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2
- 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(
-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b
^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4
*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^
3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
)))^(1/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*
c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2
*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*
d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^
2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2
)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e -
27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3
)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e +
 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b
^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1
/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(108*a^9*c^6*d^2*e - 72*a^8*b*c^6*d^3 - 3
6*a^10*c^5*e^3 + (2^(1/3)*(3^(1/2)*1i + 1)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a
```

*b*e - b^2*d + a*c*d) + (81*2^(2/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^
2)^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b
^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^
7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c -
b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2
*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c -
b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2
e(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))/4)
*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c
^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^
2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e -
5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a
^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/36 - 9
*a^6*b^5*c^4*d^3 + 54*a^7*b^3*c^5*d^3 + 9*a^9*b^2*c^4*e^3 + 108*a^9*b*c^5*d
*e^2 + 27*a^7*b^4*c^4*d^2*e - 135*a^8*b^2*c^5*d^2*e - 27*a^8*b^3*c^4*d*e^2)
)/12 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*d^3*e + 3*a*b
^2*d^2*e^2 - 4*a^2*b*d*e^3))*((3^(1/2)*1i)/2 - 1/2)*(-(b^8*d^3 - a^3*b^5*e^
3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 1
6*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 -
48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-
(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(
4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5
*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*
c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
- 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c -
b^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2))
)^(1/3) - log((2^(2/3)*(3^(1/2)*1i + 1)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^
4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e
^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*
e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)
^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27
*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 7
2*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2
*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2

```
))/(a^5*(4*a*c - b^2)^3))^(1/3)*(36*a^10*c^5*e^3 + 72*a^8*b*c^6*d^3 - 108*a
^9*c^6*d^2*e + (2^(1/3)*(3^(1/2)*1i - 1)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b
*e - b^2*d + a*c*d) - (81*2^(2/3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)
^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1
/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2
*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*
d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b
^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e
 + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9
*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))/4)*(
(b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2)
+ 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
 + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3
*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*
e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5
*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3
*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/36 + 9*a
^6*b^5*c^4*d^3 - 54*a^7*b^3*c^5*d^3 - 9*a^9*b^2*c^4*e^3 - 108*a^9*b*c^5*d*e
^2 - 27*a^7*b^4*c^4*d^2*e + 135*a^8*b^2*c^5*d^2*e + 27*a^8*b^3*c^4*d*e^2))/
12 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*d^3*e + 3*a*b^2
*d^2*e^2 - 4*a^2*b*d*e^3))*((3^(1/2)*1i)/2 + 1/2)*(-(b^8*d^3 - a^3*b^5*e^3
+ 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*
a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48
*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4
*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*
a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c
*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c
 - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^
2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b
^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^
(1/3) - log((2^(2/3)*(3^(1/2)*1i + 1)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*
d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3
 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^
2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)
^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a
^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1
/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*
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a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c
*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))
/(a^5*(4*a*c - b^2)^3))^(1/3)*(36*a^10*c^5*e^3 + 72*a^8*b*c^6*d^3 - 108*a^9
*c^6*d^2*e + (2^(1/3)*(3^(1/2)*1i - 1)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b*e
 - b^2*d + a*c*d) - (81*2^(2/3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2
*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c
^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^
2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e -
 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(
-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a
^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))/4)*((b
^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) +
8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) +
 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d
^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e
+ 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a
^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*
a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b
*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/36 + 9*a^6
*b^5*c^4*d^3 - 54*a^7*b^3*c^5*d^3 - 9*a^9*b^2*c^4*e^3 - 108*a^9*b*c^5*d*e^2
 - 27*a^7*b^4*c^4*d^2*e + 135*a^8*b^2*c^5*d^2*e + 27*a^8*b^3*c^4*d*e^2))/12
 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*d^3*e + 3*a*b^2*d
^2*e^2 - 4*a^2*b*d*e^3))*((3^(1/2)*1i)/2 + 1/2)*(-(b^8*d^3 - a^3*b^5*e^3 +
16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^
5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a
^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a
*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*
c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d
^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c -
 b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*
d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 1
2*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2
)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1
/3)
```

3.20 $\int \frac{x^8(1-x^3)}{1-x^3+x^6} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6} \, dx = -\frac{x^6}{6} - \frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] -1/6*x^6+1/6*ln(x^6-x^3+1)-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1488, 814, 648, 632, 210, 642}

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6} \, dx = -\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{x^6}{6} + \frac{1}{6}\log\left(x^6-x^3+1\right)$$

```
[In] Int[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -1/6*x^6 - ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(1-x)x^2}{1-x+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-x+\frac{x}{1-x+x^2}\right)\,dx, x, x^3\right)\\
&= -\frac{x^6}{6}+\frac{1}{3}\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx, x, x^3\right)\\
&= -\frac{x^6}{6}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)+\frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)\\
&= -\frac{x^6}{6}+\frac{1}{6}\log\left(1-x^3+x^6\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= -\frac{x^6}{6}-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^6}{6}+\frac{\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[In] Integrate[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] -1/6*x^6 + ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{x^6}{6}+\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	38
risch	$-\frac{x^6}{6}+\frac{\ln(4x^6-4x^3+4)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	40

```
[In] int(x^8*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/6*x^6+1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{6}\,x^6+\frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)+\frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

```
[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
+ 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^6}{6}+\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate(x**8*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**6/6 + log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{6}\,x^6+\frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)+\frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

```
[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
+ 1)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{6}\,x^6+\frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)+\frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

```
[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
+ 1)
```

Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx=\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}-\frac{x^6}{6}$$

```
[In] int(-(x^8*(x^3 - 1))/(x^6 - x^3 + 1),x)
[Out] log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - x^
6/6
```

3.21 $\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 31

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^3}{3} - \frac{2\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -1/3*x^3-2/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1488, 787, 632, 210}

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx = -\frac{2\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{x^3}{3}$$

```
[In] Int[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -1/3*x^3 - (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(1-x)x}{1-x+x^2}\,dx, x, x^3\right)\\
&= -\frac{x^3}{3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)\\
&= -\frac{x^3}{3} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= -\frac{x^3}{3} - \frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^3}{3} + \frac{2\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[In] Integrate[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] -1/3*x^3 + (2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{x^3}{3}+\frac{2\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	25
risch	$-\frac{x^3}{3}+\frac{2\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	25

```
[In] int(x^5*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/3*x^3+2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx=-\frac{1}{3}\,x^3+\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)$$

```
[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx=-\frac{x^3}{3}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate(x**5*(-x**3+1)/(x**6-x**3+1),x)
[Out] -x**3/3 + 2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\,x^3+\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)$$

```
[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\,x^3+\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)$$

```
[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx = -\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}-\frac{x^3}{3}$$

```
[In] int(-(x^5*(x^3 - 1))/(x^6 - x^3 + 1),x)
[Out] - (2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - x^3/3
```

3.22 $\int \frac{x^2(1-x^3)}{1-x^3+x^6} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6} \, dx = -\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] -1/6*ln(x^6-x^3+1)-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1482, 648, 632, 210, 642}

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6} \, dx = -\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6}\log\left(x^6-x^3+1\right)$$

```
[In] Int[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] -1/3*ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] - Log[1 - x^3 + x^6]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right) - \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)\\
&= -\frac{1}{6}\log\left(1-x^3+x^6\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = \frac{\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[In] Integrate[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) - Log[1 - x^3 + x^6]/6
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	33
risch	$-\frac{\ln(4x^6-4x^3+4)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	35

```
[In] int(x^2*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)$$

```
[In] integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = -\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate(x**2*(-x**3+1)/(x**6-x**3+1),x)
[Out] -log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = \frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right) - \frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

```
[In] integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = \frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right) - \frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

```
[In] integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx = -\frac{\ln\left(x^6-x^3+1\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}$$

```
[In] int(-(x^2*(x^3 - 1))/(x^6 - x^3 + 1),x)
[Out] - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

3.23 $\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 41

$$\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx=\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)+1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1488, 814, 648, 632, 210, 642}

$$\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx=\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

```
[In] Int[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1-x}{x\left(1-x+x^2\right)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{3}\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)-\frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{1-x^3}{x(1-x^3+x^6)}\,dx = \log(x) - \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&, \frac{\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

```
[In] Integrate[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^3) & ]/3
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln(x) - \frac{\ln(x^6-x^3+1)}{6} - \frac{\sqrt{3}\arctan\left(\frac{2\left(x^3-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{9}$	33
default	$-\frac{\ln(x^6-x^3+1)}{6} - \frac{\sqrt{3}\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9} + \ln(x)$	35

```
[In] int((-x^3+1)/x/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] ln(x)-1/6*ln(x^6-x^3+1)-1/9*3^(1/2)*arctan(2/3*(x^3-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1-x^3}{x(1-x^3+x^6)}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log(x)$$

```
[In] integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1-x^3}{x(1-x^3+x^6)}\,dx = \log(x) - \frac{\log(x^6-x^3+1)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

```
[In] integrate((-x**3+1)/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/9
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1-x^3}{x(1-x^3+x^6)}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \frac{1}{3}\log\left(x^3\right)$$

```
[In] integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3
*log(x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{1-x^3}{x(1-x^3+x^6)}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log\left(|x|\right)$$

```
[In] integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(abs(x))
```

Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx = \ln\left(x\right)-\frac{\ln\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}$$

```
[In] int(-(x^3 - 1)/(x*(x^6 - x^3 + 1)),x)
[Out] log(x) - log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/9
```

3.24 $\int \frac{1-x^3}{x^4(1-x^3+x^6)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 31

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx=-\frac{1}{3x^3}+\frac{2\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -1/3/x^3+2/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1488, 814, 632, 210}

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx=\frac{2\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3x^3}$$

```
[In] Int[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/3*1/x^3 + (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1-x}{x^2\left(1-x+x^2\right)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{1}{-1+x-x^2}\right)\,dx, x, x^3\right)\\ &= -\frac{1}{3x^3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+x-x^2}\,dx, x, x^3\right)\\ &= -\frac{1}{3x^3}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-2x^3\right)\\ &= -\frac{1}{3x^3}+\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx = -\frac{1}{3x^3}-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&, \frac{\log(x-\#1)}{-1+2\#1^3}\&\right]$$

```
[In] Integrate[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]
[Out] -1/3*1/x^3 - RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-1 + 2*#1^3) & ]/3
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^{3}}$	25
risch	$-\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^{3}}$	25

```
[In] int((-x^3+1)/x^4/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] -2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))-1/3/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx=-\frac{2\sqrt{3}x^3\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^3-1)\right)+3}{9\,x^3}$$

```
[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/9*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 3)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx=-\frac{2\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$$

```
[In] integrate((-x**3+1)/x**4/(x**6-x**3+1),x)
[Out] -2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9 - 1/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx=-\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\big(2\,x^3-1\big)\right)-\frac{1}{3\,x^3}$$

```
[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="maxima")
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx=-\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\big(2\,x^3-1\big)\right)-\frac{1}{3\,x^3}$$

```
[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="giac")
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3
```

Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx=\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}-\frac{1}{3\,x^3}$$

```
[In] int(-(x^3 - 1)/(x^4*(x^6 - x^3 + 1)),x)
[Out] (2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - 1/(3*x^3)
```

3.25 $\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 418

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx=-\frac{x^4}{4}-\frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] -1/4*x^4+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(
1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(
3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I
*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2
/3)+1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1
/2))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^
(2/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/6*arctan(1/3*(1+2*2^(1/3
)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1-I*3^(1/2))^(2/3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1516, 12, 1388, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = -\frac{\left(\sqrt{3}+i\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(-\sqrt{3}+i\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{x^4}{4}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[In] Int[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -1/4*x^4 - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt
[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/
((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((
3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sq
rt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9
*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3
) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])
```

```
^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^
(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^4}{4} - \frac{1}{4}\int -\frac{4x^3}{1-x^3+x^6}\,dx \\
&= -\frac{x^4}{4} + \int \frac{x^3}{1-x^3+x^6}\,dx \\
&= -\frac{x^4}{4} - \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx + \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx \\
&= -\frac{x^4}{4} + \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} \\
&\quad + \frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} \\
&\quad + \frac{\left(3+i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} \\
&\quad + \frac{\left(3+i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}
\end{aligned}
$$

$$= -\frac{x^4}{4} + \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$- \frac{\left(3-i\sqrt{3}\right)\int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}dx}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$- \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}dx}{6\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$- \frac{\left(3+i\sqrt{3}\right)\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}dx}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$- \frac{\left(3+i\sqrt{3}\right)\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}dx}{6\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$= -\frac{x^4}{4} + \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$- \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$- \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{\left(3-i\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{\left(3+i\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$= -\frac{x^4}{4} - \frac{(i+\sqrt{3})\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{(i-\sqrt{3})\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$- \frac{(3+i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$- \frac{(3-i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.11

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^4}{4} + \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1}{-1+2\#1^3}\&\right]$$

```
[In] Integrate[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] -1/4*x^4 + RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.11

method	result	size
default	$-\frac{x^4}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	46
risch	$-\frac{x^4}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	46

```
[In] int(x^6*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/4*x^4+1/3*sum(_R^3/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.64

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{4}\,x^4 - \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\sqrt{3}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(i\sqrt{-3}+i\right)+36\,x\right) + \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\sqrt{3}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(i\sqrt{-3}-i\right)+36\,x\right) + \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\sqrt{3}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(-i\sqrt{-3}+i\right)+36\,x\right) - \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\sqrt{3}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(-i\sqrt{-3}-i\right)+36\,x\right) + \frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(-i\cdot 18^{\frac{2}{3}}\sqrt{3}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}+18\,x\right) + \frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(i\cdot 18^{\frac{2}{3}}\sqrt{3}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}+18\,x\right)$$

```
[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/4*x^4 - 1/108*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)
*sqrt(3)*(I*sqrt(3) - 3)^(1/3)*(I*sqrt(-3) + I) + 36*x) + 1/108*18^(2/3)*(-
I*sqrt(3) - 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*sqrt(3)*(-I*sqrt(3) - 3)^(
1/3)*(I*sqrt(-3) - I) + 36*x) + 1/108*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(
-3) - 1)*log(18^(2/3)*sqrt(3)*(I*sqrt(3) - 3)^(1/3)*(-I*sqrt(-3) + I) + 36*
x) - 1/108*18^(2/3)*(-I*sqrt(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*sqrt
(3)*(-I*sqrt(3) - 3)^(1/3)*(-I*sqrt(-3) - I) + 36*x) + 1/54*18^(2/3)*(I*sqr
t(3) - 3)^(1/3)*log(-I*18^(2/3)*sqrt(3)*(I*sqrt(3) - 3)^(1/3) + 18*x) + 1/5
4*18^(2/3)*(-I*sqrt(3) - 3)^(1/3)*log(I*18^(2/3)*sqrt(3)*(-I*sqrt(3) - 3)^(
1/3) + 18*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.07

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^4}{4} - \operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(-1458t^4 + 9t + x\right)\right)\right)$$

```
[In] integrate(x**6*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**4/4 - RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**
4 + 9*_t + x)))
```

Maxima [F]

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = \int -\frac{(x^3-1)x^6}{x^6-x^3+1}\,dx$$

```
[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^4 + integrate(x^3/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(272) = 544.

Time = 0.32 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.54

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/4*x^4 - 1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*
pi)^2 + 2*sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*p
i)*sin(4/9*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(1/2*((-I*sqrt(
3) - 1)*cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(2*sq
rt(3)*cos(2/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*si
n(2/9*pi)^4 + 8*cos(2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + s
qrt(3)*cos(2/9*pi) + sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi)
+ 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^4
- 12*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(
1/9*pi)^3*sin(1/9*pi) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) +
 sin(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt
(3) + 1/2)*sin(1/9*pi))) - 1/18*(8*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 8*sq
rt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9*pi)^4 + 12*cos(4/9*pi)^2*sin(4/
9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt(3)*sin(4/9*pi) - cos(4/9*pi))*log((-I*sqrt
(3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(8*sqrt(3)*cos(2/9*pi)^3
*sin(2/9*pi) - 8*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - 2*cos(2/9*pi)^4 + 12*c
os(2/9*pi)^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)^4 + sqrt(3)*sin(2/9*pi) - cos(2/
9*pi))*log((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(8*sq
rt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + 2*c
os(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sin(1/9*pi)^4 - sqrt(3)*s
in(1/9*pi) - cos(1/9*pi))*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2
 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.79

$$\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx = \frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{x^4}{4} - \frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

```
[In] int(-(x^6*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*12i -
36)^(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/6)*(3^
(1/2)*12i - 36)^(1/3))/18 - x^4/4 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(- 3^
(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(4/3))/12)*(-
3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3
)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(4/
3))/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x
- (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(1/3))/4 - (2^(2/3)*3^(5/6)*(- 3^(1/2
)*1i - 3)^(1/3)*1i)/12)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/4 + (2^(2/3)*3
^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(
5/6)*1i))/36
```

3.26 $\int \frac{x^4(1-x^3)}{1-x^3+x^6} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 382

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx=-\frac{x^2}{2}+\frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}-\frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] -1/2*x^2+1/3*I*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/
2))/(1-I*3^(1/2))^(1/3)-1/3*I*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2
))^(1/3))*3^(1/2))/(1+I*3^(1/2))^(1/3)+1/9*I*2^(1/3)*ln(-2^(1/3)*x+(1-I*3^(
1/2))^(1/3))/(1-I*3^(1/2))^(1/3)*3^(1/2)-1/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1
-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*2^(1/3)/(1-I*3^(1/2))^(1/3)*3^(1/2)-
1/9*I*2^(1/3)*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))/(1+I*3^(1/2))^(1/3)*3^(1/2
)+1/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*
2^(1/3)/(1+I*3^(1/2))^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1516, 12, 1389, 298, 31, 648, 631, 210, 642}

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = \frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}-\frac{x^2}{2}-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

```
[In] Int[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -1/2*x^2 + ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((
1 - I*Sqrt[3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3
))/Sqrt[3]])/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) -
 2^(1/3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 + I*Sqrt[3
])^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1
- I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*S
qrt[3]*(1 - I*Sqrt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 +
I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sqrt[3])^(1/3))
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
```

```
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2}{2} - \frac{1}{2}\int -\frac{2x}{1-x^3+x^6}\,dx \\
&= -\frac{x^2}{2} + \int \frac{x}{1-x^3+x^6}\,dx \\
&= -\frac{x^2}{2} - \frac{i\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}} + \frac{i\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}} \\
&= -\frac{x^2}{2} + \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} \\
&\quad - \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\int \frac{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}
\end{aligned}$$

$$= -\frac{x^2}{2} + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

$$+ \frac{i\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3} + \sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x + x^2}\,dx}{2\sqrt{3}}$$

$$- \frac{i\int \frac{1}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3} + \sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x + x^2}\,dx}{2\sqrt{3}}$$

$$- \frac{i\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)} + 2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3} + \sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x + x^2}\,dx}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+ \frac{i\int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)} + 2x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3} + \sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x + x^2}\,dx}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= -\frac{x^2}{2} + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

$$- \frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3} + \sqrt[3]{2\left(1-i\sqrt{3}\right)}x + 2^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+ \frac{i\log\left(\left(1+i\sqrt{3}\right)^{2/3} + \sqrt[3]{2\left(1+i\sqrt{3}\right)}x + 2^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}$$

$$- \frac{i\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}$$

$$+ \frac{i\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

$$= -\frac{x^2}{2} + \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$
$$+ \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$
$$- \frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}$$
$$+ \frac{i\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.13

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^2}{2} + \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)}{-\#1+2\#1^4}\&\right]$$

```
[In] Integrate[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] -1/2*x^2 + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.12

method	result	size
default	$-\frac{x^2}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	44
risch	$-\frac{x^2}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	44

```
[In] int(x^4*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/2*x^2+1/3*sum(_R/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.79

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}\left(i\sqrt{-3}-i\right)+\sqrt{-3}-1\right)\left(i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}\left(-i\sqrt{-3}-i\right)-\sqrt{-3}-1\right)\left(i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}\left(i\sqrt{-3}+i\right)-\sqrt{-3}-1\right)\left(-i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{1}{3}}\left(\sqrt{3}\left(-i\sqrt{-3}+i\right)+\sqrt{-3}-1\right)\left(-i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)-\frac{1}{2}\,x^2+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(i\sqrt{3}+3\right)^{\frac{2}{3}}\left(i\sqrt{3}+1\right)+12\,x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(-i\sqrt{3}+3\right)^{\frac{2}{3}}\left(-i\sqrt{3}+1\right)+12\,x\right)$$

```
[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/108*18^(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(1/3)*(sqrt(3)*
(I*sqrt(-3) - I) + sqrt(-3) - 1)*(I*sqrt(3) + 3)^(2/3) + 24*x) + 1/108*18^(
```

```
2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-3
) - I) - sqrt(-3) - 1)*(I*sqrt(3) + 3)^(2/3) + 24*x) + 1/108*18^(2/3)*(-I*s
qrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(1/3)*(sqrt(3)*(I*sqrt(-3) + I) - s
qrt(-3) - 1)*(-I*sqrt(3) + 3)^(2/3) + 24*x) - 1/108*18^(2/3)*(-I*sqrt(3) +
3)^(1/3)*(sqrt(-3) + 1)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-3) + I) + sqrt(-3)
- 1)*(-I*sqrt(3) + 3)^(2/3) + 24*x) - 1/2*x^2 + 1/54*18^(2/3)*(I*sqrt(3) +
3)^(1/3)*log(18^(1/3)*(I*sqrt(3) + 3)^(2/3)*(I*sqrt(3) + 1) + 12*x) + 1/54*
18^(2/3)*(-I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(-I*sqrt(3) + 3)^(2/3)*(-I*sqr
t(3) + 1) + 12*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.08

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = -\frac{x^2}{2} - \operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(-6561t^5-27t^2+x\right)\right)\right)$$

```
[In] integrate(x**4*(-x**3+1)/(x**6-x**3+1),x)
[Out] -x**2/2 - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t**
5 - 27*_t**2 + x)))
```

Maxima [F]

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = \int -\frac{(x^3-1)x^4}{x^6-x^3+1}\,dx$$

```
[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
[Out] -1/2*x^2 + integrate(x/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 820 vs. 2(246) = 492.

Time = 0.35 (sec) , antiderivative size = 820, normalized size of antiderivative = 2.15

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/2*x^2 - 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi
)^2 + 5*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 1
0*cos(4/9*pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 - sqrt(3)*cos(4/9*pi)^2 + sqr
t(3)*sin(4/9*pi)^2 + 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1
)*cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*co
s(2/9*pi)^5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi
)*sin(2/9*pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi
)^3 - sin(2/9*pi)^5 - sqrt(3)*cos(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^2 + 2*cos
(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2
*I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*c
os(1/9*pi)^3*sin(1/9*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/
9*pi)^4*sin(1/9*pi) - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 + sqrt
(3)*cos(1/9*pi)^2 - sqrt(3)*sin(1/9*pi)^2 + 2*cos(1/9*pi)*sin(1/9*pi))*arct
an(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(1/9
*pi))) - 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)
^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^3
*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 - 2*sqrt(3)*cos(4/9*pi)*sin(4/
9*pi) - cos(4/9*pi)^2 + sin(4/9*pi)^2)*log((-I*sqrt(3)*cos(4/9*pi) - cos(4/
9*pi))*x + x^2 + 1) - 1/18*(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3
)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5 - 10*
cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*cos(2/9*pi)*sin(2/9*pi)^4 - 2*sqrt(3)*cos(2
/9*pi)*sin(2/9*pi) - cos(2/9*pi)^2 + sin(2/9*pi)^2)*log((-I*sqrt(3)*cos(2/9
*pi) - cos(2/9*pi))*x + x^2 + 1) - 1/18*(5*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi
) - 10*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5 - cos(1/
9*pi)^5 + 10*cos(1/9*pi)^3*sin(1/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*pi)^4 + 2*
sqrt(3)*cos(1/9*pi)*sin(1/9*pi) - cos(1/9*pi)^2 + sin(1/9*pi)^2)*log((I*sqr
t(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 10.57 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.81

$$\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx = \frac{\ln\left(x+\left(81\,x-\frac{27\left(36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(x-\left(81\,x-\frac{27\left(36+\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{4}\right)\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{x^2}{2} - \frac{2^{2/3}\ln\left(x+\frac{2^{1/3}\,3^{2/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,1\mathrm{i}}{4}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x+\frac{2^{1/3}\,3^{2/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,1\mathrm{i}}{4}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\,3^{2/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x-\frac{2^{1/3}\,3^{2/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

```
[In] int(-(x^4*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (81*x - (27*(36 - 3^(1/2)*12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162
))*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x - (81*x - (27*(3^(1/2)*12i + 36)^(
2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(3^(1/2)*12i + 36)^(1/3))/18 - x^2/2 -
 (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 + (2^(1/3)*3^
(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/
6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/12 -
 (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/3)*(3^(
1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)
^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log
(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)^(1/3)*(3^
(1/3) + 3^(5/6)*1i))/36
```

3.27 $\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 378

$$\begin{aligned}\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx &= -x-\frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\\
&+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}\\
&-\frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}\\
&+\frac{i\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}\end{aligned}$$

```
[Out] -x-1/3*I*2^(2/3)*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))/(1
-I*3^(1/2))^(2/3)+1/3*I*2^(2/3)*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/
3))*3^(1/2))/(1+I*3^(1/2))^(2/3)+1/9*I*2^(2/3)*ln(-2^(1/3)*x+(1-I*3^(1/2))^
(1/3))/(1-I*3^(1/2))^(2/3)*3^(1/2)-1/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(
1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*2^(2/3)/(1-I*3^(1/2))^(2/3)*3^(1/2)-1/9*I*
2^(2/3)*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))/(1+I*3^(1/2))^(2/3)*3^(1/2)+1/18
*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*2^(2/3
)/(1+I*3^(1/2))^(2/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1516, 1361, 206, 31, 648, 631, 210, 642}

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = -\frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}-x+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

```
[In] Int[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x - ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*
Sqrt[3])/2)^(2/3) + ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqr
t[3]])/((1 + I*Sqrt[3])/2)^(2/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/
3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/
3) - 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sq
rt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]
*(1 - I*Sqrt[3])^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt
[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\text{integral} = -x + \int \frac{1}{1-x^3+x^6}\,dx$$

$$= -x - \frac{i\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}} + \frac{i\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$

$$= -x + \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} + \frac{i\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}$$

$$- \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}} - \frac{i\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

$$= -x + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

$$- \frac{i\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$- \frac{i\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+ \frac{i\int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{i\int \frac{1}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= -x + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

$$- \frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3} + \sqrt[3]{2\left(1-i\sqrt{3}\right)}x + 2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{i\log\left(\left(1+i\sqrt{3}\right)^{2/3} + \sqrt[3]{2\left(1+i\sqrt{3}\right)}x + 2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{i\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}$$

$$- \frac{i\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 + \frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

$$= -x - \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} + \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

$$+ \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

$$- \frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3} + \sqrt[3]{2\left(1-i\sqrt{3}\right)}x + 2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{i\log\left(\left(1+i\sqrt{3}\right)^{2/3} + \sqrt[3]{2\left(1+i\sqrt{3}\right)}x + 2^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.12

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = -x + \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)}{-\#1^2+2\#1^5}\&\right]$$

```
[In] Integrate[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] -x + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) & ]/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.11

method	result	size
default	$-x+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	41
risch	$-x+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	41

```
[In] int(x^3*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] -x+1/3*sum(1/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.76

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}(\sqrt{-3}-1)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(i\sqrt{-3}-i)+3\sqrt{-3}-3\right)\left(i\sqrt{3}+3\right)^{\frac{1}{3}}+72x\right) - \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}(\sqrt{-3}+1)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(-i\sqrt{-3}-i)-3\sqrt{-3}-3\right)\left(i\sqrt{3}+3\right)^{\frac{1}{3}}+72x\right) - \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}(\sqrt{-3}+1)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(i\sqrt{-3}+i)-3\sqrt{-3}-3\right)\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}+72x\right) + \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}(\sqrt{-3}-1)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(-i\sqrt{-3}+i)+3\sqrt{-3}-3\right)\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}+72x\right) + \frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)^{\frac{4}{3}}+36x\right) + \frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)^{\frac{4}{3}}+36x\right) - x$$

```
[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*18^(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*(sqrt(3)*(
I*sqrt(-3) - I) + 3*sqrt(-3) - 3)*(I*sqrt(3) + 3)^(1/3) + 72*x) - 1/108*18^
(2/3)*(I*sqrt(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*(sqrt(3)*(-I*sqrt(-
3) - I) - 3*sqrt(-3) - 3)*(I*sqrt(3) + 3)^(1/3) + 72*x) - 1/108*18^(2/3)*(-
I*sqrt(3) + 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*(sqrt(3)*(I*sqrt(-3) + I)
- 3*sqrt(-3) - 3)*(-I*sqrt(3) + 3)^(1/3) + 72*x) + 1/108*18^(2/3)*(-I*sqrt(
3) + 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*(sqrt(3)*(-I*sqrt(-3) + I) + 3*sq
rt(-3) - 3)*(-I*sqrt(3) + 3)^(1/3) + 72*x) + 1/54*18^(2/3)*(I*sqrt(3) + 3)^
(1/3)*log(18^(2/3)*(I*sqrt(3) + 3)^(4/3) + 36*x) + 1/54*18^(2/3)*(-I*sqrt(3
) + 3)^(1/3)*log(18^(2/3)*(-I*sqrt(3) + 3)^(4/3) + 36*x) - x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.06

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = -x - \operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(729t^4+x\right)\right)\right)$$

```
[In] integrate(x**3*(-x**3+1)/(x**6-x**3+1),x)
[Out] -x - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + x))
)
```

Maxima [F]

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = \int -\frac{(x^3-1)x^3}{x^6-x^3+1}\,dx$$

```
[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
[Out] -x + integrate(1/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 635 vs. 2(244) = 488.

Time = 0.37 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.68

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^
3 - sqrt(3)*cos(4/9*pi) - sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(4/9
*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)
^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(
2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 - sqrt(3)*cos(2/9*pi) -
 sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*I*sqrt(
3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi
)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4
*cos(1/9*pi)*sin(1/9*pi)^3 + sqrt(3)*cos(1/9*pi) - sin(1/9*pi))*arctan(-1/2
*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(1/9*pi)))
- 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9
```

```
*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 - sq
rt(3)*sin(4/9*pi) + cos(4/9*pi))*log((-I*sqrt(3)*cos(4/9*pi) - cos(4/9*pi))
*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2
/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(
2/9*pi)^4 - sqrt(3)*sin(2/9*pi) + cos(2/9*pi))*log((-I*sqrt(3)*cos(2/9*pi)
- cos(2/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4
*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/
9*pi)^2 + sin(1/9*pi)^4 + sqrt(3)*sin(1/9*pi) + cos(1/9*pi))*log((I*sqrt(3)
*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - x
```

Mupad [B] (verification not implemented)

Time = 10.55 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.87

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx$$
$$= -x + \frac{\ln\left(x + \frac{2^{2/3}\,3^{1/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4} - \frac{2^{2/3}\,3^{5/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$+ \frac{\ln\left(x + \frac{2^{2/3}\,3^{1/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4} + \frac{2^{2/3}\,3^{5/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}$$
$$- \frac{2^{2/3}\,\ln\left(x - \frac{2^{2/3}\,3^{1/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2} + \frac{2^{2/3}\,3^{1/3}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$- \frac{2^{2/3}\,\ln\left(x - \frac{2^{2/3}\,3^{1/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2} + \frac{2^{2/3}\,3^{1/3}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$- \frac{2^{2/3}\,\ln\left(x + \frac{2^{2/3}\,3^{5/6}\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$- \frac{2^{2/3}\,\ln\left(x - \frac{2^{2/3}\,3^{5/6}\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

```
[In] int(-(x^3*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3 -
 3^(1/2)*1i)^(1/3)*1i)/12)*(36 - 3^(1/2)*12i)^(1/3))/18 - x + (log(x + (2^(
2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^
(1/3)*1i)/12)*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*3^(1
```

```
/3)*(3 - 3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/12
)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(
2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^
(4/3))/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log
(x + (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(1/3)*
(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i
+ 3)^(1/3)*1i)/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

3.28 $\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 411

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = \frac{(i-\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(i+\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{(3-i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{(3+i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] 1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2^(
1/3)/(1-I*3^(1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3-I*3^(1/
2))*2^(1/3)/(1-I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))
^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/18*
ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3
)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3
+I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*
3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1+I*3^(1/2))^(1/3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1524, 298, 31, 648, 631, 210, 642}

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = \frac{\left(-\sqrt{3}+i\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(\sqrt{3}+i\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[In] Int[(x*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^
(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sqr
t[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3 - I*Sqrt[
3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3
)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1
 + I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 -
```

```
 I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + (
(3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2
^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
```

```
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx - \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{(3-i\sqrt{3})\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{(3-i\sqrt{3})\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$- \frac{(3+i\sqrt{3})\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{(3+i\sqrt{3})\int \frac{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= -\frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$+ \frac{(3-i\sqrt{3})\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+\frac{1}{12}\left(-3+i\sqrt{3}\right)\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx - \frac{1}{12}\left(3+i\sqrt{3}\right)\int \frac{}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2}}$$

$$= -\frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$
$$+ \frac{(3-i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$
$$+ \frac{(3+i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$
$$+ \frac{(3-i\sqrt{3})\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$
$$+ \frac{(3+i\sqrt{3})\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= \frac{(i-\sqrt{3})\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(i+\sqrt{3})\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$
$$- \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$
$$+ \frac{(3-i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$
$$+ \frac{(3+i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.13

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1+2\#1^4}\&\right]$$

```
[In] Integrate[(x*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] -1/3*RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1 + 2
*#1^4) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.11

method	result	size
default	$-\frac{\sum_{\_R=\text{RootOf}(\_Z^6-\_Z^3+1)}\frac{(\_R^4-\_R)\ln(x-\_R)}{2\_R^5-\_R^2}}{3}$	44
risch	$\frac{\sum_{\_R=\text{RootOf}(\_Z^6-\_Z^3+1)}\frac{(-\_R^4+\_R)\ln(x-\_R)}{2\_R^5-\_R^2}}{3}$	44

```
[In] int(x*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/3*sum((_R^4-_R)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.58

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = \frac{1}{108} \cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{1}{3}}\left(i\sqrt{3}-3\right)^{\frac{2}{3}}\left(\sqrt{-3}+1\right)+12\,x\right) + \frac{1}{108} \cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{1}{3}}\left(-i\sqrt{3}-3\right)^{\frac{2}{3}}\left(\sqrt{-3}+1\right) + 12\,x\right) - \frac{1}{108} \cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(-18^{\frac{1}{3}}\left(i\sqrt{3}-3\right)^{\frac{2}{3}}\left(\sqrt{-3}-1\right) + 12\,x\right) - \frac{1}{108} \cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(-18^{\frac{1}{3}}\left(-i\sqrt{3}-3\right)^{\frac{2}{3}}\left(\sqrt{-3}-1\right) + 12\,x\right) + \frac{1}{54} \cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(6\,x-18^{\frac{1}{3}}\left(i\sqrt{3}-3\right)^{\frac{2}{3}}\right) + \frac{1}{54} \cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(6\,x-18^{\frac{1}{3}}\left(-i\sqrt{3}-3\right)^{\frac{2}{3}}\right)$$

```
[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) - 1)*log(18^(1/3)*(I*sqrt(3)
 - 3)^(2/3)*(sqrt(-3) + 1) + 12*x) + 1/108*18^(2/3)*(-I*sqrt(3) - 3)^(1/3)*
(sqrt(-3) - 1)*log(18^(1/3)*(-I*sqrt(3) - 3)^(2/3)*(sqrt(-3) + 1) + 12*x) -
 1/108*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(-18^(1/3)*(I*sqrt(
3) - 3)^(2/3)*(sqrt(-3) - 1) + 12*x) - 1/108*18^(2/3)*(-I*sqrt(3) - 3)^(1/3
)*(sqrt(-3) + 1)*log(-18^(1/3)*(-I*sqrt(3) - 3)^(2/3)*(sqrt(-3) - 1) + 12*x
) + 1/54*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*log(6*x - 18^(1/3)*(I*sqrt(3) - 3)^
(2/3)) + 1/54*18^(2/3)*(-I*sqrt(3) - 3)^(1/3)*log(6*x - 18^(1/3)*(-I*sqrt(3
) - 3)^(2/3))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.05

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = -\operatorname{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(-27t^2+x\right)\right)\right)$$

```
[In] integrate(x*(-x**3+1)/(x**6-x**3+1),x)
[Out] -RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2 + x)))
```

Maxima [F]

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = \int -\frac{(x^3-1)x}{x^6-x^3+1}\,dx$$

```
[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
[Out] -integrate((x^3 - 1)*x/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 824 vs. $2(267) = 534$.

Time = 0.37 (sec) , antiderivative size = 824, normalized size of antiderivative = 2.00

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sqr
t(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9*p
i)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 + 2*sqrt(3)*cos(4/9*pi)^2 - 2*sqrt(3)*si
n(4/9*pi)^2 - 4*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(4
/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*p
i)^5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2
/9*pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - s
in(2/9*pi)^5 + 2*sqrt(3)*cos(2/9*pi)^2 - 2*sqrt(3)*sin(2/9*pi)^2 - 4*cos(2/
9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*I*
sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(
1/9*pi)^3*sin(1/9*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*p
i)^4*sin(1/9*pi) - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 - 2*sqrt(
3)*cos(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^2 - 4*cos(1/9*pi)*sin(1/9*pi))*arc
```

```
tan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(1/
9*pi))) + 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi
)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^
3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 + 4*sqrt(3)*cos(4/9*pi)*sin(4
/9*pi) + 2*cos(4/9*pi)^2 - 2*sin(4/9*pi)^2)*log((-I*sqrt(3)*cos(4/9*pi) - c
os(4/9*pi))*x + x^2 + 1) + 1/18*(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*s
qrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5
- 10*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*cos(2/9*pi)*sin(2/9*pi)^4 + 4*sqrt(3)*
cos(2/9*pi)*sin(2/9*pi) + 2*cos(2/9*pi)^2 - 2*sin(2/9*pi)^2)*log((-I*sqrt(3
)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(5*sqrt(3)*cos(1/9*pi)^4*s
in(1/9*pi) - 10*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5
 - cos(1/9*pi)^5 + 10*cos(1/9*pi)^3*sin(1/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*p
i)^4 - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi) + 2*cos(1/9*pi)^2 - 2*sin(1/9*pi)^
2)*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 10.35 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.68

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = \frac{\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\,\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(x-\frac{\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{12}\right)\,\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^2}{24}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

```
[In] int(-(x*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*12i - 36)^(1/
3))/18 + (log(x - (- 3^(1/2)*12i - 36)^(2/3)/12)*(- 3^(1/2)*12i - 36)^(1/3)
)/18 - (2^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6
)*1i)^2)/24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)
*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(-
 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/
3)*(3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1
/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(3^(1/2)*1i - 3)
^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5
/6)*1i))/36
```

3.29 $\int \frac{1-x^3}{1-x^3+x^6} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 411

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = -\frac{(i-\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{(i+\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{(3-i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{(3+i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] -1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2^
(2/3)/(1-I*3^(1/2))^(2/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3-I*3^(1
/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2)
)^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/18
*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/
3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(
3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I
*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1436, 206, 31, 648, 631, 210, 642}

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = -\frac{\left(-\sqrt{3}+i\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(\sqrt{3}+i\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[In] Int[(1 - x^3)/(1 - x^3 + x^6),x]
```

```
[Out] -1/3*((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/
(2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*
Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 - I*Sq
rt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(
2/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)
*(1 + I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(
1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3))
```

```
+ ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x
+ 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
```

```
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx - \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$- \frac{\left(3+i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{\left(3-i\sqrt{3}\right)\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{6\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+ \frac{\left(3+i\sqrt{3}\right)\int \frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{\left(3+i\sqrt{3}\right)\int \frac{1}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{6\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= -\frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{(3-i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{(3+i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$- \frac{(3-i\sqrt{3})\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$- \frac{(3+i\sqrt{3})\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$= -\frac{(i-\sqrt{3})\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{(i+\sqrt{3})\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$- \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{(3-i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{(3+i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.14

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = -\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1^2+2\#1^5}\&\right]$$

```
[In] Integrate[(1 - x^3)/(1 - x^3 + x^6),x]
[Out] -1/3*RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2 +
 2*#1^5) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.11

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\left(-\_R^3+1\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	44
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\left(-\_R^3+1\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	44

```
[In] int((-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/3*sum((-_R^3+1)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.73

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = \frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(i\sqrt{-3}-i)+3\sqrt{-3}-3\right)\left(i\sqrt{3}-3\right)^{\frac{1}{3}}+72x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(-i\sqrt{-3}-i)-3\sqrt{-3}-3\right)\left(i\sqrt{3}-3\right)^{\frac{1}{3}}+72x\right)-\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}+1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(i\sqrt{-3}+i)-3\sqrt{-3}-3\right)\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}+72x\right)+\frac{1}{108}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\left(\sqrt{-3}-1\right)\log\left(18^{\frac{2}{3}}\left(\sqrt{3}(-i\sqrt{-3}+i)+3\sqrt{-3}-3\right)\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}+72x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(i\sqrt{3}+3\right)\left(i\sqrt{3}-3\right)^{\frac{1}{3}}+36x\right)+\frac{1}{54}\cdot 18^{\frac{2}{3}}\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}\log\left(18^{\frac{2}{3}}\left(-i\sqrt{3}+3\right)\left(-i\sqrt{3}-3\right)^{\frac{1}{3}}+36x\right)$$

```
[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*18^(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*(sqrt(3)*(
I*sqrt(-3) - I) + 3*sqrt(-3) - 3)*(I*sqrt(3) - 3)^(1/3) + 72*x) - 1/108*18^
(2/3)*(I*sqrt(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*(sqrt(3)*(-I*sqrt(-
3) - I) - 3*sqrt(-3) - 3)*(I*sqrt(3) - 3)^(1/3) + 72*x) - 1/108*18^(2/3)*(-
I*sqrt(3) - 3)^(1/3)*(sqrt(-3) + 1)*log(18^(2/3)*(sqrt(3)*(I*sqrt(-3) + I)
- 3*sqrt(-3) - 3)*(-I*sqrt(3) - 3)^(1/3) + 72*x) + 1/108*18^(2/3)*(-I*sqrt(
3) - 3)^(1/3)*(sqrt(-3) - 1)*log(18^(2/3)*(sqrt(3)*(-I*sqrt(-3) + I) + 3*sq
rt(-3) - 3)*(-I*sqrt(3) - 3)^(1/3) + 72*x) + 1/54*18^(2/3)*(I*sqrt(3) - 3)^
(1/3)*log(18^(2/3)*(I*sqrt(3) + 3)*(I*sqrt(3) - 3)^(1/3) + 36*x) + 1/54*18^
(2/3)*(-I*sqrt(3) - 3)^(1/3)*log(18^(2/3)*(-I*sqrt(3) + 3)*(-I*sqrt(3) - 3)
^(1/3) + 36*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = -\operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(729t^4 - 9t + x\right)\right)\right)$$

```
[In] integrate((-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 - 9*_t +
x)))
```

Maxima [F]

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = \int -\frac{x^3-1}{x^6-x^3+1}\,dx$$

```
[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(267) = 534$.

Time = 0.44 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.56

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = \text{Too large to display}$$

```
[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(3
)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^3
 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(
4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*
pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*c
os(2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9*
pi) + 2*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*
I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos
(1/9*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*
pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin(1/9*pi))*
arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin
(1/9*pi))) + 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*
```

```
pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9
*pi)^4 + 2*sqrt(3)*sin(4/9*pi) - 2*cos(4/9*pi))*log((-I*sqrt(3)*cos(4/9*pi)
 - cos(4/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) -
4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2
/9*pi)^2 - sin(2/9*pi)^4 + 2*sqrt(3)*sin(2/9*pi) - 2*cos(2/9*pi))*log((-I*s
qrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(1/9*pi
)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*c
os(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos(
1/9*pi))*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 10.35 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.78

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = \frac{\ln\left(x-\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(x+\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^3}{16}+27\right)}{108}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(-3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^3}{16}-27\right)}{108}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

```
[In] int(-(x^3 - 1)/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x - (((3^(1/2)*9i)/2 - 27/2)*(- 3^(1/2)*12i - 36)^(1/3))/54)*(- 3^(1/2
)*12i - 36)^(1/3))/18 + (log(x + (((3^(1/2)*9i)/2 + 27/2)*(3^(1/2)*12i - 36
)^(1/3))/54)*(3^(1/2)*12i - 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*(- 3^
(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i)*((3*(3^(1/2)*1i + 3)*(3^(1/3) +
```

```
3^(5/6)*1i)^3)/16 + 27))/108)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1
i))/36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6
)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3) - 3^(5/6)*1i)^3)/16 - 27))/108)*(3^(1/2
)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3^(5
/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(
5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)
/6)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

3.30 $\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 416

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx = -\frac{1}{x} - \frac{\left(i+\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(i-\sqrt{3}\right)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] -1/x+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2)
)*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3-I*
3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(
1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-
1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))
^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3
))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/6*arctan(1/3*(1+2*2^(1/3)*x/
(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1-I*3^(1/2))^(1/3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1518, 1388, 298, 31, 648, 631, 210, 642}

$$\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx = -\frac{(\sqrt{3}+i)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{(-\sqrt{3}+i)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{(3+i\sqrt{3})\log\left(2^{2/3}x^2+\sqrt[3]{2(1-i\sqrt{3})}x+(1-i\sqrt{3})^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{(3-i\sqrt{3})\log\left(2^{2/3}x^2+\sqrt[3]{2(1+i\sqrt{3})}x+(1+i\sqrt{3})^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{1}{x} - \frac{(3+i\sqrt{3})\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(3-i\sqrt{3})\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[In] Int[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/(
(1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3
 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqr
t[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*
2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3)
```

```
 + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^
(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(
1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
```

```
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\text{integral} = -\frac{1}{x} - \int \frac{x^4}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{x} + \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx - \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{1}{x} + -\frac{\left(-3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$- \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$+ \frac{\left(3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$- \frac{\left(3+i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$= -\frac{1}{x} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$+ -\frac{\left(-3-i\sqrt{3}\right)\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+ \frac{1}{12}\left(-3+i\sqrt{3}\right)\int \frac{1}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx + \frac{\left(3-i\sqrt{3}\right)\int \frac{\sqrt[3]{\frac{1}{2}\left(1+ \right.}}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{}}}{18\ 2^{2/3}\sqrt[3]{1+}}$$

$$= -\frac{1}{x} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$+ \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+ \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$+ -\frac{\left(-3-i\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+ \frac{\left(3-i\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= -\frac{1}{x} - \frac{(i+\sqrt{3})\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+ \frac{(i-\sqrt{3})\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$- \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$+ \frac{(3+i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$+ \frac{(3-i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.11

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx = -\frac{1}{x} - \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1^2}{-1+2\#1^3}\&\right]$$

```
[In] Integrate[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
[Out] -x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) & ]/
3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.10

method	result	size
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(27\_Z^{6}-9\_Z^{3}+1\right)}\_R\ln\left(-27\_R^{5}+6\_R^{2}+x\right)\right)}{3}$	40
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}-\frac{1}{x}$	46

```
[In] int((-x^3+1)/x^2/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/x+1/3*sum(_R*ln(-27*_R^5+6*_R^2+x),_R=RootOf(27*_Z^6-9*_Z^3+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.75

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx = \frac{18^{\frac{2}{3}}\left(\sqrt{-3}x-x\right)\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(18^{\frac{1}{3}}\left(\sqrt{3}\left(i\sqrt{-3}+i\right)-\sqrt{-3}-1\right)\left(i\sqrt{3}+3\right)^{\frac{2}{3}}+24\,x\right)-18^{\frac{2}{3}}\left(\sqrt{-3}x+\right.}{}$$

```
[In] integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="fricas")
[Out] 1/108*(18^(2/3)*(sqrt(-3)*x - x)*(I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(sqrt(3
)*(I*sqrt(-3) + I) - sqrt(-3) - 1)*(I*sqrt(3) + 3)^(2/3) + 24*x) - 18^(2/3)
*(sqrt(-3)*x + x)*(I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-3)
+ I) + sqrt(-3) - 1)*(I*sqrt(3) + 3)^(2/3) + 24*x) - 18^(2/3)*(sqrt(-3)*x +
 x)*(-I*sqrt(3) + 3)^(1/3)*log(18^(1/3)*(sqrt(3)*(I*sqrt(-3) - I) + sqrt(-3
) - 1)*(-I*sqrt(3) + 3)^(2/3) + 24*x) + 18^(2/3)*(sqrt(-3)*x - x)*(-I*sqrt(
3) + 3)^(1/3)*log(18^(1/3)*(sqrt(3)*(-I*sqrt(-3) - I) - sqrt(-3) - 1)*(-I*s
qrt(3) + 3)^(2/3) + 24*x) + 2*18^(2/3)*x*(-I*sqrt(3) + 3)^(1/3)*log(18^(1/3
)*(I*sqrt(3) + 1)*(-I*sqrt(3) + 3)^(2/3) + 12*x) + 2*18^(2/3)*x*(I*sqrt(3)
+ 3)^(1/3)*log(18^(1/3)*(I*sqrt(3) + 3)^(2/3)*(-I*sqrt(3) + 1) + 12*x) - 10
8)/x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.07

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx = -\operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(6561t^5+54t^2+x\right)\right)\right) - \frac{1}{x}$$

```
[In] integrate((-x**3+1)/x**2/(x**6-x**3+1),x)
[Out] -RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 + 54*_t*
*2 + x))) - 1/x
```

Maxima [F]

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx = \int -\frac{x^3-1}{(x^6-x^3+1)x^2}\,dx$$

```
[In] integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="maxima")
[Out] -1/x - integrate(x^4/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(272) = 544$.

Time = 0.32 (sec) , antiderivative size = 832, normalized size of antiderivative = 2.00

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="giac")
[Out] 1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*
sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos(4
/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(3)*
sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos
(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*cos(2
/9*pi)^5 - 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/9*pi)*
sin(2/9*pi)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(2/9*pi)
^3 - 2*sin(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^2 - 2*co
s(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/
2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^5 - 20*sqrt(3
```

```
)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 10*c
os(1/9*pi)^4*sin(1/9*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sin(1/9*pi)^5
 - sqrt(3)*cos(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*pi
))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*
sin(1/9*pi))) + 1/18*(10*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 20*sqrt(3)*cos
(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqrt(3)*sin(4/9*pi)^5 + 2*cos(4/9*pi)^5 - 20*c
os(4/9*pi)^3*sin(4/9*pi)^2 + 10*cos(4/9*pi)*sin(4/9*pi)^4 + 2*sqrt(3)*cos(4
/9*pi)*sin(4/9*pi) + cos(4/9*pi)^2 - sin(4/9*pi)^2)*log((-I*sqrt(3)*cos(4/9
*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(2/9*pi)^4*sin(2/9*p
i) - 20*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + 2*sqrt(3)*sin(2/9*pi)^5 + 2*c
os(2/9*pi)^5 - 20*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*cos(2/9*pi)*sin(2/9*pi)^
4 + 2*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) + cos(2/9*pi)^2 - sin(2/9*pi)^2)*log(
(-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(
1/9*pi)^4*sin(1/9*pi) - 20*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sqrt(3)*
sin(1/9*pi)^5 - 2*cos(1/9*pi)^5 + 20*cos(1/9*pi)^3*sin(1/9*pi)^2 - 10*cos(1
/9*pi)*sin(1/9*pi)^4 - 2*sqrt(3)*cos(1/9*pi)*sin(1/9*pi) + cos(1/9*pi)^2 -
sin(1/9*pi)^2)*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/x
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.75

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx = \frac{\ln\left(-x+\left(162\,x+\frac{27\,{\left(36+\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,{\left(36+\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18} + \frac{\ln\left(-x-\left(162\,x+\frac{27\,{\left(36-\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,{\left(36-\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18} - \frac{1}{x} - \frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,1\mathrm{i}}{4}\right)\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,1\mathrm{i}}{4}\right)\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36} - \frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$

```
[In] int(-(x^3 - 1)/(x^2*(x^6 - x^3 + 1)),x)
```

```
[Out] (log((162*x + (27*(3^(1/2)*12i + 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162) -
 x)*(3^(1/2)*12i + 36)^(1/3))/18 + (log(- x - (162*x + (27*(36 - 3^(1/2)*12
i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162))*(36 - 3^(1/2)*12i)^(1/3))/18 - 1/x
 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 - (2^(1/3)*
3^(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(
5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/12
 + (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/3)*(3
^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1
i)^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*l
og(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)^(1/3)*(
3^(1/3) - 3^(5/6)*1i))/36
```

3.31 $\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 418

$$\int \frac{1-x^3}{x^3(1-x^3+x^6)}\,dx = -\frac{1}{2x^2} + \frac{(i+\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{(i-\sqrt{3})\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{(3+i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{(3-i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] -1/2/x^2-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(
1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(
3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I
*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2
/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1
/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^
(2/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/6*arctan(1/3*(1+2*2^(1/3
)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1-I*3^(1/2))^(2/3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1518, 12, 1388, 206, 31, 648, 631, 210, 642}

$$\int \frac{1-x^3}{x^3(1-x^3+x^6)}\,dx = \frac{(\sqrt{3}+i)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(-\sqrt{3}+i)\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} - \frac{1}{2x^2} + \frac{(3+i\sqrt{3})\log\left(2^{2/3}x^2+\sqrt[3]{2(1-i\sqrt{3})}x+(1-i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(3-i\sqrt{3})\log\left(2^{2/3}x^2+\sqrt[3]{2(1+i\sqrt{3})}x+(1+i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} - \frac{(3+i\sqrt{3})\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3})\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}}$$

```
[In] Int[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/2*1/x^2 + ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sq
rt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I - Sqrt[3])*ArcTan[(1 + (2*x
)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) -
((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*
Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/
(9*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2
/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3
```

```
])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{2x^2} - \frac{1}{2}\int \frac{2x^3}{1-x^3+x^6}\,dx \\
&= -\frac{1}{2x^2} - \int \frac{x^3}{1-x^3+x^6}\,dx \\
&= -\frac{1}{2x^2} + \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx - \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx \\
&= -\frac{1}{2x^2} - \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} \\
&\quad - \frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} \\
&\quad - \frac{\left(3+i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} \\
&\quad - \frac{\left(3+i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}
\end{aligned}$$

$$= -\frac{1}{2x^2} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{\left(3-i\sqrt{3}\right)\int\frac{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}dx}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{\left(3-i\sqrt{3}\right)\int\frac{1}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}dx}{6\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$+ \frac{\left(3+i\sqrt{3}\right)\int\frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}dx}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{\left(3+i\sqrt{3}\right)\int\frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}dx}{6\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$= -\frac{1}{2x^2} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/3} + \sqrt[3]{2\left(1-i\sqrt{3}\right)}x + 2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$+ \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1+i\sqrt{3}\right)^{2/3} + \sqrt[3]{2\left(1+i\sqrt{3}\right)}x + 2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$- \frac{\left(3-i\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$- \frac{\left(3+i\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$= -\frac{1}{2x^2} + \frac{(i+\sqrt{3})\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$
$$- \frac{(i-\sqrt{3})\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$
$$- \frac{(3+i\sqrt{3})\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{(3-i\sqrt{3})\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$
$$+ \frac{(3+i\sqrt{3})\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$
$$+ \frac{(3-i\sqrt{3})\log\left(\left(1+i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+2^{2/3}x^2\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.11

$$\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx = -\frac{1}{2x^2} - \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&, \frac{\log(x-\#1)\#1}{-1+2\#1^3}\&\right]$$

```
[In] Integrate[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
[Out] -1/2*1/x^2 - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]
/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.09

method	result	size
risch	$-\frac{1}{2x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(27\_Z^6-9\_Z^3+1\right)}\_R\ln\left(-18\_R^4+3\_R+x\right)\right)}{3}$	38
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}-\frac{1}{2x^2}$	46

```
[In] int((-x^3+1)/x^3/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2+1/3*sum(_R*ln(-18*_R^4+3*_R+x),_R=RootOf(27*_Z^6-9*_Z^3+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.72

$$\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx$$
$$=\frac{2\cdot 18^{\frac{2}{3}}x^2\left(i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(-i\cdot 18^{\frac{2}{3}}\sqrt{3}\left(i\sqrt{3}+3\right)^{\frac{1}{3}}+18\,x\right)+2\cdot 18^{\frac{2}{3}}x^2\left(-i\sqrt{3}+3\right)^{\frac{1}{3}}\log\left(i\cdot 18^{\frac{2}{3}}\sqrt{3}\left(-i\sqrt{3}\right.\right.}{}$$

```
[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*x^2*(I*sqrt(3) + 3)^(1/3)*log(-I*18^(2/3)*sqrt(3)*(I*sqrt
(3) + 3)^(1/3) + 18*x) + 2*18^(2/3)*x^2*(-I*sqrt(3) + 3)^(1/3)*log(I*18^(2/
3)*sqrt(3)*(-I*sqrt(3) + 3)^(1/3) + 18*x) - 18^(2/3)*(sqrt(-3)*x^2 + x^2)*(
I*sqrt(3) + 3)^(1/3)*log(18^(2/3)*sqrt(3)*(I*sqrt(3) + 3)^(1/3)*(I*sqrt(-3)
 + I) + 36*x) + 18^(2/3)*(sqrt(-3)*x^2 - x^2)*(-I*sqrt(3) + 3)^(1/3)*log(18
^(2/3)*sqrt(3)*(-I*sqrt(3) + 3)^(1/3)*(I*sqrt(-3) - I) + 36*x) + 18^(2/3)*(
sqrt(-3)*x^2 - x^2)*(I*sqrt(3) + 3)^(1/3)*log(18^(2/3)*sqrt(3)*(I*sqrt(3) +
 3)^(1/3)*(-I*sqrt(-3) + I) + 36*x) - 18^(2/3)*(sqrt(-3)*x^2 + x^2)*(-I*sqr
t(3) + 3)^(1/3)*log(18^(2/3)*sqrt(3)*(-I*sqrt(3) + 3)^(1/3)*(-I*sqrt(-3) -
I) + 36*x) - 54)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.08

$$\int \frac{1-x^3}{x^3 (1-x^3+x^6)} \, dx = -\operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log\left(-1458t^4 - 9t + x\right)\right)\right) - \frac{1}{2x^2}$$

```
[In] integrate((-x**3+1)/x**3/(x**6-x**3+1),x)
[Out] -RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 - 9*_t
+ x))) - 1/(2*x**2)
```

Maxima [F]

$$\int \frac{1-x^3}{x^3 (1-x^3+x^6)} \, dx = \int -\frac{x^3-1}{(x^6-x^3+1)x^3} \, dx$$

```
[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="maxima")
[Out] -1/2/x^2 - integrate(x^3/(x^6 - x^3 + 1), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(272) = 544.

Time = 0.32 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.54

$$\int \frac{1-x^3}{x^3 (1-x^3+x^6)} \, dx = \text{Too large to display}$$

```
[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="giac")
[Out] 1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + 2*s
qrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*pi)*sin(4/9*
pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos
(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*cos(2
/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*pi)^4
 + 8*cos(2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3)*cos(
2/9*pi) + sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/
2*I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 12*sqrt(3
)*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/9*pi)^3*s
in(1/9*pi) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) + sin(1/9*pi
```

```
))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*
sin(1/9*pi))) + 1/18*(8*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 8*sqrt(3)*cos(4
/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9*pi)^4 + 12*cos(4/9*pi)^2*sin(4/9*pi)^2 - 2
*sin(4/9*pi)^4 + sqrt(3)*sin(4/9*pi) - cos(4/9*pi))*log((-I*sqrt(3)*cos(4/9
*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(8*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi
) - 8*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - 2*cos(2/9*pi)^4 + 12*cos(2/9*pi)^
2*sin(2/9*pi)^2 - 2*sin(2/9*pi)^4 + sqrt(3)*sin(2/9*pi) - cos(2/9*pi))*log(
(-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) - 1/18*(8*sqrt(3)*cos(1
/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + 2*cos(1/9*pi)^
4 - 12*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sin(1/9*pi)^4 - sqrt(3)*sin(1/9*pi)
- cos(1/9*pi))*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/2
/x^2
```

Mupad [B] (verification not implemented)

Time = 10.54 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.79

$$\int \frac{1-x^3}{x^3(1-x^3+x^6)}\,dx$$
$$= \frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} + \frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{6}\right)\,\left(36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18} - \frac{1}{2\,x^2}$$
$$- \frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$- \frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{4/3}}{12}\right)\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$- \frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\,\left(3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$
$$- \frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,1\mathrm{i}}{12}\right)\,\left(3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}$$

```
[In] int(-(x^3 - 1)/(x^3*(x^6 - x^3 + 1)),x)
```

```
[Out] (log(x + (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(36 - 3^(1/2)*12i)^
(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3)*1i)/6)*(3^(1/2
```

```
)*12i + 36)^(1/3))/18 - 1/(2*x^2) - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3 -
3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/12)*(3 - 3^
(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(1
/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(4/3))/12
)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(
2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^
(1/3)*1i)/12)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*
log(x + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1
/2)*1i + 3)^(1/3)*1i)/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

3.32 $\int \frac{x^2(-2+x^3)}{1-x^3+x^6} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6} \, dx = \frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] 1/6*ln(x^6-x^3+1)+1/3*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1482, 648, 632, 210, 642}

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6} \, dx = \frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{6}\log\left(x^6-x^3+1\right)$$

```
[In] Int[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[1 - x^3 + x^6]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{-2+x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{6}\log\left(1-x^3+x^6\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= -\frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = -\frac{\arctan\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[In] Integrate[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
[Out] -(ArcTan[(-1 + 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{3}$	33
risch	$\frac{\ln\left(4x^{6}-4x^{3}+4\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{3}$	35

```
[In] int(x^2*(x^3-2)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/6*ln(x^6-x^3+1)-1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx=-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)+\frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

```
[In] integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx=\frac{\log\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate(x**2*(x**3-2)/(x**6-x**3+1),x)
[Out] log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)+\frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

```
[In] integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right)+\frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

```
[In] integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx = \frac{\ln\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{3}$$

```
[In] int((x^2*(x^3 - 2))/(x^6 - x^3 + 1),x)
[Out] log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/3
```

3.33 $\int \frac{1+x^3}{x\left(1-x^3+x^6\right)} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 39

$$\int \frac{1+x^3}{x\left(1-x^3+x^6\right)} \, dx = -\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x) - \frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)-1/3*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1488, 814, 648, 632, 210, 642}

$$\int \frac{1+x^3}{x\left(1-x^3+x^6\right)} \, dx = -\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log(x)$$

```
[In] Int[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] -(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1+x}{x\,(1-x+x^2)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{2-x}{1-x+x^2}\right)\,dx, x, x^3\right)\\
&= \log(x)+\frac{1}{3}\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= \frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.41

$$\int \frac{1+x^3}{x(1-x^3+x^6)}\,dx = \log(x) - \frac{1}{3}\text{RootSum}\Big[1-\#1^3 + \#1^6\&, \frac{-2\log(x-\#1)+\log(x-\#1)\#1^3}{-1+2\#1^3}\&\Big]$$

```
[In] Integrate[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1]*#1^3)/(-
1 + 2*#1^3) & ]/3
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
risch	$\ln(x)-\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\arctan\left(\frac{2\left(x^3-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	33
default	$-\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{3}+\ln(x)$	35

```
[In] int((x^3+1)/x/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] ln(x)-1/6*ln(x^6-x^3+1)+1/3*3^(1/2)*arctan(2/3*(x^3-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1+x^3}{x(1-x^3+x^6)}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log(x)$$

```
[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1+x^3}{x\,(1-x^3+x^6)}\,dx = \log{(x)} - \frac{\log{(x^6-x^3+1)}}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((x**3+1)/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{1+x^3}{x\,(1-x^3+x^6)}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right) - \frac{1}{6}\,\log\left(x^6-x^3+1\right) + \frac{1}{3}\,\log\left(x^3\right)$$

```
[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3*
log(x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{1+x^3}{x\,(1-x^3+x^6)}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^3-1)\right) - \frac{1}{6}\,\log\left(x^6-x^3+1\right) + \log\left(|x|\right)$$

```
[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx = \ln\left(x\right)-\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{3}$$

```
[In] int((x^3 + 1)/(x*(x^6 - x^3 + 1)),x)
[Out] log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/3
```

3.34 $\int \frac{1+x^3}{x-x^4+x^7} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 39

$$\int \frac{1+x^3}{x-x^4+x^7} \, dx = -\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x) - \frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)-1/3*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1608, 1488, 814, 648, 632, 210, 642}

$$\int \frac{1+x^3}{x-x^4+x^7} \, dx = -\frac{\arctan\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log(x)$$

```
[In] Int[(1 + x^3)/(x - x^4 + x^7),x]
```

```
[Out] -(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx \\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1+x}{x\left(1-x+x^2\right)}\,dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{2-x}{1-x+x^2}\right)\,dx, x, x^3\right)
\end{aligned}
$$

$$= \log(x) + \frac{1}{3}\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, x^3\right)$$

$$= \log(x) - \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)$$

$$= \log(x) - \frac{1}{6}\log\left(1-x^3+x^6\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)$$

$$= \frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x) - \frac{1}{6}\log\left(1-x^3+x^6\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.41

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx = \log(x) - \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&, \frac{-2\log(x-\#1)+\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

```
[In] Integrate[(1 + x^3)/(x - x^4 + x^7),x]
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1]*#1^3)/(-
1 + 2*#1^3) & ]/3
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
risch	$\ln\left(x\right)-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^{3}-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	33
default	$-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{3}+\ln\left(x\right)$	35

```
[In] int((x^3+1)/(x^7-x^4+x),x,method=_RETURNVERBOSE)
[Out] ln(x)-1/6*ln(x^6-x^3+1)+1/3*3^(1/2)*arctan(2/3*(x^3-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log\left(x\right)$$

```
[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx = \log\left(x\right) - \frac{\log\left(x^6-x^3+1\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

```
[In] integrate((x**3+1)/(x**7-x**4+x),x)
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/3
```

Maxima [F]

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx = \int \frac{x^3+1}{x^7-x^4+x}\,dx$$

```
[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="maxima")
[Out] -integrate((x^5 - 2*x^2)/(x^6 - x^3 + 1), x) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log\left(|x|\right)$$

```
[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1+x^3}{x-x^4+x^7}\,dx = \ln(x) - \frac{\ln(x^6-x^3+1)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}\,x^3}{3}\right)}{3}$$

```
[In] int((x^3 + 1)/(x - x^4 + x^7),x)
[Out] log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/3
```

3.35 $\int \left(d+ex^3\right)^{5/2} \left(a+bx^3+cx^6\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 396

$$\int \left(d+ex^3\right)^{5/2} \left(a+bx^3+cx^6\right) \, dx = \frac{54d^2(16cd^2-58bde+667ae^2)\,x\sqrt{d+ex^3}}{124729e^2}$$
$$+\frac{30d(16cd^2-58bde+667ae^2)\,x(d+ex^3)^{3/2}}{124729e^2}+\frac{2(16cd^2-58bde+667ae^2)\,x(d+ex^3)^{5/2}}{11339e^2}$$
$$-\frac{2(8cd-29be)x(d+ex^3)^{7/2}}{667e^2}+\frac{2cx^4(d+ex^3)^{7/2}}{29e}$$
$$+\frac{54\;3^{3/4}\sqrt{2+\sqrt{3}}d^3(16cd^2-58bde+667ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{(1-\sqrt{\ldots}}{(1+\sqrt{\ldots}}\right.\right.}{124729e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

```
[Out] 30/124729*d*(667*a*e^2-58*b*d*e+16*c*d^2)*x*(e*x^3+d)^(3/2)/e^2+2/11339*(66
7*a*e^2-58*b*d*e+16*c*d^2)*x*(e*x^3+d)^(5/2)/e^2-2/667*(-29*b*e+8*c*d)*x*(e
*x^3+d)^(7/2)/e^2+2/29*c*x^4*(e*x^3+d)^(7/2)/e+54/124729*d^2*(667*a*e^2-58*
b*d*e+16*c*d^2)*x*(e*x^3+d)^(1/2)/e^2+54/124729*3^(3/4)*d^3*(667*a*e^2-58*b
*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^(1/2))
)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*
((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)
^(1/2)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1
/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1425, 396, 201, 224}

$$\int (d+ex^3)^{5/2}(a+bx^3+cx^6)\,dx = \frac{54\,3^{3/4}\sqrt{2+\sqrt{3}}d^3\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}(667ae^2-58bde+16cd^2)\,\text{EllipticF}\left(\text{arcsi}\right.}{124729e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$
$$+\frac{2x(d+ex^3)^{5/2}(667ae^2-58bde+16cd^2)}{11339e^2}+\frac{30dx(d+ex^3)^{3/2}(667ae^2-58bde+16cd^2)}{124729e^2}$$
$$+\frac{54d^2x\sqrt{d+ex^3}(667ae^2-58bde+16cd^2)}{124729e^2}-\frac{2x(d+ex^3)^{7/2}(8cd-29be)}{667e^2}+\frac{2cx^4(d+ex^3)^{7/2}}{29e}$$

```
[In] Int[(d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (54*d^2*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*Sqrt[d + e*x^3])/(124729*e^2) +
 (30*d*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*(d + e*x^3)^(3/2))/(124729*e^2)
+ (2*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*(d + e*x^3)^(5/2))/(11339*e^2) - (
2*(8*c*d - 29*b*e)*x*(d + e*x^3)^(7/2))/(667*e^2) + (2*c*x^4*(d + e*x^3)^(7
/2))/(29*e) + (54*3^(3/4)*Sqrt[2 + Sqrt[3]]*d^3*(16*c*d^2 - 58*b*d*e + 667*
a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^
2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d
^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/
(124729*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3)
 + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1425

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2cx^4(d+ex^3)^{7/2}}{29e} + \frac{2\int (d+ex^3)^{5/2}\left(\frac{29ae}{2} - \left(4cd - \frac{29be}{2}\right)x^3\right)\,dx}{29e} \\
&= -\frac{2(8cd-29be)x(d+ex^3)^{7/2}}{667e^2} + \frac{2cx^4(d+ex^3)^{7/2}}{29e} \\
&\quad - \frac{1}{667}\left(-667a - \frac{2d(8cd-29be)}{e^2}\right)\int (d+ex^3)^{5/2}\,dx \\
&= \frac{2\left(667a + \frac{2d(8cd-29be)}{e^2}\right)x(d+ex^3)^{5/2}}{11339} - \frac{2(8cd-29be)x(d+ex^3)^{7/2}}{667e^2} \\
&\quad + \frac{2cx^4(d+ex^3)^{7/2}}{29e} + \frac{\left(15d\left(667a + \frac{2d(8cd-29be)}{e^2}\right)\right)\int (d+ex^3)^{3/2}\,dx}{11339} \\
&= \frac{30d\left(667a + \frac{2d(8cd-29be)}{e^2}\right)x(d+ex^3)^{3/2}}{124729} + \frac{2\left(667a + \frac{2d(8cd-29be)}{e^2}\right)x(d+ex^3)^{5/2}}{11339} \\
&\quad - \frac{2(8cd-29be)x(d+ex^3)^{7/2}}{667e^2} + \frac{2cx^4(d+ex^3)^{7/2}}{29e} \\
&\quad + \frac{\left(135d^2\left(667a + \frac{2d(8cd-29be)}{e^2}\right)\right)\int \sqrt{d+ex^3}\,dx}{124729}
\end{aligned}$$

$$= \frac{54d^2\left(667a + \frac{2d(8cd-29be)}{e^2}\right)x\sqrt{d+ex^3}}{124729} + \frac{30d\left(667a + \frac{2d(8cd-29be)}{e^2}\right)x(d+ex^3)^{3/2}}{124729} + \frac{2\left(667a + \frac{2d(8cd-29be)}{e^2}\right)x(d+ex^3)^{5/2}}{11339} - \frac{2(8cd-29be)x(d+ex^3)^{7/2}}{667e^2} + \frac{2cx^4(d+ex^3)^{7/2}}{29e} + \frac{\left(81d^3\left(667a + \frac{2d(8cd-29be)}{e^2}\right)\right)\int \frac{1}{\sqrt{d+ex^3}}\,dx}{124729}$$

$$= \frac{54d^2\left(667a + \frac{2d(8cd-29be)}{e^2}\right)x\sqrt{d+ex^3}}{124729} + \frac{30d\left(667a + \frac{2d(8cd-29be)}{e^2}\right)x(d+ex^3)^{3/2}}{124729} + \frac{2\left(667a + \frac{2d(8cd-29be)}{e^2}\right)x(d+ex^3)^{5/2}}{11339} - \frac{2(8cd-29be)x(d+ex^3)^{7/2}}{667e^2} + \frac{2cx^4(d+ex^3)^{7/2}}{29e} + \frac{54\,3^{3/4}\sqrt{2+\sqrt{3}}d^3(16cd^2-58bde+667ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^2 \cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e} \cdots\right.}}}{124729e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d \cdots}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.26

$$\int \left(d+ex^3\right)^{5/2}\left(a+bx^3+cx^6\right)\,dx = \frac{x\sqrt{d+ex^3}\left(-2(d+ex^3)^3(8cd-29be-23cex^3) + \frac{(16cd^4+29d^2e(-2bd+23ae))\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{3},\frac{4}{3},- \cdots\right)}{\sqrt{1+\frac{ex^3}{d}}}\right)}{667e^2}$$

```
[In] Integrate[(d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)^3*(8*c*d - 29*b*e - 23*c*e*x^3) + ((16*c
*d^4 + 29*d^2*e*(-2*b*d + 23*a*e))*Hypergeometric2F1[-5/2, 1/3, 4/3, -((e*x
^3)/d)])/Sqrt[1 + (e*x^3)/d]))/(667*e^2)
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.10

method	result
risch	$\frac{2x\left(4301e^{4}c\,x^{12}+5423b\,e^{4}x^{9}+11407d\,e^{3}c\,x^{9}+7337a\,e^{4}x^{6}+15631bd\,e^{3}x^{6}+8591c\,d^{2}e^{2}x^{6}+24679d\,e^{3}a\,x^{3}+14123b\,d^{2}e^{2}x^{3}+405d^{3}ec\,x^{3}\right.}{124729e^{2}}$
elliptic	$\frac{2c\,e^{2}x^{13}\sqrt{e\,x^{3}+d}}{29}+\frac{2\left(b\,e^{3}+\frac{61}{29}cd\,e^{2}\right)x^{10}\sqrt{e\,x^{3}+d}}{23e}+\frac{2\left(a\,e^{3}+3d\,e^{2}b+3c\,d^{2}e-\frac{20d\left(b\,e^{3}+\frac{61}{29}cd\,e^{2}\right)}{23e}\right)x^{7}\sqrt{e\,x^{3}+d}}{17e}+\frac{2\left(3d\,e^{2}a+3b\,d^{2}\right.}{}$
default	Expression too large to display

```
[In] int((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/124729/e^2*x*(4301*c*e^4*x^12+5423*b*e^4*x^9+11407*c*d*e^3*x^9+7337*a*e^4
*x^6+15631*b*d*e^3*x^6+8591*c*d^2*e^2*x^6+24679*a*d*e^3*x^3+14123*b*d^2*e^2
*x^3+405*c*d^3*e*x^3+35351*a*d^2*e^2+2349*b*d^3*e-648*c*d^4)*(e*x^3+d)^(1/2
)-54/124729*I*d^3*(667*a*e^2-58*b*d*e+16*c*d^2)/e^3*3^(1/2)*(-d*e^2)^(1/3)*
(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^
2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2
)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*
e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3
^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e
/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1
/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.43

$$\int \left(d+ex^3\right)^{5/2}\left(a+bx^3+cx^6\right)\,dx = \frac{2\left(81\left(16\,cd^5-58\,bd^4e+667\,ad^3e^2\right)\sqrt{e}\text{weierstrassPInverse}\left(0,-\frac{4\,d}{e},x\right)+\left(4301\,ce^5x^{13}+187\,(61\,c\right.\right.}{}$$

```
[In] integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 2/124729*(81*(16*c*d^5 - 58*b*d^4*e + 667*a*d^3*e^2)*sqrt(e)*weierstrassPIn
verse(0, -4*d/e, x) + (4301*c*e^5*x^13 + 187*(61*c*d*e^4 + 29*b*e^5)*x^10 +
 11*(781*c*d^2*e^3 + 1421*b*d*e^4 + 667*a*e^5)*x^7 + (405*c*d^3*e^2 + 14123
*b*d^2*e^3 + 24679*a*d*e^4)*x^4 - (648*c*d^4*e - 2349*b*d^3*e^2 - 35351*a*d
^2*e^3)*x)*sqrt(e*x^3 + d))/e^3
```

Sympy [A] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.01

$$\int \left(d+ex^3\right)^{5/2}\left(a+bx^3+cx^6\right)\,dx = \frac{ad^{\frac{5}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2ad^{\frac{3}{2}}ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a\sqrt{d}e^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{bd^{\frac{5}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2bd^{\frac{3}{2}}ex^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{b\sqrt{d}e^2x^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{13}{3}\right)} + \frac{cd^{\frac{5}{2}}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{2cd^{\frac{3}{2}}ex^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{13}{3}\right)} + \frac{c\sqrt{d}e^2x^{13}\Gamma\left(\frac{13}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{3}\\ \frac{16}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{16}{3}\right)}$$

```
[In] integrate((e*x**3+d)**(5/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**(5/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/d
)/(3*gamma(4/3)) + 2*a*d**(3/2)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,)
, e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + a*sqrt(d)*e**2*x**7*gamma(7/3)
*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + b*
d**(5/2)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/
d)/(3*gamma(7/3)) + 2*b*d**(3/2)*e*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3
,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + b*sqrt(d)*e**2*x**10*gamma(
10/3)*hyper((-1/2, 10/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3)
) + c*d**(5/2)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar
(I*pi)/d)/(3*gamma(10/3)) + 2*c*d**(3/2)*e*x**10*gamma(10/3)*hyper((-1/2, 1
0/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3)) + c*sqrt(d)*e**2*x
**13*gamma(13/3)*hyper((-1/2, 13/3), (16/3,), e*x**3*exp_polar(I*pi)/d)/(3*
gamma(16/3))
```

Maxima [F]

$$\int (d+ex^3)^{5/2}\,(a+bx^3+cx^6)\;dx=\int (cx^6+bx^3+a)\,(ex^3+d)^{\frac{5}{2}}\,dx$$

```
[In] integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(5/2), x)
```

Giac [F]

$$\int (d+ex^3)^{5/2}\,(a+bx^3+cx^6)\;dx=\int (cx^6+bx^3+a)\,(ex^3+d)^{\frac{5}{2}}\,dx$$

```
[In] integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex^3)^{5/2}\,(a+bx^3+cx^6)\;dx=\int (e\,x^3+d)^{5/2}\,(c\,x^6+b\,x^3+a)\;dx$$

```
[In] int((d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x)
```

```
[Out] int((d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6), x)
```

3.36 $\int (d+ex^3)^{3/2} (a+bx^3+cx^6) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 356

$$\int (d+ex^3)^{3/2} (a+bx^3+cx^6) \, dx = \frac{18d(16cd^2-46bde+391ae^2)\,x\sqrt{d+ex^3}}{21505e^2} + \frac{2(16cd^2-46bde+391ae^2)\,x(d+ex^3)^{3/2}}{4301e^2} - \frac{2(8cd-23be)x(d+ex^3)^{5/2}}{391e^2} + \frac{2cx^4(d+ex^3)^{5/2}}{23e}$$

$$+ \frac{18\, 3^{3/4}\sqrt{2+\sqrt{3}}d^2(16cd^2-46bde+391ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\cdots}{\left(1+\sqrt{3}\right)\cdots}\right.\right.}{21505e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

```
[Out] 2/4301*(391*a*e^2-46*b*d*e+16*c*d^2)*x*(e*x^3+d)^(3/2)/e^2-2/391*(-23*b*e+8
*c*d)*x*(e*x^3+d)^(5/2)/e^2+2/23*c*x^4*(e*x^3+d)^(5/2)/e+18/21505*d*(391*a*
e^2-46*b*d*e+16*c*d^2)*x*(e*x^3+d)^(1/2)/e^2+18/21505*3^(3/4)*d^2*(391*a*e^
2-46*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^
(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(
1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2
)))^2)^(1/2)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*
x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1425, 396, 201, 224}

$$\int (d+ex^3)^{3/2}(a+bx^3+cx^6)\,dx = \frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}d^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}(391ae^2-46bde+16cd^2)\,\text{EllipticF}\left(\text{arc}\right.}{21505e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$
$$+\frac{2x(d+ex^3)^{3/2}(391ae^2-46bde+16cd^2)}{4301e^2}+\frac{18dx\sqrt{d+ex^3}(391ae^2-46bde+16cd^2)}{21505e^2}$$
$$-\frac{2x(d+ex^3)^{5/2}(8cd-23be)}{391e^2}+\frac{2cx^4(d+ex^3)^{5/2}}{23e}$$

```
[In] Int[(d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (18*d*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*x*Sqrt[d + e*x^3])/(21505*e^2) + (2
*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*x*(d + e*x^3)^(3/2))/(4301*e^2) - (2*(8*
c*d - 23*b*e)*x*(d + e*x^3)^(5/2))/(391*e^2) + (2*c*x^4*(d + e*x^3)^(5/2))/
(23*e) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*d^2*(16*c*d^2 - 46*b*d*e + 391*a*e^2
)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((
1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3
) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(2150
5*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(
1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1425

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2cx^4(d+ex^3)^{5/2}}{23e} + \frac{2\int (d+ex^3)^{3/2}\left(\frac{23ae}{2} - \left(4cd - \frac{23be}{2}\right)x^3\right)\,dx}{23e} \\
&= -\frac{2(8cd-23be)x(d+ex^3)^{5/2}}{391e^2} + \frac{2cx^4(d+ex^3)^{5/2}}{23e} \\
&\quad - \frac{1}{391}\left(-391a - \frac{2d(8cd-23be)}{e^2}\right)\int \left(d+ex^3\right)^{3/2}\,dx \\
&= \frac{2\left(391a + \frac{2d(8cd-23be)}{e^2}\right)x(d+ex^3)^{3/2}}{4301} - \frac{2(8cd-23be)x(d+ex^3)^{5/2}}{391e^2} \\
&\quad + \frac{2cx^4(d+ex^3)^{5/2}}{23e} + \frac{\left(9d\left(391a + \frac{2d(8cd-23be)}{e^2}\right)\right)\int \sqrt{d+ex^3}\,dx}{4301} \\
&= \frac{18d\left(391a + \frac{2d(8cd-23be)}{e^2}\right)x\sqrt{d+ex^3}}{21505} + \frac{2\left(391a + \frac{2d(8cd-23be)}{e^2}\right)x(d+ex^3)^{3/2}}{4301} \\
&\quad - \frac{2(8cd-23be)x(d+ex^3)^{5/2}}{391e^2} + \frac{2cx^4(d+ex^3)^{5/2}}{23e} \\
&\quad + \frac{\left(27d^2\left(391a + \frac{2d(8cd-23be)}{e^2}\right)\right)\int \frac{1}{\sqrt{d+ex^3}}\,dx}{21505}
\end{aligned}$$

$$= \frac{18d\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x\sqrt{d+ex^3}}{21505}+\frac{2\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x(d+ex^3)^{3/2}}{4301} - \frac{2(8cd-23be)x(d+ex^3)^{5/2}}{391e^2}+\frac{2cx^4(d+ex^3)^{5/2}}{23e} + \frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}d^2(16cd^2-46bde+391ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}F\left(\sin^{-1}\left(\frac{(1-}{(1-}\right.\right.}{21505e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.72 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.28

$$\int (d+ex^3)^{3/2}\left(a+bx^3+cx^6\right)dx = \frac{x\sqrt{d+ex^3}\left(-2(d+ex^3)^2\left(8cd-23be-17cex^3\right)+\frac{(16cd^3+23de(-2bd+17ae))\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\right.}{\sqrt{1+\frac{ex^3}{d}}}\right.}{391e^2}$$

```
[In] Integrate[(d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x]
[Out] (x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)^2*(8*c*d - 23*b*e - 17*c*e*x^3) + ((16*c
*d^3 + 23*d*e*(-2*b*d + 17*a*e))*Hypergeometric2F1[-3/2, 1/3, 4/3, -((e*x^3
)/d)])/Sqrt[1 + (e*x^3)/d]))/(391*e^2)
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.12

method	result
risch	$\frac{2x\left(935e^{3}c\,x^{9}+1265b\,e^{3}x^{6}+1430cd\,e^{2}x^{6}+1955a\,e^{3}x^{3}+2300bd\,e^{2}x^{3}+135c\,d^{2}e\,x^{3}+5474d\,e^{2}a+621b\,d^{2}e-216d^{3}c\right)\sqrt{e\,x^{3}+d}}{21505e^{2}}-\frac{18id^{2}(39}{}$
elliptic	$\frac{2ec\,x^{10}\sqrt{e\,x^{3}+d}}{23}+\frac{2\left(b\,e^{2}+\frac{26}{23}dce\right)x^{7}\sqrt{e\,x^{3}+d}}{17e}+\frac{2\left(a\,e^{2}+2bde+c\,d^{2}-\frac{14d\left(b\,e^{2}+\frac{26}{23}dce\right)}{17e}\right)x^{4}\sqrt{e\,x^{3}+d}}{11e}+\frac{2\left(2eda+b\,d^{2}-\frac{8d\left(a\,e^{2}+2b\right.}{}\right.}{}$
default	Expression too large to display

```
[In] int((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21505/e^2*x*(935*c*e^3*x^9+1265*b*e^3*x^6+1430*c*d*e^2*x^6+1955*a*e^3*x^3
+2300*b*d*e^2*x^3+135*c*d^2*e*x^3+5474*a*d*e^2+621*b*d^2*e-216*c*d^3)*(e*x^
3+d)^(1/2)-18/21505*I*d^2*(391*a*e^2-46*b*d*e+16*c*d^2)/e^3*3^(1/2)*(-d*e^2
)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*
e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*
I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2
)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3
^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)
^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.38

$$\int \left(d+ex^{3}\right)^{3/2}\left(a+bx^{3}+cx^{6}\right)\,dx=\frac{2\left(27\left(16\,cd^{4}-46\,bd^{3}e+391\,ad^{2}e^{2}\right)\sqrt{e}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,d}{e},x\right)+\left(935\,ce^{4}x^{10}+55\,(26\,cde\right.}{21}$$

```
[In] integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 2/21505*(27*(16*c*d^4 - 46*b*d^3*e + 391*a*d^2*e^2)*sqrt(e)*weierstrassPInv
erse(0, -4*d/e, x) + (935*c*e^4*x^10 + 55*(26*c*d*e^3 + 23*b*e^4)*x^7 + 5*(
27*c*d^2*e^2 + 460*b*d*e^3 + 391*a*e^4)*x^4 - (216*c*d^3*e - 621*b*d^2*e^2
- 5474*a*d*e^3)*x)*sqrt(e*x^3 + d))/e^3
```

Sympy [A] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.72

$$\int \left(d+ex^3\right)^{3/2}\left(a+bx^3+cx^6\right)\,dx = \frac{ad^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a\sqrt{d}ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{bd^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{b\sqrt{d}ex^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{cd^{\frac{3}{2}}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{c\sqrt{d}ex^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

```
[In] integrate((e*x**3+d)**(3/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**(3/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/d
)/(3*gamma(4/3)) + a*sqrt(d)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e
*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + b*d**(3/2)*x**4*gamma(4/3)*hyper(
(-1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + b*sqrt(d)*e
*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*g
amma(10/3)) + c*d**(3/2)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3
*exp_polar(I*pi)/d)/(3*gamma(10/3)) + c*sqrt(d)*e*x**10*gamma(10/3)*hyper((
-1/2, 10/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3))
```

Maxima [F]

$$\int \left(d+ex^3\right)^{3/2}\left(a+bx^3+cx^6\right)\,dx=\int \left(cx^6+bx^3+a\right)\left(ex^3+d\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(3/2), x)
```

Giac [F]

$$\int \left(d+ex^3\right)^{3/2}\left(a+bx^3+cx^6\right)\,dx=\int \left(cx^6+bx^3+a\right)\left(ex^3+d\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^3\right)^{3/2}\left(a+bx^3+cx^6\right)\,dx=\int {\left(e\,x^3+d\right)}^{3/2}\,\left(c\,x^6+b\,x^3+a\right)\,dx$$

```
[In] int((d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x)
[Out] int((d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6), x)
```

3.37 $\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 316

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx$$
$$=\frac{2\left(16cd^2-34bde+187ae^2\right)x\sqrt{d+ex^3}}{935e^2}-\frac{2(8cd-17be)x\left(d+ex^3\right)^{3/2}}{187e^2}+\frac{2cx^4\left(d+ex^3\right)^{3/2}}{17e}$$
$$+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}d\left(16cd^2-34bde+187ae^2\right)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{(1-\cdots}{(1+\cdots}\right.\right.}{935e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

```
[Out] -2/187*(-17*b*e+8*c*d)*x*(e*x^3+d)^(3/2)/e^2+2/17*c*x^4*(e*x^3+d)^(3/2)/e+2
/935*(187*a*e^2-34*b*d*e+16*c*d^2)*x*(e*x^3+d)^(1/2)/e^2+2/935*3^(3/4)*d*(1
87*a*e^2-34*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3
)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1
+3^(1/2)))^2)^(1/2)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e
^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1425, 396, 201, 224}

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx$$

$$= \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}d\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(187ae^2-34bde+16cd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+(1}{\sqrt[3]{e}x+(1}\right.\right.}{935e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

$$+\frac{2x\sqrt{d+ex^3}\left(187ae^2-34bde+16cd^2\right)}{935e^2}-\frac{2x\left(d+ex^3\right)^{3/2}(8cd-17be)}{187e^2}+\frac{2cx^4\left(d+ex^3\right)^{3/2}}{17e}$$

```
[In] Int[Sqrt[d + e*x^3]*(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*(16*c*d^2 - 34*b*d*e + 187*a*e^2)*x*Sqrt[d + e*x^3])/(935*e^2) - (2*(8*c
*d - 17*b*e)*x*(d + e*x^3)^(3/2))/(187*e^2) + (2*c*x^4*(d + e*x^3)^(3/2))/(
17*e) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*d*(16*c*d^2 - 34*b*d*e + 187*a*e^2)*(d
^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 +
Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) +
e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*e^(7
/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x
)^2]*Sqrt[d + e*x^3])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1425

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2cx^4(d+ex^3)^{3/2}}{17e} + \frac{2\int \sqrt{d+ex^3}\left(\frac{17ae}{2} - \left(4cd - \frac{17be}{2}\right)x^3\right)\,dx}{17e}$$

$$= -\frac{2(8cd-17be)x(d+ex^3)^{3/2}}{187e^2} + \frac{2cx^4(d+ex^3)^{3/2}}{17e} - \frac{1}{187}\left(-187a - \frac{2d(8cd-17be)}{e^2}\right)\int \sqrt{d+ex^3}\,dx$$

$$= \frac{2}{935}\left(187a + \frac{2d(8cd-17be)}{e^2}\right)x\sqrt{d+ex^3} - \frac{2(8cd-17be)x(d+ex^3)^{3/2}}{187e^2} + \frac{2cx^4(d+ex^3)^{3/2}}{17e} + \frac{1}{935}\left(3d\left(187a + \frac{2d(8cd-17be)}{e^2}\right)\right)\int \frac{1}{\sqrt{d+ex^3}}\,dx$$

$$= \frac{2}{935}\left(187a+\frac{2d(8cd-17be)}{e^2}\right)x\sqrt{d+ex^3} - \frac{2(8cd-17be)x(d+ex^3)^{3/2}}{187e^2} + \frac{2cx^4(d+ex^3)^{3/2}}{17e} + \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}d(16cd^2-34bde+187ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}F\left(\sin^{-1}\left(\frac{(1-\sqrt{\cdots}}{(1+\sqrt{\cdots}}\right.\right.}{935e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.95 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.31

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx$$
$$=\frac{x\sqrt{d+ex^3}\left(-2(d+ex^3)\left(8cd-17be-11cex^3\right)+\frac{(16cd^2+17e(-2bd+11ae))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{ex^3}{d}\right)}{\sqrt{1+\frac{ex^3}{d}}}\right)}{187e^2}$$

```
[In] Integrate[Sqrt[d + e*x^3]*(a + b*x^3 + c*x^6),x]
```

```
[Out] (x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)*(8*c*d - 17*b*e - 11*c*e*x^3) + ((16*c*d
^2 + 17*e*(-2*b*d + 11*a*e))*Hypergeometric2F1[-1/2, 1/3, 4/3, -((e*x^3)/d)
])/Sqrt[1 + (e*x^3)/d]))/(187*e^2)
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.15

method	result
risch	$\frac{2x\left(55c\,e^{2}x^{6}+85b\,e^{2}x^{3}+15dc\,x^{3}e+187a\,e^{2}+51bde-24c\,d^{2}\right)\sqrt{e\,x^{3}+d}}{935e^{2}}-\frac{2id\left(187a\,e^{2}-34bde+16c\,d^{2}\right)\sqrt{3}\left(-d\,e^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-d\,e^{2}\right)^{\frac{1}{3}}}{2e}-\cdots\right)}{\left(-d\cdots\right.}}\cdots}{\cdots}$
elliptic	$\frac{2c\,x^{7}\sqrt{e\,x^{3}+d}}{17}+\frac{2\left(be+\frac{3cd}{17}\right)x^{4}\sqrt{e\,x^{3}+d}}{11e}+\frac{2\left(ae+bd-\frac{8d\left(be+\frac{3cd}{17}\right)}{11e}\right)x\sqrt{e\,x^{3}+d}}{5e}-\frac{2i\left(da-\frac{2d\left(ae+bd-\frac{8d\left(be+\frac{3cd}{17}\right)}{11e}\right)}{5e}\right)\sqrt{3}\left(-d\,e^{2}\right)\cdots}{\cdots}$
default	Expression too large to display

```
[In] int((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/935*x*(55*c*e^2*x^6+85*b*e^2*x^3+15*c*d*e*x^3+187*a*e^2+51*b*d*e-24*c*d^2
)/e^2*(e*x^3+d)^(1/2)-2/935*I*d*(187*a*e^2-34*b*d*e+16*c*d^2)/e^3*3^(1/2)*(
-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^
(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3
```

```
)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*
3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1
/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-
d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.33

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx$$
$$= \frac{2\left(3\left(16\,cd^3-34\,bd^2e+187\,ade^2\right)\sqrt{e}\text{weierstrassPInverse}\left(0,-\frac{4\,d}{e},x\right)+\left(55\,ce^3x^7+5\left(3\,cde^2+17\,be^3\right)x^4-\right.}{935\,e^3}$$

```
[In] integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 2/935*(3*(16*c*d^3 - 34*b*d^2*e + 187*a*d*e^2)*sqrt(e)*weierstrassPInverse(
0, -4*d/e, x) + (55*c*e^3*x^7 + 5*(3*c*d*e^2 + 17*b*e^3)*x^4 - (24*c*d^2*e
- 51*b*d*e^2 - 187*a*e^3)*x)*sqrt(e*x^3 + d))/e^3
```

Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.39

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx = \frac{a\sqrt{d}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{b\sqrt{d}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{c\sqrt{d}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((e*x**3+d)**(1/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*sqrt(d)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/d)
/(3*gamma(4/3)) + b*sqrt(d)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e*x*
*3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + c*sqrt(d)*x**7*gamma(7/3)*hyper((-1/
2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3))
```

Maxima [F]

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx = \int \left(cx^6+bx^3+a\right)\sqrt{ex^3+d}\,dx$$

```
[In] integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d), x)
```

Giac [F]

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx = \int \left(cx^6+bx^3+a\right)\sqrt{ex^3+d}\,dx$$

```
[In] integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx = \int \sqrt{e\,x^3+d}\,\left(c\,x^6+b\,x^3+a\right)\,dx$$

```
[In] int((d + e*x^3)^(1/2)*(a + b*x^3 + c*x^6),x)
[Out] int((d + e*x^3)^(1/2)*(a + b*x^3 + c*x^6), x)
```

3.38 $\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 278

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = -\frac{2(8cd-11be)x\sqrt{d+ex^3}}{55e^2}+\frac{2cx^4\sqrt{d+ex^3}}{11e} + \frac{2\sqrt{2+\sqrt{3}}(16cd^2-22bde+55ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}\ldots}{\left(1+\sqrt{3}\right)\sqrt[3]{d}\ldots}\right.\right.}{55\sqrt[4]{3}e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

```
[Out] -2/55*(-11*b*e+8*c*d)*x*(e*x^3+d)^(1/2)/e^2+2/11*c*x^4*(e*x^3+d)^(1/2)/e+2/
165*(55*a*e^2-22*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d
^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(
1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+
e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used

$= \{1425, 396, 224\}$

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}(55ae^2-22bde+16cd^2)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right),-7-4\sqrt{3}\right)}{55\sqrt[4]{3}e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$
$$-\frac{2x\sqrt{d+ex^3}(8cd-11be)}{55e^2}+\frac{2cx^4\sqrt{d+ex^3}}{11e}$$

```
[In] Int[(a + b*x^3 + c*x^6)/Sqrt[d + e*x^3],x]
```

```
[Out] (-2*(8*c*d - 11*b*e)*x*Sqrt[d + e*x^3])/(55*e^2) + (2*c*x^4*Sqrt[d + e*x^3]
)/(11*e) + (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 22*b*d*e + 55*a*e^2)*(d^(1/3) +
 e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])
*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*
x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*3^(1/4)*e^(7/
3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)
^2]*Sqrt[d + e*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1425

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
```

```
^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2cx^4\sqrt{d+ex^3}}{11e} + \frac{2\int \frac{\frac{11ae}{2}-\left(4cd-\frac{11be}{2}\right)x^3}{\sqrt{d+ex^3}}\,dx}{11e}$$
$$= -\frac{2(8cd-11be)x\sqrt{d+ex^3}}{55e^2} + \frac{2cx^4\sqrt{d+ex^3}}{11e} - \frac{1}{55}\left(-55a - \frac{2d(8cd-11be)}{e^2}\right)\int \frac{1}{\sqrt{d+ex^3}}\,dx$$
$$= -\frac{2(8cd-11be)x\sqrt{d+ex^3}}{55e^2} + \frac{2cx^4\sqrt{d+ex^3}}{11e}$$
$$+ \frac{2\sqrt{2+\sqrt{3}}(16cd^2-22bde+55ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+}\right.\right.}{55\sqrt[4]{3}e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.35

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx$$
$$= \frac{x\left(-2(d+ex^3)(8cd-11be-5cex^3)+(16cd^2+11e(-2bd+5ae))\sqrt{1+\frac{ex^3}{d}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\right.\right.}{55e^2\sqrt{d+ex^3}}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)/Sqrt[d + e*x^3],x]
[Out] (x*(-2*(d + e*x^3)*(8*c*d - 11*b*e - 5*c*e*x^3) + (16*c*d^2 + 11*e*(-2*b*d
+ 5*a*e))*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)
]))/(55*e^2*Sqrt[d + e*x^3])
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.20

method	result
risch	$\frac{2x\left(5c\,x^{3}e+11be-8cd\right)\sqrt{e\,x^{3}+d}}{55e^{2}}-\frac{2i\left(55a\,e^{2}-22bde+16c\,d^{2}\right)\sqrt{3}\,\left(-d\,e^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-d\,e^{2}\right)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\,\left(-d\,e^{2}\right)^{\frac{1}{3}}}{2e}\right)\sqrt{3}\,e}{\left(-d\,e^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-d\,e^{2}\right)^{\frac{1}{3}}}{e}}{-\frac{3\left(-d\,e^{2}\right)^{\frac{1}{3}}}{2e}+\frac{i\sqrt{3}\cdots}{\cdots}}}\cdots}{\cdots}$
elliptic	$\frac{2c\,x^{4}\sqrt{e\,x^{3}+d}}{11e}+\frac{2\left(b-\frac{8cd}{11e}\right)x\sqrt{e\,x^{3}+d}}{5e}-\frac{2i\left(a-\frac{2d\left(b-\frac{8cd}{11e}\right)}{5e}\right)\sqrt{3}\,\left(-d\,e^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-d\,e^{2}\right)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\,\left(-d\,e^{2}\right)^{\frac{1}{3}}}{2e}\right)\sqrt{3}\,e}{\left(-d\,e^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-\cdots\right)}{\cdots}}{-\frac{3\left(-d\,e^{2}\right)^{\frac{1}{3}}}{2e}+\cdots}}\cdots}{\cdots}$
default	Expression too large to display

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/55*x*(5*c*e*x^3+11*b*e-8*c*d)/e^2*(e*x^3+d)^(1/2)-2/165*I*(55*a*e^2-22*b*
d*e+16*c*d^2)/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3
^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(
1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+
1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/
3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-
1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/
e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1
/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.26

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = \frac{2\left(\left(16\,cd^2-22\,bde+55\,ae^2\right)\sqrt{e}\,\text{weierstrassPInverse}\left(0,-\frac{4\,d}{e},x\right)+\left(5\,ce^2x^4-\left(8\,cde-11\,be^2\right)x\right)\sqrt{ex^3+d}\right)}{55\,e^3}$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/55*((16*c*d^2 - 22*b*d*e + 55*a*e^2)*sqrt(e)*weierstrassPInverse(0, -4*d/
e, x) + (5*c*e^2*x^4 - (8*c*d*e - 11*b*e^2)*x)*sqrt(e*x^3 + d))/e^3
```

Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.43

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = \frac{ax\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(1/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(
d)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), e*x**3*exp_pol
ar(I*pi)/d)/(3*sqrt(d)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((1/2, 7/3), (1
0/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(10/3))
```

Maxima [F]

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = \int \frac{cx^6+bx^3+a}{\sqrt{ex^3+d}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)
```

Giac [F]

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = \int \frac{cx^6+bx^3+a}{\sqrt{ex^3+d}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + bx^3 + cx^6}{\sqrt{d + ex^3}}\, dx = \int \frac{c\, x^6 + b\, x^3 + a}{\sqrt{e\, x^3 + d}}\, dx$$

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(1/2),x)
[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(1/2), x)
```

3.39 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 289

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx = \frac{2(cd^2-bde+ae^2)\,x}{3de^2\sqrt{d+ex^3}}+\frac{2cx\sqrt{d+ex^3}}{5e^2} - \frac{2\sqrt{2+\sqrt{3}}(16cd^2-5e(2bd+ae))\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}\right)\right.}{15\sqrt[4]{3}de^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

```
[Out] 2/3*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(1/2)+2/5*c*x*(e*x^3+d)^(1/2)/e^2
-2/45*(16*c*d^2-5*e*(a*e+2*b*d))*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d
^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(
1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3
)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used

$= \{1423, 396, 224\}$

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}(16cd^2-5e(ae+2bd))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right.\right.}{15\sqrt[4]{3}de^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

$$+\frac{2x(ae^2-bde+cd^2)}{3de^2\sqrt{d+ex^3}}+\frac{2cx\sqrt{d+ex^3}}{5e^2}$$

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*Sqrt[d + e*x^3]) + (2*c*x*Sqrt[d + e
*x^3])/(5*e^2) - (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 5*e*(2*b*d + a*e))*(d^(1/
3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt
[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1
/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(15*3^(1/4)*d
*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1
/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Sim
p[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{2(cd^2-bde+ae^2)\,x}{3de^2\sqrt{d+ex^3}} - \frac{2\int \frac{\frac{1}{2}(2cd^2-e(2bd+ae))-\frac{3}{2}cdex^3}{\sqrt{d+ex^3}}\,dx}{3de^2}$$

$$= \frac{2(cd^2-bde+ae^2)\,x}{3de^2\sqrt{d+ex^3}} + \frac{2cx\sqrt{d+ex^3}}{5e^2} - \frac{(16cd^2-5e(2bd+ae))\int \frac{1}{\sqrt{d+ex^3}}\,dx}{15de^2}$$

$$= \frac{2(cd^2-bde+ae^2)\,x}{3de^2\sqrt{d+ex^3}} + \frac{2cx\sqrt{d+ex^3}}{5e^2} - \frac{2\sqrt{2+\sqrt{3}}(16cd^2-5e(2bd+ae))\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{\cdot}}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{\cdot}}\right.\right.}{15\sqrt[4]{3}de^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.35

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx = \frac{x\left(2(5e(-bd+ae)+cd(8d+3ex^3))+(-16cd^2+5e(2bd+ae))\sqrt{1+\frac{ex^3}{d}}\,\text{Hypergeo}\right.}{15de^2\sqrt{d+ex^3}}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x]
[Out] (x*(2*(5*e*(-(b*d) + a*e) + c*d*(8*d + 3*e*x^3)) + (-16*c*d^2 + 5*e*(2*b*d
+ a*e))*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)])
)/(15*d*e^2*Sqrt[d + e*x^3])
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.32

method	result
elliptic	$\frac{2x(a\,e^2-bde+c\,d^2)}{3e^2d\sqrt{\left(x^3+\frac{d}{e}\right)e}}+\frac{2cx\sqrt{e\,x^3+d}}{5e^2}-\frac{2i\left(\frac{be-cd}{e^2}+\frac{a\,e^2-bde+c\,d^2}{3e^2d}-\frac{2cd}{5e^2}\right)\sqrt{3}\,(-d\,e^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-d\,e^2)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\,(-d\,e^2)^{\frac{1}{3}}}{2e}\right)\sqrt{3}\,e}{(-d\,e^2)^{\frac{1}{3}}}}\sqrt{\frac{}{-\frac{3(-}{}}}}{}$
default	Expression too large to display
risch	Expression too large to display

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/e^2/d*x*(a*e^2-b*d*e+c*d^2)/((x^3+1/e*d)*e)^(1/2)+2/5*c*x*(e*x^3+d)^(1/
2)/e^2-2/3*I*((b*e-c*d)/e^2+1/3*(a*e^2-b*d*e+c*d^2)/e^2/d-2/5*c/e^2*d)*3^(1
/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1
/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^
2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)
+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^
(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*
e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3
/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.43

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx =$$

$$-\frac{2\left((16\,cd^3-10\,bd^2e-5\,ade^2+(16\,cd^2e-10\,bde^2-5\,ae^3)x^3)\sqrt{e}\mathrm{weierstrassPInverse}\left(0,-\frac{4\,d}{e},x\right)-(3\,cde^2x\right.}{15\,(de^4x^3+d^2e^3)}$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*((16*c*d^3 - 10*b*d^2*e - 5*a*d*e^2 + (16*c*d^2*e - 10*b*d*e^2 - 5*a*
e^3)*x^3)*sqrt(e)*weierstrassPInverse(0, -4*d/e, x) - (3*c*d*e^2*x^4 + (8*c
*d^2*e - 5*b*d*e^2 + 5*a*e^3)*x)*sqrt(e*x^3 + d))/(d*e^4*x^3 + d^2*e^3)
```

Sympy [A] (verification not implemented)

Time = 6.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.41

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx = \frac{ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(3/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(3
/2)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), e*x**3*exp_po
lar(I*pi)/d)/(3*d**(3/2)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((3/2, 7/3),
(10/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(3/2)*gamma(10/3))
```

Maxima [F]

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx = \int \frac{cx^6+bx^3+a}{(ex^3+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(3/2), x)
```

Giac [F]

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx = \int \frac{cx^6+bx^3+a}{(ex^3+d)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx=\int \frac{c\,x^6+b\,x^3+a}{(e\,x^3+d)^{3/2}}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x)
[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2), x)
```

3.40 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 309

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}} \, dx = \frac{2(cd^2-bde+ae^2)\,x}{9de^2\,(d+ex^3)^{3/2}} - \frac{2(11cd^2-2bde-7ae^2)\,x}{27d^2e^2\sqrt{d+ex^3}}$$
$$+ \frac{2\sqrt{2+\sqrt{3}}(16cd^2+e(2bd+7ae))\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\ldots}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\ldots}\right.\right.}{27\sqrt[4]{3}d^2e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

```
[Out] 2/9*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(3/2)-2/27*(-7*a*e^2-2*b*d*e+11*c
*d^2)*x/d^2/e^2/(e*x^3+d)^(1/2)+2/81*(16*c*d^2+e*(7*a*e+2*b*d))*(d^(1/3)+e^
(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*
x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^2/e^(7/3)
/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)
))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1423, 393, 224}

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(e(7ae+2bd)+16cd^2\right)\text{EllipticF}\left(\text{arcsi}\right.}{27\sqrt[4]{3}d^2e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

$$-\frac{2x(-7ae^2-2bde+11cd^2)}{27d^2e^2\sqrt{d+ex^3}}+\frac{2x(ae^2-bde+cd^2)}{9de^2\left(d+ex^3\right)^{3/2}}$$

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(9*d*e^2*(d + e*x^3)^(3/2)) - (2*(11*c*d^2 -
2*b*d*e - 7*a*e^2)*x)/(27*d^2*e^2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(
16*c*d^2 + e*(2*b*d + 7*a*e))*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)
*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)
*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*d^2*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(
1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
```

```
e^2*n*(q + 1))), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Sim
p[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{2(cd^2-bde+ae^2)x}{9de^2(d+ex^3)^{3/2}} - \frac{2\int \frac{\frac{1}{2}(2cd^2-e(2bd+7ae))-\frac{9}{2}cdex^3}{(d+ex^3)^{3/2}}\,dx}{9de^2}$$

$$= \frac{2(cd^2-bde+ae^2)x}{9de^2(d+ex^3)^{3/2}} - \frac{2(11cd^2-2bde-7ae^2)x}{27d^2e^2\sqrt{d+ex^3}} - \frac{\left(4\left(-\frac{9}{2}cd^2e+\frac{1}{4}e(2cd^2-e(2bd+7ae))\right)\right)\int \frac{1}{\sqrt{d+ex^3}}\,dx}{27d^2e^3}$$

$$= \frac{2(cd^2-bde+ae^2)x}{9de^2(d+ex^3)^{3/2}} - \frac{2(11cd^2-2bde-7ae^2)x}{27d^2e^2\sqrt{d+ex^3}} + \frac{2\sqrt{2+\sqrt{3}}(16cd^2+e(2bd+7ae))\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{\cdots}}\right.\right.}{27\sqrt[4]{3}d^2e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.42

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}}\,dx = \frac{-2x(cd^2(8d+11ex^3)+e(bd(d-2ex^3)-ae(10d+7ex^3)))+(16cd^2+e(2bd+7ae)\cdots}{27d^2e^2(d+ex^3)^{3/2}}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x]
```

```
[Out] (-2*x*(c*d^2*(8*d + 11*e*x^3) + e*(b*d*(d - 2*e*x^3) - a*e*(10*d + 7*e*x^3)
)) + (16*c*d^2 + e*(2*b*d + 7*a*e))*x*(d + e*x^3)*Sqrt[1 + (e*x^3)/d]*Hyper
geometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)])/(27*d^2*e^2*(d + e*x^3)^(3/2))
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.30

method	result
elliptic	$\frac{2x(a\,e^2-bde+c\,d^2)\sqrt{e\,x^3+d}}{9d\,e^4\left(x^3+\frac{d}{e}\right)^2}+\frac{2x(7a\,e^2+2bde-11c\,d^2)}{27e^2d^2\sqrt{\left(x^3+\frac{d}{e}\right)e}}-\frac{2i\left(\frac{c}{e^2}+\frac{7a\,e^2+2bde-11c\,d^2}{27e^2d^2}\right)\sqrt{3}\,(-d\,e^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-d\,e^2)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\,(-d\,e^2)^{\frac{1}{3}}}{2e}\right.}{(-d\,e^2)^{\frac{1}{3}}}}}{}$
default	Expression too large to display

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9/d*x/e^4*(a*e^2-b*d*e+c*d^2)*(e*x^3+d)^(1/2)/(x^3+1/e*d)^2+2/27/e^2/d^2*
x*(7*a*e^2+2*b*d*e-11*c*d^2)/((x^3+1/e*d)*e)^(1/2)-2/3*I*(c/e^2+1/27/e^2/d^
2*(7*a*e^2+2*b*d*e-11*c*d^2))*3^(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)
^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-
1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))
^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*
e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(
-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/
2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e
^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.61

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}}\,dx = \frac{2\left(\left(\left(16\,cd^2e^2+2\,bde^3+7\,ae^4\right)x^6+16\,cd^4+2\,bd^3e+7\,ad^2e^2+2\left(16\,cd^3e+2\,bd^2e^2+7\right.\right.\right.}{}$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/27*(((16*c*d^2*e^2 + 2*b*d*e^3 + 7*a*e^4)*x^6 + 16*c*d^4 + 2*b*d^3*e + 7*
a*d^2*e^2 + 2*(16*c*d^3*e + 2*b*d^2*e^2 + 7*a*d*e^3)*x^3)*sqrt(e)*weierstra
ssPInverse(0, -4*d/e, x) - ((11*c*d^2*e^2 - 2*b*d*e^3 - 7*a*e^4)*x^4 + (8*c
*d^3*e + b*d^2*e^2 - 10*a*d*e^3)*x)*sqrt(e*x^3 + d))/(d^2*e^5*x^6 + 2*d^3*e
^4*x^3 + d^4*e^3)
```

Sympy [A] (verification not implemented)

Time = 34.98 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.39

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}}\,dx = \frac{ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{7}{3},\frac{5}{2}\\ \frac{10}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(5/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(5
/2)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), e*x**3*exp_po
lar(I*pi)/d)/(3*d**(5/2)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((7/3, 5/2),
(10/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(5/2)*gamma(10/3))
```

Maxima [F]

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}}\,dx = \int \frac{cx^6+bx^3+a}{(ex^3+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(5/2), x)
```

Giac [F]

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}}\,dx = \int \frac{cx^6+bx^3+a}{(ex^3+d)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}}\,dx=\int \frac{c\,x^6+b\,x^3+a}{(e\,x^3+d)^{5/2}}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x)
[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2), x)
```

3.41 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 349

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx = \frac{2(cd^2-bde+ae^2)\,x}{15de^2\,(d+ex^3)^{5/2}} - \frac{2(17cd^2-2bde-13ae^2)\,x}{135d^2e^2\,(d+ex^3)^{3/2}} + \frac{2(16cd^2+14bde+91ae^2)\,x}{405d^3e^2\sqrt{d+ex^3}} + \frac{2\sqrt{2+\sqrt{3}}(16cd^2+14bde+91ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{}}\right.\right.}{405\sqrt[4]{3}d^3e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

```
[Out] 2/15*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(5/2)-2/135*(-13*a*e^2-2*b*d*e+1
7*c*d^2)*x/d^2/e^2/(e*x^3+d)^(3/2)+2/405*(91*a*e^2+14*b*d*e+16*c*d^2)*x/d^3
/e^2/(e*x^3+d)^(1/2)+2/1215*(91*a*e^2+14*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x
)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/
3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^3/e^(7/3)/(e*x^3
+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1423, 393, 205, 224}

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}(91ae^2+14bde+16cd^2)\,\text{EllipticF}\left(\text{arc}\right.}{405\sqrt[4]{3}d^3e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

$$-\frac{2x(-13ae^2-2bde+17cd^2)}{135d^2e^2(d+ex^3)^{3/2}}+\frac{2x(ae^2-bde+cd^2)}{15de^2(d+ex^3)^{5/2}}+\frac{2x(91ae^2+14bde+16cd^2)}{405d^3e^2\sqrt{d+ex^3}}$$

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(15*d*e^2*(d + e*x^3)^(5/2)) - (2*(17*c*d^2 -
 2*b*d*e - 13*a*e^2)*x)/(135*d^2*e^2*(d + e*x^3)^(3/2)) + (2*(16*c*d^2 + 14
*b*d*e + 91*a*e^2)*x)/(405*d^3*e^2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]]*
(16*c*d^2 + 14*b*d*e + 91*a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1
/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1
/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*d^3*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) +
 e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Sim
p[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{2(cd^2-bde+ae^2)\,x}{15de^2\left(d+ex^3\right)^{5/2}} - \frac{2\int \frac{\frac{1}{2}\left(2cd^2-e(2bd+13ae)\right)-\frac{15}{2}cdex^3}{\left(d+ex^3\right)^{5/2}}\,dx}{15de^2}$$

$$= \frac{2(cd^2-bde+ae^2)\,x}{15de^2\left(d+ex^3\right)^{5/2}} - \frac{2(17cd^2-2bde-13ae^2)\,x}{135d^2e^2\left(d+ex^3\right)^{3/2}} + \frac{(16cd^2+14bde+91ae^2)\int \frac{1}{\left(d+ex^3\right)^{3/2}}\,dx}{135d^2e^2}$$

$$= \frac{2(cd^2-bde+ae^2)\,x}{15de^2\left(d+ex^3\right)^{5/2}} - \frac{2(17cd^2-2bde-13ae^2)\,x}{135d^2e^2\left(d+ex^3\right)^{3/2}} + \frac{2(16cd^2+14bde+91ae^2)\,x}{405d^3e^2\sqrt{d+ex^3}} + \frac{(16cd^2+14bde+91ae^2)\int \frac{1}{\sqrt{d+ex^3}}\,dx}{405d^3e^2}$$

$$= \frac{2(cd^2-bde+ae^2)\,x}{15de^2\left(d+ex^3\right)^{5/2}} - \frac{2(17cd^2-2bde-13ae^2)\,x}{135d^2e^2\left(d+ex^3\right)^{3/2}} + \frac{2(16cd^2+14bde+91ae^2)\,x}{405d^3e^2\sqrt{d+ex^3}} + \frac{2\sqrt{2+\sqrt{3}}(16cd^2+14bde+91ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+}\right.\right.}{405\sqrt[4]{3}d^3e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.48

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx = \frac{2x(cd^2(-8d^2-19dex^3+16e^2x^6)+e(bd(-7d^2+34dex^3+14e^2x^6)+ae(157d^2+221d}{40}$$

[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x]

[Out] (2*x*(c*d^2*(-8*d^2 - 19*d*e*x^3 + 16*e^2*x^6) + e*(b*d*(-7*d^2 + 34*d*e*x^3 + 14*e^2*x^6) + a*e*(157*d^2 + 221*d*e*x^3 + 91*e^2*x^6))) + (16*c*d^2 + 7*e*(2*b*d + 13*a*e))*x*(d + e*x^3)^2*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)])/(405*d^3*e^2*(d + e*x^3)^(5/2))

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.25

method	result
elliptic	$\frac{2x(a e^2-bde+c d^2)\sqrt{e x^3+d}}{15d e^5\left(x^3+\frac{d}{e}\right)^3}+\frac{2x(13a e^2+2bde-17c d^2)\sqrt{e x^3+d}}{135d^2e^4\left(x^3+\frac{d}{e}\right)^2}+\frac{2x(91a e^2+14bde+16c d^2)}{405e^2d^3\sqrt{\left(x^3+\frac{d}{e}\right)e}}-\frac{2i(91a e^2+14bde+16c d^2)\sqrt{3}\,(-}{}$
default	Expression too large to display

[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x,method=_RETURNVERBOSE)

[Out] 2/15/d*x/e^5*(a*e^2-b*d*e+c*d^2)*(e*x^3+d)^(1/2)/(x^3+1/e*d)^3+2/135/d^2*x*(13*a*e^2+2*b*d*e-17*c*d^2)/e^4*(e*x^3+d)^(1/2)/(x^3+1/e*d)^2+2/405/e^2/d^3*x*(91*a*e^2+14*b*d*e+16*c*d^2)/((x^3+1/e*d)*e)^(1/2)-2/1215*I*(91*a*e^2+14*b*d*e+16*c*d^2)/d^3/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.77

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx = \frac{2\left(\left(\left(16\,cd^2e^3+14\,bde^4+91\,ae^5\right)x^9+3\left(16\,cd^3e^2+14\,bd^2e^3+91\,ade^4\right)x^6+16\,cd^5+1\right.\right.}{}$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/405*(((16*c*d^2*e^3 + 14*b*d*e^4 + 91*a*e^5)*x^9 + 3*(16*c*d^3*e^2 + 14*b
*d^2*e^3 + 91*a*d*e^4)*x^6 + 16*c*d^5 + 14*b*d^4*e + 91*a*d^3*e^2 + 3*(16*c
*d^4*e + 14*b*d^3*e^2 + 91*a*d^2*e^3)*x^3)*sqrt(e)*weierstrassPInverse(0, -
4*d/e, x) + ((16*c*d^2*e^3 + 14*b*d*e^4 + 91*a*e^5)*x^7 - (19*c*d^3*e^2 - 3
4*b*d^2*e^3 - 221*a*d*e^4)*x^4 - (8*c*d^4*e + 7*b*d^3*e^2 - 157*a*d^2*e^3)*
x)*sqrt(e*x^3 + d))/(d^3*e^6*x^9 + 3*d^4*e^5*x^6 + 3*d^5*e^4*x^3 + d^6*e^3)
```

Sympy [A] (verification not implemented)

Time = 146.82 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.34

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx = \frac{ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{7}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{7}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{7}{2}\\ \frac{7}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{7}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{7}{3},\frac{7}{2}\\ \frac{10}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{7}{2}}\Gamma\left(\frac{10}{3}\right)}$$

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(7/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 7/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(7
/2)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((4/3, 7/2), (7/3,), e*x**3*exp_po
lar(I*pi)/d)/(3*d**(7/2)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((7/3, 7/2),
(10/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(7/2)*gamma(10/3))
```

Maxima [F]

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx = \int \frac{cx^6+bx^3+a}{(ex^3+d)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(7/2), x)
```

Giac [F]

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx = \int \frac{cx^6+bx^3+a}{(ex^3+d)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx = \int \frac{c\,x^6+b\,x^3+a}{{(e\,x^3+d)}^{7/2}}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x)
[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2), x)
```

3.42 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 389

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}}\,dx = \frac{2(cd^2-bde+ae^2)\,x}{21de^2\,(d+ex^3)^{7/2}} - \frac{2(23cd^2-2bde-19ae^2)\,x}{315d^2e^2\,(d+ex^3)^{5/2}}$$
$$+\frac{2(16cd^2+26bde+247ae^2)\,x}{2835d^3e^2\,(d+ex^3)^{3/2}} + \frac{2(16cd^2+26bde+247ae^2)\,x}{1215d^4e^2\sqrt{d+ex^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}(16cd^2+26bde+247ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}\right),\ldots\right)}{1215\sqrt[4]{3}d^4e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

```
[Out] 2/21*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(7/2)-2/315*(-19*a*e^2-2*b*d*e+2
3*c*d^2)*x/d^2/e^2/(e*x^3+d)^(5/2)+2/2835*(247*a*e^2+26*b*d*e+16*c*d^2)*x/d
^3/e^2/(e*x^3+d)^(3/2)+2/1215*(247*a*e^2+26*b*d*e+16*c*d^2)*x/d^4/e^2/(e*x^
3+d)^(1/2)+2/3645*(247*a*e^2+26*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*Ellipti
cF((e^(1/3)*x+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(
e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^4/e^(7/3)/(e*x^3+d)^(1/2)
/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1423, 393, 205, 224}

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}(247ae^2+26bde+16cd^2)\,\text{EllipticF}\left(\text{ar}\right.}{1215\sqrt[4]{3}d^4e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

$$-\frac{2x(-19ae^2-2bde+23cd^2)}{315d^2e^2\,(d+ex^3)^{5/2}}+\frac{2x(ae^2-bde+cd^2)}{21de^2\,(d+ex^3)^{7/2}}$$
$$+\frac{2x(247ae^2+26bde+16cd^2)}{1215d^4e^2\sqrt{d+ex^3}}+\frac{2x(247ae^2+26bde+16cd^2)}{2835d^3e^2\,(d+ex^3)^{3/2}}$$

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(21*d*e^2*(d + e*x^3)^(7/2)) - (2*(23*c*d^2 -
 2*b*d*e - 19*a*e^2)*x)/(315*d^2*e^2*(d + e*x^3)^(5/2)) + (2*(16*c*d^2 + 26
*b*d*e + 247*a*e^2)*x)/(2835*d^3*e^2*(d + e*x^3)^(3/2)) + (2*(16*c*d^2 + 26
*b*d*e + 247*a*e^2)*x)/(1215*d^4*e^2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]
]*(16*c*d^2 + 26*b*d*e + 247*a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d
^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e
^(1/3)*x)], -7 - 4*Sqrt[3]])/(1215*3^(1/4)*d^4*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/
3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Sim
p[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(cd^2-bde+ae^2)\,x}{21de^2\,(d+ex^3)^{7/2}} - \frac{2\int \frac{\frac{1}{2}(2cd^2-e(2bd+19ae))-\frac{21}{2}cdex^3}{(d+ex^3)^{7/2}}\,dx}{21de^2} \\
&= \frac{2(cd^2-bde+ae^2)\,x}{21de^2\,(d+ex^3)^{7/2}} - \frac{2(23cd^2-2bde-19ae^2)\,x}{315d^2e^2\,(d+ex^3)^{5/2}} + \frac{(16cd^2+26bde+247ae^2)\int \frac{1}{(d+ex^3)^{5/2}}\,dx}{315d^2e^2} \\
&= \frac{2(cd^2-bde+ae^2)\,x}{21de^2\,(d+ex^3)^{7/2}} - \frac{2(23cd^2-2bde-19ae^2)\,x}{315d^2e^2\,(d+ex^3)^{5/2}} \\
&\quad + \frac{2(16cd^2+26bde+247ae^2)\,x}{2835d^3e^2\,(d+ex^3)^{3/2}} + \frac{(16cd^2+26bde+247ae^2)\int \frac{1}{(d+ex^3)^{3/2}}\,dx}{405d^3e^2} \\
&= \frac{2(cd^2-bde+ae^2)\,x}{21de^2\,(d+ex^3)^{7/2}} - \frac{2(23cd^2-2bde-19ae^2)\,x}{315d^2e^2\,(d+ex^3)^{5/2}} + \frac{2(16cd^2+26bde+247ae^2)\,x}{2835d^3e^2\,(d+ex^3)^{3/2}} \\
&\quad + \frac{2(16cd^2+26bde+247ae^2)\,x}{1215d^4e^2\sqrt{d+ex^3}} + \frac{(16cd^2+26bde+247ae^2)\int \frac{1}{\sqrt{d+ex^3}}\,dx}{1215d^4e^2}
\end{aligned}$$

$$= \frac{2(cd^2 - bde + ae^2)\,x}{21de^2\,(d+ex^3)^{7/2}} - \frac{2(23cd^2 - 2bde - 19ae^2)\,x}{315d^2e^2\,(d+ex^3)^{5/2}} + \frac{2(16cd^2 + 26bde + 247ae^2)\,x}{2835d^3e^2\,(d+ex^3)^{3/2}} + \frac{2(16cd^2 + 26bde + 247ae^2)\,x}{1215d^4e^2\sqrt{d+ex^3}} + \frac{2\sqrt{2+\sqrt{3}}(16cd^2 + 26bde + 247ae^2)\left(\sqrt[3]{d} + \sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3} - \sqrt[3]{d}\sqrt[3]{e}x + e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d} + \sqrt[3]{e}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d} + \cdots}{\left(1+\sqrt{3}\right)\sqrt[3]{d} + \cdots}\right.\right.}{1215\sqrt[4]{3}d^4e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d} + \sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d} + \sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.51

$$\int \frac{a + bx^3 + cx^6}{(d+ex^3)^{9/2}}\,dx = \frac{2x(cd^2(-56d^3 - 189d^2ex^3 + 384de^2x^6 + 112e^3x^9) + e(bd(-91d^3 + 756d^2ex^3 + 624de^2 \cdots}{\cdots}$$

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x]
```

```
[Out] (2*x*(c*d^2*(-56*d^3 - 189*d^2*e*x^3 + 384*d*e^2*x^6 + 112*e^3*x^9) + e*(b*
d*(-91*d^3 + 756*d^2*e*x^3 + 624*d*e^2*x^6 + 182*e^3*x^9) + a*e*(3388*d^3 +
 7182*d^2*e*x^3 + 5928*d*e^2*x^6 + 1729*e^3*x^9))) + 7*(16*c*d^2 + 13*e*(2*
b*d + 19*a*e))*x*(d + e*x^3)^3*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1
/2, 4/3, -((e*x^3)/d)])/(8505*d^4*e^2*(d + e*x^3)^(7/2))
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.24

method	result
elliptic	$\frac{2x\left(a\,e^2-bde+c\,d^2\right)\sqrt{e\,x^3+d}}{21d\,e^6\left(x^3+\frac{d}{e}\right)^4}+\frac{2x\left(19a\,e^2+2bde-23c\,d^2\right)\sqrt{e\,x^3+d}}{315d^2e^5\left(x^3+\frac{d}{e}\right)^3}+\frac{2x\left(247a\,e^2+26bde+16c\,d^2\right)\sqrt{e\,x^3+d}}{2835d^3e^4\left(x^3+\frac{d}{e}\right)^2}+\frac{2x\left(247a\,e^2+26bde+\cdots\right.}{1215e^2d^4\sqrt{\left(x^3+\cdots\right.}}$
default	Expression too large to display

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/d*x/e^6*(a*e^2-b*d*e+c*d^2)*(e*x^3+d)^(1/2)/(x^3+1/e*d)^4+2/315/d^2*x*
(19*a*e^2+2*b*d*e-23*c*d^2)/e^5*(e*x^3+d)^(1/2)/(x^3+1/e*d)^3+2/2835/d^3*x*
(247*a*e^2+26*b*d*e+16*c*d^2)/e^4*(e*x^3+d)^(1/2)/(x^3+1/e*d)^2+2/1215/e^2/
d^4*x*(247*a*e^2+26*b*d*e+16*c*d^2)/((x^3+1/e*d)*e)^(1/2)-2/3645*I*(247*a*e
^2+26*b*d*e+16*c*d^2)/d^4/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(
1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/
e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(
1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/
(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d
*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)
,(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2
)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}}\,dx = \frac{2\left(7\left((16\,cd^2e^4+26\,bde^5+247\,ae^6)x^{12}+4\,(16\,cd^3e^3+26\,bd^2e^4+247\,ade^5)x^9+16\,cd\right.\right.}{}$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/8505*(7*((16*c*d^2*e^4 + 26*b*d*e^5 + 247*a*e^6)*x^12 + 4*(16*c*d^3*e^3 +
 26*b*d^2*e^4 + 247*a*d*e^5)*x^9 + 16*c*d^6 + 26*b*d^5*e + 247*a*d^4*e^2 +
6*(16*c*d^4*e^2 + 26*b*d^3*e^3 + 247*a*d^2*e^4)*x^6 + 4*(16*c*d^5*e + 26*b*
d^4*e^2 + 247*a*d^3*e^3)*x^3)*sqrt(e)*weierstrassPInverse(0, -4*d/e, x) + (
7*(16*c*d^2*e^4 + 26*b*d*e^5 + 247*a*e^6)*x^10 + 24*(16*c*d^3*e^3 + 26*b*d^
2*e^4 + 247*a*d*e^5)*x^7 - 189*(c*d^4*e^2 - 4*b*d^3*e^3 - 38*a*d^2*e^4)*x^4
 - 7*(8*c*d^5*e + 13*b*d^4*e^2 - 484*a*d^3*e^3)*x)*sqrt(e*x^3 + d))/(d^4*e^
7*x^12 + 4*d^5*e^6*x^9 + 6*d^6*e^5*x^6 + 4*d^7*e^4*x^3 + d^8*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}}\,dx = \int \frac{cx^6+bx^3+a}{(ex^3+d)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(9/2), x)
```

Giac [F]

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}}\,dx = \int \frac{cx^6+bx^3+a}{(ex^3+d)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}}\,dx = \int \frac{c\,x^6+b\,x^3+a}{(e\,x^3+d)^{9/2}}\,dx$$

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x)
[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2), x)
```

3.43 $\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 433

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{ex}{c} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

```
[Out] e*x/c-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(c*d-b*e+
(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^
(1/2))^(3/4)-1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(
c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a
*c+b^2)^(1/2))^(3/4)-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(
1/4))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(
-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)
```

```
^(1/2))^(1/4))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/
c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1516, 1436, 218, 214, 211}

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = -\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{ex}{c}$$

```
[In] Int[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (e*x)/c - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-
b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])
/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((c*d - b*e + (b*c*d
- b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
- ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/
4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqr
t[b^2 - 4*a*c])^(3/4))
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{ex}{c} - \frac{\int \frac{ae-(cd-be)x^4}{a+bx^4+cx^8}\,dx}{c} \\
&= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} \\
&\quad + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}
\end{aligned}
$$

$$= \frac{ex}{c} - \frac{\left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}} - \sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} - \frac{\left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}} + \sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} - \frac{\left(cd - be + \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}} - \sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b-\sqrt{b^2-4ac}}} - \frac{\left(cd - be + \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}} + \sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b-\sqrt{b^2-4ac}}}$$

$$= \frac{ex}{c} - \frac{\left(cd - be + \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(cd - be + \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(cd - be - \frac{bcd - b^2e + 2ace}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.20

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{ex}{c} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{ae\log(x-\#1) - cd\log(x-\#1)\#1^4 + be\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4c}$$

```
[In] Integrate[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (e*x)/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*e*Log[x - #1] - c*d*Log[x - #1
]*#1^4 + b*e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.15

method	result	size
default	$\frac{ex}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(\left(-be+cd\right)\_R^{4}-ae\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4c}$	67
risch	$\frac{ex}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(\left(-be+cd\right)\_R^{4}-ae\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4c}$	67

```
[In] int(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] e*x/c+1/4/c*sum(((-b*e+c*d)*_R^4-a*e)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(
_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12866 vs. 2(353) = 706.

Time = 7.77 (sec) , antiderivative size = 12866, normalized size of antiderivative = 29.71

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx=\text{Too large to display}$$

```
[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx=\text{Timed out}$$

```
[In] integrate(x**4*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \int \frac{(ex^4+d)x^4}{cx^8+bx^4+a}\,dx$$

```
[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] e*x/c - integrate(-((c*d - b*e)*x^4 - a*e)/(c*x^8 + b*x^4 + a), x)/c
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

```
[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 50213, normalized size of antiderivative = 115.97

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

```
[In] int((x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
[Out] atan((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^
2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 163
84*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b
^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 1
28*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4
 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d
^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2
) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)
^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 -
 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 32
0*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*
d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*
c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*
```

d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c

```
^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + ((((4*x*
(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b
^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e
 - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c
^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2
) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3
*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3
*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*
b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c
^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*
b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c
^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*
d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c
^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a
^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*
(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d
^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*
d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^
2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8
*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b
^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*
a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^
2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*
c^8)))^(3/4) + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b
^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*
c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^
2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 2
0*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*
c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^
5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(
1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*
d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*
b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13
*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*
(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^
4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) -
 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^
2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c
^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*
a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^
```

4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/(((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) -

(16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - ((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^

(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^
3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(
16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^
7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*
d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*
b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^
2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2)
+ 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*
e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c -
 b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-
(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*
c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5
*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
 + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 -
16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) + (16*(a^3*b^6*e^5
 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a
^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6
*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22
*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c
^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^
2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c -
b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2
*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b
^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*
a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 +
240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*
b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^
3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^
5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d
*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4
*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4
) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2
*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*
a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^
3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d
*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^
3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*
e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c
^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*
c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^
2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4
*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^

```
5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48
*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(
-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 20
0*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c
^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*
b^2*c^8)))^(1/4)))*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1
/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4
 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3
*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)
^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^
3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*
(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^
3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) -
 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288
*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 +
b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + ata
n((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 -
 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*
a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b^9*
e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b
^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*
a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 -
120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*
e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) +
 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 12
8*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a
^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e
^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 9
6*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5
*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4
 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*
a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 1
28*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e
^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*
e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3
*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*
d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3
*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3
+ 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 2
```

56*a^3*b^2*c^8)))^(3/4) - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^
5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 +
13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3
*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*
c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 -
19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b
^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 1
28*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4
 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d
^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2
) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)
^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 -
 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 32
0*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*
d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5
*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6
+ a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d
^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^
2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*
b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^
3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^
4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^
4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*
a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2
) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^
2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c
 - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*
(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^
5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c
^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
- 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + ((((4*x*(40
96*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*
c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e -
1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c^4*
d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e
- 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^
3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3

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*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e
^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*
b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(
4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4
+ 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e
^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e
^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*
d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*
c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c
 - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b
^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(3/4) + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*
c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2
*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 -
 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a
^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4
*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c
^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2
) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3
*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3
*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*
b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c
^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*
b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c
^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*
d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c
^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c
^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*
e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^
3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3
*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e
- 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(
b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c
 - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 +
128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^
4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*
d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*
```

c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/
2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2
)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2
- 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 3
20*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2
*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/(((((4*x*(4096*a^4*b*c^7*d
^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*
a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c
^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*
(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d
^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*
d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^
2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8
*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b
^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*
a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^
2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*
c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12
288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5
)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6
*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3
*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c -
b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^
2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*
e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*
b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/
2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 -
 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^
9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) - (1
6*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^
7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c
^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^
2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4
 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^
4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e
^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^
5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c
^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2
*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*
b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5

)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - ((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^

2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*
a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3
*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^
2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*
a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4) + (16*(a^3*b^6*e^5 -
4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*
c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^
2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^
4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*
d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 +
5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2
)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*
c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*
c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c
- b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240
*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2
*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 +
4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^
3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4
*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) +
(4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^
3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5
*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c
^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^
5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b
^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4
*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*
d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5
*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c
^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^
8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*
b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a
^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
c^8)))^(1/4)))(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2)
- c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 +
80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e
+ 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)
^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c

^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(
4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d
^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66
*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^
3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8
*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + 2*atan
((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 -
2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a
^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - ((-(b^9*e^4
+ b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^
5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*
c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*
a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
- 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*
a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^
2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b
^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^
2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d +
 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4
 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*
a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 1
28*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e
^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*
e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3
*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*
d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3
*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3
- 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2
)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 2
56*a^3*b^2*c^8)))^(3/4)*1i + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5
*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5
 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*
b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b
^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3
 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(
-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4
+ 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*
e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^
2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^

2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(
1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b
^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^
2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 +
320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c
^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c
^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a
^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^
4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4
*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*
c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 -
4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*
c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2
) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 +
80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e
+ 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5
)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*
c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-
(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 6
6*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a
^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^
8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + ((((4*x*
(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b
^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e
- 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + ((-(b^9*e^4 + b^5*c^4*
d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) -
8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e
- 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^
3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3
*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e
^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*
b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4
*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*
d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5
*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c
^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^
8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^

(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*
b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a
^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2
*c^8)))^(3/4)*1i - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a
^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*
b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^
3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4
 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*
b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4
 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^
5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*
c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*
a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*
a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^
2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b
^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^
2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6
 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^
2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e
^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^
4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*
c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^
3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*
c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*
b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) +
6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^
2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b
^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4
*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^
3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d
^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4))/(((((4*x*(4096*a^4*
b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2
 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^
3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - ((-(b^9*e^4 + b^5*c^4*d^4 + b^4*
e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c

```
^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*
c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^
2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4
*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48
*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(
-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 20
0*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c
^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*
b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d
- 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c
- b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a
^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4
*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(
4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3
+ 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*
a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*
e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2
)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3
*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(25
6*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3
/4)*1i + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e
^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^
5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22
*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*
b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^
4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*
d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e
- 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^
3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3
*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e
^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c
^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*
e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^
3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3
*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e
```

- 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(
b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c
 - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 +
128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^
4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*
d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*
c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/
2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2
)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2
- 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 3
20*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2
*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - ((((4*x*(4096*a^4*b*c^7*d
^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*
a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c
^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + ((-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(
4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4
+ 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e
^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e
^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*
d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*
c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c
 - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b
^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288
*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^
5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^
6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^
3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c -
 b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a
^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c
e^4(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4
*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1
/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3
- 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c
^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i
- (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^
2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a
^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^
4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d
*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 3

2*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b
^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^
3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a
^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 +
a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4
- 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e +
48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e -
200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*
a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)
^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a
^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6
- 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2
*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c
^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*
d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2
*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4
+ b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)
^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3
*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120
*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2
- 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2
e^2(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40
*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a
^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*
b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a
^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i))*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e
^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^
5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c
^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2
*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*
b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*
a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-
(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200
*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^
3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b
^2*c^8)))^(1/4) + 2*atan((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2
+ 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a
^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^
6*d*e))/c - ((-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) -

c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*
a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 6
1*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1
/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*
d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a
*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*
e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*
b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b
*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^
5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8
*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*
(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4
 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2
*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c
^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b
^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e
^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2
+ 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*
c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*
c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i + (16*(a^3*b^6*e^5 - 4*a
^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5
*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b
^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b
^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4
*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*
a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5
)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6
*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3
*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c -
b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^
2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*
e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*
b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/
2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 -
 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^
9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i -
 (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^
3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5
*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c
^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^

5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b ^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4 *(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5* d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5 *d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c ^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^ 8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^ (1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a* b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4 *a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a ^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3* d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2 *c^8)))^(1/4) + ((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2* b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5 *e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + ((-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(- (4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4* e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5* c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^ 7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^ 5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^ 2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e ^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a *b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b ^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a ^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2) ^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3 *c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120 *a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2 *e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40 *a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1 /2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a ^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3* b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3* (-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a ^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*1i - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a ^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3 *e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2 *e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2

*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^
2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) -
c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*
a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 6
1*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1
/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*
d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a
*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*
e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*
b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b
*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^
5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4
*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^
5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e
^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3
 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*
b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e
^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c
- b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a
^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4
*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(
4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3
+ 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*
a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*
e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2
)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3
*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(25
6*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1
/4))/(((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^
2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 163
84*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - ((-(b^9*
e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b
^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*
a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 -
120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*
e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) +
 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 12
8*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a
^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e
^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 9
6*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5

*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4
*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e
- 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c
^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^
7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*
a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4
*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^
3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5
*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*
e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(3/4)*1i + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b
*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4
*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*
a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a
^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2
*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/
c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-
(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*
e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*
c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^
7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 -
6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^
5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^
2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e
^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a
*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b
^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 -
2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^
2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2
*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*
b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^
5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^
4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^
(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d
^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d
^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^
2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*
b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^
4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*
c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
- 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 2
88*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b

^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (
(((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 204
8*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*
c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + ((-(b^9*e^4 + b
^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^
(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6
*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3
*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 1
3*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b
^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
+ 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b
^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*
c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b
^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 30
72*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4 -
b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b
^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*
a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4
- a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4
- 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e
+ 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e
^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e
- 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6
*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*
a^3*b^2*c^8)))^(3/4)*1i - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^
5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 +
13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3
*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*
c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 -
19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b
^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 1
28*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4
- 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d
^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2
) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)
^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 -
128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 32
0*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*

```
d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a^3*
c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d
^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^
2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2
*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a
^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3
*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) -
 c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80
*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e +
61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(
1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4
*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*
a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3
*e*(-(4*a*c - b^2)^5)^(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a
*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*
b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c
^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i))*(-(b^9*e
^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - c^4*d^4*(-(4*a*c - b^
2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a
^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 1
20*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e
^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) +
40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128
*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^
3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b*c^2*d*e^
3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96
*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (e*x)/c
```

3.44 $\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = -\frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}} + \frac{e\log\left(a+bx^4+cx^8\right)}{8c}$$

```
[Out] 1/8*e*ln(c*x^8+b*x^4+a)/c-1/4*(-b*e+2*c*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)
^(1/2))/c/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1482, 648, 632, 212, 642}

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{e\log\left(a+bx^4+cx^8\right)}{8c} - \frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}$$

```
[In] Int[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] -1/4*((2*c*d - b*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 -
 4*a*c]) + (e*Log[a + b*x^4 + c*x^8])/(8*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{d+ex}{a+bx+cx^2}\,dx, x, x^4\right)\\
&= \frac{e\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8c} + \frac{(2cd-be)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)}{8c}\\
&= \frac{e\log\left(a+bx^4+cx^8\right)}{8c} - \frac{(2cd-be)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)}{4c}\\
&= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}} + \frac{e\log\left(a+bx^4+cx^8\right)}{8c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{-\frac{2(-2cd+be)\arctan\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+e\log\left(a+bx^4+cx^8\right)}{8c}$$

```
[In] Integrate[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
[Out] ((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] + e*Log[a + b*x^4 + c*x^8])/(8*c)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

method	result
default	$\frac{e\ln(c\,x^8+b\,x^4+a)}{8c}+\frac{\left(d-\frac{be}{2c}\right)\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}}$
risch	$\frac{\ln\left(\left(\left(-4abce+8a\,c^2d+b^3e-2b^2cd+\sqrt{-(be-2cd)^2(4ac-b^2)}\,b\right)x^4+2\sqrt{-(be-2cd)^2(4ac-b^2)}\,a\right)ae}{8ac-2b^2}-\frac{\ln\left(\left(-4abce+8a\,c^2d+b^3e-2b^2c\right.\right.}{}$

```
[In] int(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/8*e*ln(c*x^8+b*x^4+a)/c+1/2*(d-1/2/c*b*e)/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^4+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.00

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \left[\frac{(b^2-4\,ac)e\log\left(cx^8+bx^4+a\right)-\sqrt{b^2-4\,ac}(2\,cd-be)\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)}{8\left(b^2c-4\,ac^2\right)},\ \frac{(b^2-4}{}\right.$$

```
[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] [1/8*((b^2 - 4*a*c)*e*log(c*x^8 + b*x^4 + a) - sqrt(b^2 - 4*a*c)*(2*c*d - b
*e)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a
*c))/(c*x^8 + b*x^4 + a)))/(b^2*c - 4*a*c^2), 1/8*((b^2 - 4*a*c)*e*log(c*x^
```

```
8 + b*x^4 + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^4 + b)*s
qrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(63) = 126$.

Time = 119.86 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.99

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \left(\frac{e}{8c} - \frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right)\log\left(x^4 + \frac{-16ac\left(\frac{e}{8c} - \frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right) + 2ae + 4b^2\left(\frac{e}{8c} - \frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right) -}{be-2cd}\right) + \left(\frac{e}{8c} + \frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right)\log\left(x^4 + \frac{-16ac\left(\frac{e}{8c} + \frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right) + 2ae + 4b^2\left(\frac{e}{8c} + \frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right) -}{be-2cd}\right)$$

```
[In] integrate(x**3*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] (e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2)))*log(x**4
 + (-16*a*c*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2
))) + 2*a*e + 4*b**2*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a
*c - b**2))) - b*d)/(b*e - 2*c*d)) + (e/(8*c) + sqrt(-4*a*c + b**2)*(b*e -
2*c*d)/(8*c*(4*a*c - b**2)))*log(x**4 + (-16*a*c*(e/(8*c) + sqrt(-4*a*c + b
**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) + 2*a*e + 4*b**2*(e/(8*c) + sqrt(-
4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 1.83 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{e\log\left(cx^8+bx^4+a\right)}{8\,c} + \frac{(2\,cd-be)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}c}$$

```
[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*e*log(c*x^8 + b*x^4 + a)/c + 1/4*(2*c*d - b*e)*arctan((2*c*x^4 + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

Mupad [B] (verification not implemented)

Time = 10.39 (sec) , antiderivative size = 3704, normalized size of antiderivative = 51.44

$$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

```
[In] int((x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

```
[Out] - (log(a + b*x^4 + c*x^8)*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*c)) -
 (atan((8*x^4*(((a*c - b^2)*(((((4*b^2*e - 16*a*c*e)*(((b*e - 2*c*d)*(448*b
^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 1
6*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a*c*e)*(b*
e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16
*b^2*c)) + ((b*e - 2*c*d)*(96*b*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(448*b^3*c^
3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2
*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 144*b^3*c^2*e^2 - 240*b^2*c^3*d*e))/(8*c*
(4*a*c - b^2)^(1/2)))*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*c)) - (((
(b*e - 2*c*d)*(((b*e - 2*c*d)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4
*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) +
(32*b^3*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a
*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2)) + (4*b^3*c^2*(4*b^2*e - 16*a*c
*e)*(b*e - 2*c*d)^2)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)))*(b*e - 2*c*d))/
(8*c*(4*a*c - b^2)^(1/2)) + ((b*e - 2*c*d)*(((4*b^2*e - 16*a*c*e)*(96*b*c^4
*d^2 + ((4*b^2*e - 16*a*c*e)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*
(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 1
44*b^3*c^2*e^2 - 240*b^2*c^3*d*e))/(2*(64*a*c^2 - 16*b^2*c)) - 8*c^4*d^3 +
20*b^3*c*e^3 - 48*b^2*c^2*d*e^2 + 36*b*c^3*d^2*e))/(8*c*(4*a*c - b^2)^(1/2)
) - (b^3*c*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)^3)/(2*(64*a*c^2 - 16*b^2*c)*(
4*a*c - b^2)^(3/2))))/(8*a^3*c^2) + ((b^3 - 3*a*b*c)*(b^3*e^4 + (b^3*(b*e -
 2*c*d)^4)/(8*(4*a*c - b^2)^2) - c^3*d^3*e - ((((b*e - 2*c*d)*(((b*e - 2*c*
```

```
d)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*
a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a
*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*
a*c - b^2)^(1/2)) + (4*b^3*c^2*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)^2)/((64*a
*c^2 - 16*b^2*c)*(4*a*c - b^2)))*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^
2*c)) + ((4*b^2*e - 16*a*c*e)*(((4*b^2*e - 16*a*c*e)*(96*b*c^4*d^2 + ((4*b^
2*e - 16*a*c*e)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16
*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 144*b^3*c^2*e^
2 - 240*b^2*c^3*d*e))/(2*(64*a*c^2 - 16*b^2*c)) - 8*c^4*d^3 + 20*b^3*c*e^3
- 48*b^2*c^2*d*e^2 + 36*b*c^3*d^2*e))/(2*(64*a*c^2 - 16*b^2*c)) + 3*b*c^2*d
^2*e^2 - ((((4*b^2*e - 16*a*c*e)*(((b*e - 2*c*d)*(448*b^3*c^3*e - 384*b^2*c
^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a
*c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c
^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) + ((b*e - 2
*c*d)*(96*b*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(448*b^3*c^3*e - 384*b^2*c^4*d
+ (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 -
 16*b^2*c)) + 144*b^3*c^2*e^2 - 240*b^2*c^3*d*e))/(8*c*(4*a*c - b^2)^(1/2))
)*(b*e - 2*c*d))/(8*c*(4*a*c - b^2)^(1/2)) - 3*b^2*c*d*e^3))/(8*a^3*c^2*(4*
a*c - b^2)^(1/2)))*(4*a*c - b^2)^2)/(b^4*e^4 + 16*c^4*d^4 + 24*b^2*c^2*d^2*
e^2 - 32*b*c^3*d^3*e - 8*b^3*c*d*e^3) + ((a*c - b^2)*(4*a*c - b^2)^2*(((4*b
^2*e - 16*a*c*e)*(((((b*e - 2*c*d)*(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*
a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^
(1/2)) + (64*a*b^2*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*
b^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*c)
) + ((b*e - 2*c*d)*(64*a*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(768*a*b^2*c^3*e -
 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)
))/(2*(64*a*c^2 - 16*b^2*c)) + 208*a*b^2*c^2*e^2 - 256*a*b*c^3*d*e))/(8*c*(
4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) - ((b*e - 2*c*d)*(((((b*e -
 2*c*d)*(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c
*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (64*a*b^2*c^3*(4*b
^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))
)*(b*e - 2*c*d))/(8*c*(4*a*c - b^2)^(1/2)) + (8*a*b^2*c^2*(4*b^2*e - 16*a*c
*e)*(b*e - 2*c*d)^2)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))))/(8*c*(4*a*c -
b^2)^(1/2)) + ((b*e - 2*c*d)*(((4*b^2*e - 16*a*c*e)*(64*a*c^4*d^2 + ((4*b^2
*e - 16*a*c*e)*(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e -
 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 208*a*b^2*c
^2*e^2 - 256*a*b*c^3*d*e))/(2*(64*a*c^2 - 16*b^2*c)) + 24*a*b^2*c*e^3 + 16*
a*c^3*d^2*e - 40*a*b*c^2*d*e^2))/(8*c*(4*a*c - b^2)^(1/2)) - (a*b^2*c*(4*b^
2*e - 16*a*c*e)*(b*e - 2*c*d)^3)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(3/2)
)))/(a^3*c^2*(b^4*e^4 + 16*c^4*d^4 + 24*b^2*c^2*d^2*e^2 - 32*b*c^3*d^3*e -
8*b^3*c*d*e^3)) + ((4*a*c - b^2)^(3/2)*(b^3 - 3*a*b*c)*(a*b^2*e^4 - ((4*b^2
*e - 16*a*c*e)*(((((b*e - 2*c*d)*(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*
b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1
/2)) + (64*a*b^2*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^
2*c)*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(8*c*(4*a*c - b^2)^(1/2)) + (8*a*
```

```
b^2*c^2*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)^2)/((64*a*c^2 - 16*b^2*c)*(4*a*c
 - b^2))))/(2*(64*a*c^2 - 16*b^2*c)) + ((4*b^2*e - 16*a*c*e)*(((4*b^2*e - 1
6*a*c*e)*(64*a*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(768*a*b^2*c^3*e - 512*a*b*c
^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*
a*c^2 - 16*b^2*c)) + 208*a*b^2*c^2*e^2 - 256*a*b*c^3*d*e))/(2*(64*a*c^2 - 1
6*b^2*c)) + 24*a*b^2*c*e^3 + 16*a*c^3*d^2*e - 40*a*b*c^2*d*e^2))/(2*(64*a*c
^2 - 16*b^2*c)) + a*c^2*d^2*e^2 - ((((((b*e - 2*c*d)*(768*a*b^2*c^3*e - 512
*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(
8*c*(4*a*c - b^2)^(1/2)) + (64*a*b^2*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)
)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*e - 16*a*c*e))/(2*(64
*a*c^2 - 16*b^2*c)) + ((b*e - 2*c*d)*(64*a*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*
(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64
*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 208*a*b^2*c^2*e^2 - 256*a*
b*c^3*d*e))/(8*c*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(8*c*(4*a*c - b^2)^(1
/2)) + (a*b^2*(b*e - 2*c*d)^4)/(4*(4*a*c - b^2)^2) - 2*a*b*c*d*e^3))/(a^3*c
^2*(b^4*e^4 + 16*c^4*d^4 + 24*b^2*c^2*d^2*e^2 - 32*b*c^3*d^3*e - 8*b^3*c*d*
e^3)))*(b*e - 2*c*d))/(4*c*(4*a*c - b^2)^(1/2))
```

3.45 $\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 375

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(e+(b*e-2*c*d)/
(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/4*arcta
nh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(e+(b*e-2*c*d)/(-4*a*c+
b^2)^(1/2))*2^(1/4)/c^(3/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/4*arctan(2^(1/4
)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/
2))*2^(1/4)/c^(3/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/4*arctanh(2^(1/4)*c^(1/
```

```
4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(
1/4)/c^(3/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1524, 304, 211, 214}

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}} + \frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

```
[In] Int[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
+ ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + S
qrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)
) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
- Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1
/4)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])
^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx \\ &\quad+\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx \\ &= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}} \\ &\quad-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}+\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}\end{aligned}$$

$$= \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\left(e + \frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}} - \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(e + \frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4}c^{3/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.16

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]$$

```
[In] Integrate[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b*#1
+ 2*c*#1^5) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^6e+\_R^2d\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	51
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^6e+\_R^2d\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	51

```
[In] int(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4*sum((_R^6*e+_R^2*d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+
a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15561 vs. $2(295) = 590$.

Time = 49.64 (sec) , antiderivative size = 15561, normalized size of antiderivative = 41.50

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(e*x**4+d)/(c*x**8+b*x**4+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \int \frac{(ex^4+d)x^2}{cx^8+bx^4+a}\,dx$$

[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")

[Out] integrate((e*x^4 + d)*x^2/(c*x^8 + b*x^4 + a), x)

Giac [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] Timed out

Mupad [B] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 29445, normalized size of antiderivative = 78.52

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

[In] int((x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x)

[Out] 2*atan(((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3

*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*
a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*
c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d
*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*
c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 102
4*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*
b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^
4*d*e - 16384*a^3*b^3*c^5*d*e)*1i - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 -
4096*a^3*b*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b
^4*c^3*e^3 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^
5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) -
11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e
^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*
a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b
^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*
a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) + (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2
*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5
*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e
+ 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*
c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*
d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*
a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*
a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128
*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 -
48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*
b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c
^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*
b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*
(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(
-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(
-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^
3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e +
6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*
b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c
^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*
c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 -
32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 3276
8*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e)*1i + 4096*a
^5*c^5*e^3 + 256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e -

2048*a^2*b^3*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4))/((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e)*1i - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c

^5 - 256*a^4*b^2*c^6)))^(1/4)*1i - (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e)*1i + 4096*a^5*c^5*e^3 + 256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*1i + 2*a*c^5*d^7 + 2*a^4*c^2*d*e^6 + 6*a^2*c^4*d^5*e^2 + 6*a^3*c^3*d^3*e^4 - 2*a^4*b*c*e^7 - 8*a*b*c^4*d^6*e + 18*a^2*b^2*c^2*d^3*e^4 + 2*a*b^4*c*d^3*e^4 + 6*a^3*b^2*c*d*e^6 + 12*a*b^2*c^3*d^5*e^2 - 8*a*b^3*c^2*d^4*e^3 - 18*a^2*b*c^3*d^4*e^3 - 6*a^2*b^3*c*d^2*e^5 - 12*a^3*b*c^2*d^2*e^5))*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^

2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^
(1/4) - atan(-((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e +
16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*
e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a
^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4
*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^
5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c
^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c
^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c
^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 -
8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4
*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(2
56*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)
))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) -
8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) -
11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^
3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a
^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^
2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a
^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^
2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 20
48*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2
*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^
3 - 4096*a^3*b*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a
^3*b^4*c^3*e^3 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^
2*c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2)
- 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2)
- 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*
e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64
*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*
b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-
(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96
*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*1i + (x*(4*a^3*b^3*c*e^6 - 12*a^4*b
*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b
*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d
^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*
b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*
c^2*d*e^5) - (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) -

11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) + 4096*a^5*c^5*e^3 + 256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*1i)/((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3

*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*
c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*
e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6
*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^
(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 25
6*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*
c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*
e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e -
16384*a^3*b^3*c^5*d*e) - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*
c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3
+ 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e))*(
-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*
d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*
e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d
^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^
3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^
3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 +
6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 -
256*a^4*b^2*c^6)))^(1/4) - (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*
c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*
b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d
^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 -
36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*
b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4
+ 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4
- 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e
+ 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^
3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e
+ 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a
*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*
a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2
)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)
^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)
^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*
a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^
2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2
- 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*
c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^
6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 3276
8*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*
c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*
e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) + 4096*a^5*c^5*e^3 + 256*
a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5

*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) + 2*a*c^5*d^7 + 2*a^4*c^2*d*e^6 + 6*a^2*c^4*d^5*e^2 + 6*a^3*c^3*d^3*e^4 - 2*a^4*b*c*e^7 - 8*a*b*c^4*d^6*e + 18*a^2*b^2*c^2*d^3*e^4 + 2*a*b^4*c*d^3*e^4 + 6*a^3*b^2*c*d*e^6 + 12*a*b^2*c^3*d^5*e^2 - 8*a*b^3*c^2*d^4*e^3 - 18*a^2*b*c^3*d^4*e^3 - 6*a^2*b^3*c*d^2*e^5 - 12*a^3*b*c^2*d^2*e^5))*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*2i - atan(-((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4

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*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e -
 16384*a^3*b^3*c^5*d*e) - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b
*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3
 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e))*
(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4
*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c
*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*
d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b
^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d
^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3
 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^
5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 -
 256*a^4*b^2*c^6)))^(1/4)*1i + (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*
a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*
a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5
*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^
3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (
-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*
d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*
e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d
^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^
3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^
3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 -
 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 -
256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c -
 b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c -
b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c -
b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4
 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^
2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*
e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*
a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^
2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 -
32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*
b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^
6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) + 4096*a^5*c^5*e^3 +
256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3
*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2
*e + 6144*a^3*b^2*c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c
 - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c
- b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c
- b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^
4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*
c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^
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2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*1i)/((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) - (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3

*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4
*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*
a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c
^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(
-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256
*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))
^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8
*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11
*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 12
8*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3
- 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2
*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*
c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a
c - b^2)^5)^(1/2))/(512(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3
*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2
- 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048
*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b
^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) + 4096*a^5*c^5*e^3 + 256*a*b^5*c^4*d^3
+ 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5*d^3 + 256*a^3
*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 6144*a^3*b^2*
c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) -
8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) -
11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^
3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a
^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^
2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a
^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) + 2*a*c^5*d^7 + 2*a^4*c^2*d*e^6 + 6*a
^2*c^4*d^5*e^2 + 6*a^3*c^3*d^3*e^4 - 2*a^4*b*c*e^7 - 8*a*b*c^4*d^6*e + 18*a
^2*b^2*c^2*d^3*e^4 + 2*a*b^4*c*d^3*e^4 + 6*a^3*b^2*c*d*e^6 + 12*a*b^2*c^3*d
^5*e^2 - 8*a*b^3*c^2*d^4*e^3 - 18*a^2*b*c^3*d^4*e^3 - 6*a^2*b^3*c*d^2*e^5 -
12*a^3*b*c^2*d^2*e^5))*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2
)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)
^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)
^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*
a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^
2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2
- 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*
c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^
6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*2i + 2*atan(((x*(4*a^3*b^
3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c
^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^
4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c
^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2

*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*
c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c
 - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c
 - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e
^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5
*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d
^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16
*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 +
b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b
*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b
*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4
*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2
- 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b
^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e
^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*
(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^
6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10
240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*
a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^
3*c^5*d*e)*1i - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*c^6*d^3 +
 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 + 2048*a^
4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^
7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 +
16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 -
48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e +
 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*
d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e +
40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2
))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^
4*b^2*c^6)))^(1/4) + (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^
5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^
3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4
 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2
*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^
4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a
^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a
^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128
*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*
e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a
^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(
512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^
2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(

1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1
/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1
/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*
c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2
+ 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*
a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^
3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4
+ 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*
c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^
2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 20
48*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e)*1i + 4096*a^5*c^5*e^3 + 256*a*b
^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5*d^
3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 61
44*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b
^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^
2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^
2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 -
4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*
d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^
2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*
b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*
b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4))/((x*(4*a^3*b^3*c*e^6 -
12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^
3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*
b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^
2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16
*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^
5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5
)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5
)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*
b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*
e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 -
128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*
d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*
c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d
^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4
+ a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4
- a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^
3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4
*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*
e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*
c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4
)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b
^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c

```
^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e
)*1i - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*c^6*d^3 + 12288*a^
4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 + 2048*a^4*b^2*c^4
*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b
^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*
c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*
c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*
c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 -
 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^
4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^
2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(
256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6
)))^(1/4)*1i - (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e +
16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*
e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a
^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4
*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^
5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c
^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c
^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c
^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 -
8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4
*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(2
56*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)
))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) -
11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^
3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a
^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^
2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a
^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^
2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 20
48*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2
*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e)*1i + 4096*a^5*c^5*e^3 + 256*a*b^5*c^4
*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5*d^3 + 25
6*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 6144*a^3
*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^
6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^
2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 12
```

```
8*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*
e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^
4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*1i + 2*a*c^5*d^7 + 2*a^4*c^2*
d*e^6 + 6*a^2*c^4*d^5*e^2 + 6*a^3*c^3*d^3*e^4 - 2*a^4*b*c*e^7 - 8*a*b*c^4*d
^6*e + 18*a^2*b^2*c^2*d^3*e^4 + 2*a*b^4*c*d^3*e^4 + 6*a^3*b^2*c*d*e^6 + 12*
a*b^2*c^3*d^5*e^2 - 8*a*b^3*c^2*d^4*e^3 - 18*a^2*b*c^3*d^4*e^3 - 6*a^2*b^3*
c*d^2*e^5 - 12*a^3*b*c^2*d^2*e^5))*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(
4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4
*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4
*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c
^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a
*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c
^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3
- 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)
```

3.46 $\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 184

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e+(-b*e+2*c*d
)/(-4*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arct
an(x^2*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*c*d)/(-4*a*c
+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1504, 1180, 211}

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[In] Int[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((
e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx, x, x^2\right)\\
&= \frac{1}{4}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)\\
&\quad+\frac{1}{4}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)\\
&= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.97

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{\frac{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[In] Integrate[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b
^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.91

method	result
default	$2c\left(\frac{\left(e\sqrt{-4ac+b^2}+be-2cd\right)\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(e\sqrt{-4ac+b^2}-be+2cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)$
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16c^3a^3-8a^2b^2c^2+a\,b^4c\right)\_Z^4+\left(-4a^2bc\,e^2+16a^2c^2de+b^3e^2a-4b^2cdea-4b\,c^2d^2a+b^3c\,d^2\right)\_Z^2+a^2e^4-2abd\,e^3+2ac\,d^2e^2+b^2d^2\right.}\right.$

```
[In] int(x*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c*(1/8*(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))-1/8*(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1535 vs. $2(144) = 288$.

Time = 0.41 (sec) , antiderivative size = 1535, normalized size of antiderivative = 8.34

$$\int \frac{x(d + ex^4)}{a + bx^4 + cx^8} \, dx = \text{Too large to display}$$

```
[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*
sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))*log(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 + 1/2
*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4
*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2
+ a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2
 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*
c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 -
 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2
+ a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-(c^2*d^4
 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 - 1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*
d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c -
 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^
3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt(
(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c -
4*a^2*c^2))) + 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*
c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^
3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2
*e^4)*x^2 + 1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2
+ ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 -
 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*
c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2
*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/4*sqrt(1/2)*s
qrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 -
 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)
)*log(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 - 1/2*sqrt(1/2)*((b^
2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d -
 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2
*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4
*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3
)))/(a*b^2*c - 4*a^2*c^2)))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

```
[In] integrate(x*(e*x**4+d)/(c*x**8+b*x**4+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \int \frac{(ex^4+d)x}{cx^8+bx^4+a}\,dx$$

```
[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] integrate((e*x^4 + d)*x/(c*x^8 + b*x^4 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(144) = 288$.

Time = 2.36 (sec) , antiderivative size = 1402, normalized size of antiderivative = 7.62

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] 1/8*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)
*b*c^2)*d + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^
```

```
2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^
2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/8*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^
3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*
b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*
(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d + 2*(2*a*b^2*c^2 - 8*a^2*c^3
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arc
tan(2*sqrt(1/2)*x^2/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c
- 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

Mupad [B] (verification not implemented)

Time = 12.42 (sec) , antiderivative size = 4501, normalized size of antiderivative = 24.46

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

```
[In] int((x*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan((b^4*c*d^3*x^2*1i + a^2*b^3*e^3*x^2*1i + a^2*c^3*d^3*x^2*8i - a^2*e^3*
x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - a^3*b*c*e^3
*x^2*4i - a*b^4*d*e^2*x^2*1i - b*c*d^3*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c
^2 - 12*a*b^4*c)^(1/2)*1i - a*b^2*c^2*d^3*x^2*6i - a^3*c^2*d*e^2*x^2*8i + a
*b*d*e^2*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a*
c*d^2*e*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2
*b*c^2*d^2*e*x^2*4i + a^2*b^2*c*d*e^2*x^2*6i - a*b^3*c*d^2*e*x^2*1i)/(8*a^2
*b^4*e^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4
*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)
/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) - 1024*a^3*b^3*c^2*(-(
a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4
*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) -
4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^
3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) - 64*a^3*c^3*d^2*(-(a*b^3*e^2 + b^
3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*
d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2
```

- 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) + 64*a^4*c^2*e^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) + 128*a^2*b^5*c*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) + 2048*a^4*b*c^3*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) - 48*a^3*b^2*c*e^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) + 16*a^2*b^2*c^2*d^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) - 16*a^2*b^3*c*d*e*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) + 64*a^3*b*c^2*d*e*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2)))*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2)*2i + atan((b^4*c*d^3*x^2*1i + a^2*b^3*e^3*x^2*1i + a^2*c^3*d^3*x^2*8i + a^2*e^3*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - a^3*b*c*e^3*x^2*4i - a*b^4*d*e^2*x^2*1i + b*c*d^3*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - a*b^2*c^2*d^3*x^2*6i - a^3*c^2*d*e^2*x^2*8i - a*b*d*e^2*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - a*c*d^2*e*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*b*c^2*d^2*e*x^2*4i + a^2*b^2*c*d*e^2*x^2*6i - a*b^3*c*d^2*e*x^2*1i)/(8*a^2*b^4*e^2*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) - 1024*a^3*b^3*c^2*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d

```
^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2
*b^2*c^2 + 32*a*b^4*c))^(3/2) - 64*a^3*c^3*d^2*(-(a*b^3*e^2 + b^3*c*d^2 - a
*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c
*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*
a*b^4*c))^(1/2) + 64*a^4*c^2*e^2*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 4
8*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*
c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2
) + 128*a^2*b^5*c*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b
^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) + 2048*a^4*b*
c^3*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1
2*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512
*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) - 48*a^3*b^2*c*e^2*(-(a*b^3
*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b
*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 2
56*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) + 16*a^2*b^2*c^2*d^2*(-(a*b^3*e^2 + b^3
*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d
^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 -
 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2
*c^2 + 32*a*b^4*c))^(1/2) - 16*a^2*b^3*c*d*e*(-(a*b^3*e^2 + b^3*c*d^2 - a*e
^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e
^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*
b^4*c))^(1/2) + 64*a^3*b*c^2*d*e*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 4
8*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*
c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2
)))*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1
2*a*b^4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512
*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2)*2i
```

3.47 $\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 375

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

```
[Out] -1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(e+(b*e-2*c*d)
/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/4*arct
anh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(e+(b*e-2*c*d)/(-4*a*c
+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/4*arctan(2^(1/
4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1
/2))*2^(3/4)/c^(1/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/4*arctanh(2^(1/4)*c^(1
/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^
(3/4)/c^(1/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1436, 218, 214, 211}

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = -\frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[In] Int[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
[Out] -1/2*((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
- Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4
)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
+ Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3
/4)) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(
-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])
^(3/4)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*
c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx \\
&\quad +\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx \\
&= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{-b-\sqrt{b^2-4ac}}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{-b-\sqrt{b^2-4ac}}} \\
&\quad -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{-b+\sqrt{b^2-4ac}}}-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{-b+\sqrt{b^2-4ac}}}
\end{aligned}$$

$$= -\frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(e + \frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(e + \frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.16

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]$$

```
[In] Integrate[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b*#1^
3 + 2*c*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^4e+d\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	47
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^4e+d\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	47

```
[In] int((e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9245 vs. $2(295) = 590$.

Time = 2.70 (sec) , antiderivative size = 9245, normalized size of antiderivative = 24.65

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**4+d)/(c*x**8+b*x**4+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \int \frac{ex^4+d}{cx^8+bx^4+a}\,dx$$

```
[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] integrate((e*x^4 + d)/(c*x^8 + b*x^4 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{d + ex^4}{a + bx^4 + cx^8} \, dx = \text{Timed out}$$

```
[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 13.41 (sec) , antiderivative size = 36707, normalized size of antiderivative = 97.89

$$\int \frac{d + ex^4}{a + bx^4 + cx^8} \, dx = \text{Too large to display}$$

```
[In] int((d + e*x^4)/(a + b*x^4 + c*x^8),x)
```

```
[Out] - atan((((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11
*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*
a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128
*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e -
 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*
b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c
^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*
b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(
-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(
-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-
(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3
*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 4
0*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*
b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*
c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*
c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 4
9152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b
*c^7*d) + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 +
 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*
a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^
5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a
*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*
c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*
```

d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2
*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3
*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
+ 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 1
6*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 -
16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e -
16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*
d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3
 - 16*a^2*b^2*c^4*d*e^4) + x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 +
 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^
3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c
^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^
5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*
d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e
^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d
*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^
3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^
3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a
^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(
1/4)*1i - ((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) -
11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) -
8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 1
28*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e
 - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^
2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2
*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*
a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^
5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4
*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b
^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 +
 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^
4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^
8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^
5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e +
 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4
*b*c^7*d) - x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2
 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 819
2*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*
c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4
*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4
*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*

a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^
3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a
^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c
^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c -
16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5
- 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e
- 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^
3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e
^3 - 16*a^2*b^2*c^4*d*e^4) - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2
+ 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*
d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3
*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*
b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^
4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2
*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3
*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*
a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*
c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(
-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256
*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))
^(1/4)*1i)/(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2)
- 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3
*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*
a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b
^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*
a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d
^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2
*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3
+ 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*
a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*
b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*
a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e
+ 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a
^4*b*c^7*d) + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d
^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8
192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^
4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-
(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-

(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*
c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40
*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b
*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1
/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c
 - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^
5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*
e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*
c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2
*e^3 - 16*a^2*b^2*c^4*d*e^4) + x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e
^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^
4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b
^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^
3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*
c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c
^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c
^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 -
8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^
2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(2
56*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)
))^(1/4) + ((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) -
 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*
e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a
^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^
2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a
^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^
4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4
*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*
b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3
+ 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a
^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b
^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a
^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e
+ 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^
4*b*c^7*d) - x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^
2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 81
92*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4
*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(

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4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(
4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4
*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c
^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*
a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*
c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c
- 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5
 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e
 - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c
^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*
e^3 - 16*a^2*b^2*c^4*d*e^4) - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^
2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4
*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^
3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3
*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c
^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^
2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^
3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8
*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2
*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*
(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(25
6*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4))
)^(1/4)))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 1
1*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8
*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 12
8*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e
- 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2
*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*
c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a
*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5
*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*2i - atan((((-(b^7*c*d^4 + a^3*b^5*e^4
- a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 -
a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b
^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 +
 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*
c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*
e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5
 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(
((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c
^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*
c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4
*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*
b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^
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2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d) + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3 - 16*a^2*b^2*c^4*d*e^4) + x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i - ((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^

3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d) - x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3 - 16*a^2*b^2*c^4*d*e^4) - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i)/(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4

*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*
a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*
c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d
*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*
c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 19
6608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*
b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d) + x*(1024*b^7*c^4*d^2 -
11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2
*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*
d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*
(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^
2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c
*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*
d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b
^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2
*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3
+ 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^
5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 -
256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^
5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6
*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 6
4*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3 - 16*a^2*b^2*c^4*d*e^4) + x*(8*
c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d
^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b
*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 1
6*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^
5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^
5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*
b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d
^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 +
64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*
d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*
c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4) + ((-(b^7*c*d^4 + a^3*b^5*e
^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4
- a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4
+ b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^
3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b
^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d
^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*
c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4
)*(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^
5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b

^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*
c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a
^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c
*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*
e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b
^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c
^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196
608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b
^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d) - x*(1024*b^7*c^4*d^2 - 1
1264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*
b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d
*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(
-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2
*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*
e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d
^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^
3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*
e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 +
6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 -
256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5
- 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*
d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64
*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3 - 16*a^2*b^2*c^4*d*e^4) - x*(8*c
^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^
2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*
c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16
*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5
)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5
)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b
^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^
3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 +
64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d
^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c
^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)))*(-(b^7*c*d^4 + a^3*b^5*e^4
- a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 -
a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 +
b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3
+ 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4
*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3
*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^
5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*
2i - 2*atan((((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2)

- 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d)*1i + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4)*1i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d*e^4 - 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b*c^6*d^4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e^3 - 256*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^

6*b^2*c^4)))^(1/4) - ((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d)*1i - x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4)*1i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d*e^4 - 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b*c^6*d^4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e^3 - 256*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i + x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)

^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4))/(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d)*1i + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4)*1i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d*e^4 - 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b*c^6*d^4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e^3 - 256*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c

^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*
b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i + ((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d
^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2
*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3
 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*
a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*
b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7
*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4
- 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 +
 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e
- 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2
*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 -
 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^
2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a
^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^
3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e +
 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d)*1i - x*(1024*b^7*c^4*d^2 - 11264*a*
b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6
*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 20
48*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c
*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 -
48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 1
6*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e -
128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d
^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 1
28*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*
c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6
*b^2*c^4)))^(3/4)*1i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 1
92*a^3*c^5*d*e^4 - 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*
e^2 + 64*a*b*c^6*d^4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b
^3*c^4*d^2*e^3 - 256*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i + x*(8*c^
7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2
*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c
^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*
a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)
^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^
(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^
6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3

*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 6
4*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^
3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^
2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i))*(-(b^7*c*d^4 + a^3*b^5*e
^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4
 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4
- b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^
3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b
^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d
^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*
c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4
) - 2*atan((((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2)
- 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3
*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*
a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b
^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*
a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d
^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2
*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3
 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*
a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*
b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*
a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e
 + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a
^4*b*c^7*d)*1i + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^
7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2
- 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2
*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4
*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b
^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 +
 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^
4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^
8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4)*1i - 64*a*
c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d*e^4 - 16*b^3*c^
5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b*c^6*d^4*e - 16*

a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e^3 - 256*a^2*b*c
^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*
c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 -
16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^
3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c
*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 -
48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 1
6*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e -
128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d
^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 1
28*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*
c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6
*b^2*c^4)))^(1/4) - ((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5
)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5
)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b
^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^
3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 +
64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d
^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c
^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4
 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 -
 a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 +
b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3
+ 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4
*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3
*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c
 - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^
5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*
(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b
^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d -
262144*a^4*b*c^7*d)*1i - x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*
a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*
c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 2
0480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 -
 a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a
*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^
2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 +
40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c
*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e
 + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5
+ a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4)*1i
 - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d*e^4 - 1

6*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b*c^6*d^4 *e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e^3 - 256 *a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i + x*(8*c^7*d^6 - 8*a^3*c^4*e^ 6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^ 4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^ 5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))* (-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^ 2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c *e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4* d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b ^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2 *e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^ 5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4))/(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^ 4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^ 2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4 *a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a ^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3 *b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c ^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^ 2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^ 3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8 *a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2 *c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2* (-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(25 6*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 40 96*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7* c^4*d - 262144*a^4*b*c^7*d)*1i + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024* a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4 *d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b ^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4 *d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2* e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3* d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a ^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c ^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(- (4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256* a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^

```
(3/4)*1i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d
*e^4 - 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b
*c^6*d^4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e
^3 - 256*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i - x*(8*c^7*d^6 - 8*a^
3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^
2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 1
6*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^
2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*
a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a
^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*
a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e -
48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b
^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^
2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b
^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i + ((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^
4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4
*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*
b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3
+ 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a
^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b
^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*
c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 -
 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 +
16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e -
 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*
d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 -
128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2
*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2
))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^
6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3
*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e +
4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d)*1i - x*(1024*b^7*c^4*d^2 - 11264*a*b
^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*
d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 204
8*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*
d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 4
8*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16
*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 1
28*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^
2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 12
8*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))
```

```
/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*
b^2*c^4)))^(3/4)*1i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 19
2*a^3*c^5*d*e^4 - 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e
^2 + 64*a*b*c^6*d^4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^
3*c^4*d^2*e^3 - 256*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i + x*(8*c^7
*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*
e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^
6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a
*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^
(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(
1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6
*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*
e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64
*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3
*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2
 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i))*(-(b^7*c*d^4 + a^3*b^5*e^
4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4
- a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 +
 b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3
 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^
4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^
3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c
^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)
```

3.48 $\int \frac{d+ex^4}{x(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \frac{d+ex^4}{x(a+bx^4+cx^8)}\,dx=\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{d\log(a+bx^4+cx^8)}{8a}$$

```
[Out] d*ln(x)/a-1/8*d*ln(c*x^8+b*x^4+a)/a+1/4*(-2*a*e+b*d)*arctanh((2*c*x^4+b)/(-
4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1488, 814, 648, 632, 212, 642}

$$\int \frac{d+ex^4}{x(a+bx^4+cx^8)}\,dx=\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{d\log(a+bx^4+cx^8)}{8a}+\frac{d\log(x)}{a}$$

```
[In] Int[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqrt[b^2 - 4*
a*c]) + (d*Log[x])/a - (d*Log[a + b*x^4 + c*x^8])/(8*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{d+ex}{x\left(a+bx+cx^2\right)}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{d}{ax}+\frac{-bd+ae-cdx}{a\left(a+bx+cx^2\right)}\right)\,dx,x,x^4\right)\\
&= \frac{d\log(x)}{a}+\frac{\text{Subst}\left(\int \frac{-bd+ae-cdx}{a+bx+cx^2}\,dx,x,x^4\right)}{4a}\\
&= \frac{d\log(x)}{a}-\frac{d\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^4\right)}{8a}+\frac{(-bd+2ae)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^4\right)}{8a}\\
&= \frac{d\log(x)}{a}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}-\frac{(-bd+2ae)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^4\right)}{4a}
\end{aligned}$$

$$= \frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}} + \frac{d\log(x)}{a} - \frac{d\log\left(a+bx^4+cx^8\right)}{8a}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx$$
$$= \frac{d\log(x)}{a} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^4}{b+2c\#1^4}\&\right]}{4a}$$

```
[In] Integrate[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
[Out] (d*Log[x])/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[x
 - #1] + c*d*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

method	result
default	$\frac{d\ln(x)}{a} + \frac{-\frac{d\ln\left(c\,x^8+b\,x^4+a\right)}{4}+\frac{\left(ae-\frac{bd}{2}\right)\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a}$
risch	$\frac{d\ln(x)}{a} + \frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4c\,a^2-b^2a\right)\_Z^2+\left(4acd-b^2d\right)\_Z+a\,e^2-bde+c\,d^2\right)} \_R\ln\left(\left(\left(18ac-5b^2\right)\_R^2+\left(-be+9cd\right)\_R+4e^2\right)x^4-b\_\right.\right.}{4}$

```
[In] int((e*x^4+d)/x/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
[Out] d*ln(x)/a+1/2/a*(-1/4*d*ln(c*x^8+b*x^4+a)+(a*e-1/2*b*d)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.76 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.08

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx = \left[-\frac{(b^2-4\,ac)d\log\left(cx^8+bx^4+a\right)-8\,(b^2-4\,ac)d\log\left(x\right)+\sqrt{b^2-4\,ac}(bd-2\,ae)\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac}{cx^8+b}\right.}{8\,(ab^2-4\,a^2c)} - \frac{(b^2-4\,ac)d\log\left(cx^8+bx^4+a\right)-8\,(b^2-4\,ac)d\log\left(x\right)-2\,\sqrt{-b^2+4\,ac}(bd-2\,ae)\arctan\left(-\frac{(2\,cx^4+b)\sqrt{}}{b^2-4}\right.}{8\,(ab^2-4\,a^2c)}\right.$$

```
[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2 - 4*a*c)*d*log(c*x^8 + b*x^4 + a) - 8*(b^2 - 4*a*c)*d*log(x) +
sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c -
(2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)))/(a*b^2 - 4*a^2*c), -
1/8*((b^2 - 4*a*c)*d*log(c*x^8 + b*x^4 + a) - 8*(b^2 - 4*a*c)*d*log(x) - 2*
sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**4+d)/x/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 1.88 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx = -\frac{d\log\left(cx^8+bx^4+a\right)}{8\,a} + \frac{d\log\left(x^4\right)}{4\,a} - \frac{\left(bd-2\,ae\right)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}a}$$

```
[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/8*d*log(c*x^8 + b*x^4 + a)/a + 1/4*d*log(x^4)/a - 1/4*(b*d - 2*a*e)*arct
an((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

Mupad [B] (verification not implemented)

Time = 10.97 (sec) , antiderivative size = 8454, normalized size of antiderivative = 108.38

$$\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

```
[In] int((d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x)
```

```
[Out] (d*log(x))/a - (log(a + b*x^4 + c*x^8)*(4*b^2*d - 16*a*c*d))/(2*(16*a*b^2 -
 64*a^2*c)) + (atan((128*a^5*x^4*(((c^4*e^5 - ((4*b^2*d - 16*a*c*d)*(11*b*c
^4*e^4 + 9*c^5*d*e^3 - ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^
2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*
(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e)
)/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*c
^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^2*c^4*e^3 + 108*b*
c^5*d*e^2))/(2*(16*a*b^2 - 64*a^2*c))))/(2*(16*a*b^2 - 64*a^2*c)) - ((4*b^2
*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*d)*(((4*b^2*d - 16*a*c
*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^
5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b
^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a
*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1
/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^
2)/(128*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/(2*(16*a*b^2 - 64*a^2*c)
) + ((2*a*e - b*d)*(((((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 -
 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*
e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1
280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a*b^2 - 64*a^2*c)*(4
*a*c - b^2)^(1/2)))*(4*b^2*d - 16*a*c*d))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a
*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 460
8*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e +
```

3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^
5*e^2 - 432*b^2*c^5*d*e))/(8*a*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b^2)^(1
/2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((((((((2*a*e - b*d)*(((4*b^2*d - 16*a*c
d)(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^
5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b
^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a
*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1
/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^
2)/(128*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)))*(2*a*e - b*d))/(8*a*(4*a*
c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*
a*e - b*d)^3)/(1024*a^3*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2)))*(2*a*e
- b*d))/(8*a*(4*a*c - b^2)^(1/2)) - ((((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*
d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 6
4*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c
- b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a
*e - b*d))/(16*a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*d - 16*
a*c*d))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(
((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^
2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(2*(16*a*b^2 -
64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*c^5*d*e))/(8*a*(4*
a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d -
16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 46
08*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e +
3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c
^5*e^2 - 432*b^2*c^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^
2*c^4*e^3 + 108*b*c^5*d*e^2))/(8*a*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*
a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c
^5)*(2*a*e - b*d)^4)/(8192*a^4*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^2))*(5*b
^5*d - a^3*c^2*e - a*b^4*e - 24*a*b^3*c*d + 23*a^2*b*c^2*d + 3*a^2*b^2*c*e)
)/(32*a^5*c^4*(a^2*e^2 - 20*b^2*d^2 + 81*a*c*d^2 - a*b*d*e)) - ((((4*b^2*d
- 16*a*c*d)*(((((((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608
*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3
456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b
^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a*b^2 - 64*a^2*c)*(4*a*c
- b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*
c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^2)/(128*a^2*(16*a*b^2 -
64*a^2*c)*(4*a*c - b^2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^
2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^3)/(1024*a^3*
(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2))))/(2*(16*a*b^2 - 64*a^2*c)) - ((
4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*d)*(((4*b^2*d - 1
6*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b
^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) +
((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*
(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*d - 16*a*c*d))/(2*(16*a*
b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*

c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*c^5*d*e))/(8*a*(4*a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*c^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^2*c^4*e^3 + 108*b*c^5*d*e^2))/(8*a*(4*a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(11*b*c^4*e^4 + 9*c^5*d*e^3 - ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*c^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^2*c^4*e^3 + 108*b*c^5*d*e^2))/(2*(16*a*b^2 - 64*a^2*c))))/(8*a*(4*a*c - b^2)^(1/2)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^2)/(128*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*d - 16*a*c*d))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*c^5*d*e))/(8*a*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b^2)^(1/2)) - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^5)/(32768*a^5*(4*a*c - b^2)^(5/2)))*(144*a^3*c^3*d - 40*b^6*d + 8*a*b^5*e - 488*a^2*b^2*c^2*d + 272*a*b^4*c*d - 40*a^2*b^3*c*e + 40*a^3*b*c^2*e))/(256*a^5*c^4*(4*a*c - b^2)^(1/2)*(a^2*e^2 - 20*b^2*d^2 + 81*a*c*d^2 - a*b*d*e)))*(4*a*c - b^2)^(5/2))/(16*a^4*c^4*e^4 + b^4*c^4*d^4 + 24*a^2*b^2*c^4*d^2*e^2 - 8*a*b^3*c^4*d^3*e - 32*a^3*b*c^4*d*e^3) + (4*(4*a*c - b^2)^(5/2)*(5*b^5*d - a^3*c^2*e - a*b^4*e - 24*a*b^3*c*d + 23*a^2*b*c^2*d + 3*a^2*b^2*c*e)*(c^4*d*e^4 + ((4*b^2*d - 16*a*c*d)*(a*c^4*e^4 + ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 96*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3))/(2*(16*a*b^2 - 64*a^2*c)) - 16*b*c^4*d*e^3))/(2*(16*a*b^2 - 64*a^2*c)) - ((((4*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(((2*a*e - b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128

```
*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^2)
^(1/2)) + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64*a
^2*c)*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b^2)^(1/2)) + (2*b^4*c^4*(4*b^2*
d - 16*a*c*d)*(2*a*e - b*d)^2)/(a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/(2
*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((2*a*e -
b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d)
)/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*c^4*(4*b^2*d
- 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2))))/(2
*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^
4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64
*a^2*c)))/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/
(8*a*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b^2)^(1/2)))*(4*b^2*d - 16*a*c*d)
)/(2*(16*a*b^2 - 64*a^2*c)) - ((((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d
)*(((2*a*e - b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*
d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*
c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2
)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d
)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(
16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b
^3*c^4*d*e))/(8*a*(4*a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*
e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 256
*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))
/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/(2*(16*a*
b^2 - 64*a^2*c)) + 96*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3))/(8*a*(4*a*c - b^2)^(
1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1/2)) + ((2*a*e - b*d)*(((2*a*e -
 b*d)*(((2*a*e - b*d)*(((2*a*e - b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (1
28*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^
2)^(1/2)) + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64
*a^2*c)*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b^2)^(1/2)) + (2*b^4*c^4*(4*b^
2*d - 16*a*c*d)*(2*a*e - b*d)^2)/(a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/
(8*a*(4*a*c - b^2)^(1/2)) + (b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d)^3)/
(4*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2))))/(8*a*(4*a*c - b^2)^(1/2
)) + (b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d)^4)/(32*a^3*(16*a*b^2 - 64*
a^2*c)*(4*a*c - b^2)^2)))/(c^4*(a^2*e^2 - 20*b^2*d^2 + 81*a*c*d^2 - a*b*d*e
)*(16*a^4*c^4*e^4 + b^4*c^4*d^4 + 24*a^2*b^2*c^4*d^2*e^2 - 8*a*b^3*c^4*d^3*
e - 32*a^3*b*c^4*d*e^3)) + ((4*a*c - b^2)^2*(((4*b^2*d - 16*a*c*d)*(((4*b^2
*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(256*b^4*c^4*d - 256*
a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/
(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))
/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) +
((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (1
28*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^2 - 6
4*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/(8*a*(4*a*c - b^2)^(1/2)))
)/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^
2*d - 16*a*c*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d
```

```
- 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c
^4*e^2 - 256*b^3*c^4*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 96*b^2*c^4*d*e^2 - 1
6*a*b*c^4*e^3))/(8*a*(4*a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) - ((4
*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(((2*a*e - b*d)*(((2*a*e - b*d)*(256*b^4
*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 -
 64*a^2*c)))/(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(
2*a*e - b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b
^2)^(1/2)) + (2*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d)^2)/(a*(16*a*b^2
- 64*a^2*c)*(4*a*c - b^2))))/(8*a*(4*a*c - b^2)^(1/2)) + (b^4*c^4*(4*b^2*d
- 16*a*c*d)*(2*a*e - b*d)^3)/(4*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/
2))))/(2*(16*a*b^2 - 64*a^2*c)) - ((((4*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(
((2*a*e - b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d -
 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*c^4
*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(
1/2))))/(8*a*(4*a*c - b^2)^(1/2)) + (2*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e
- b*d)^2)/(a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/(2*(16*a*b^2 - 64*a^2*c
)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(256*b^4*c^4*d -
 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*
c)))/(8*a*(4*a*c - b^2)^(1/2)) + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e -
b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c
)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e
 + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^
2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/(8*a*(4*a*c - b^2)^(1
/2))))/(8*a*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1/2))
+ ((2*a*e - b*d)*(a*c^4*e^4 + ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*
(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4
*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^2 - 64*a^2*c)) + 96*
a*b^2*c^4*e^2 - 256*b^3*c^4*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 96*b^2*c^4*d*
e^2 - 16*a*b*c^4*e^3))/(2*(16*a*b^2 - 64*a^2*c)) - 16*b*c^4*d*e^3))/(8*a*(4
*a*c - b^2)^(1/2)) + (b^4*c^4*(2*a*e - b*d)^5)/(128*a^4*(4*a*c - b^2)^(5/2)
))*(144*a^3*c^3*d - 40*b^6*d + 8*a*b^5*e - 488*a^2*b^2*c^2*d + 272*a*b^4*c*
d - 40*a^2*b^3*c*e + 40*a^3*b*c^2*e))/(2*c^4*(a^2*e^2 - 20*b^2*d^2 + 81*a*c
*d^2 - a*b*d*e)*(16*a^4*c^4*e^4 + b^4*c^4*d^4 + 24*a^2*b^2*c^4*d^2*e^2 - 8*
a*b^3*c^4*d^3*e - 32*a^3*b*c^4*d*e^3)))*(2*a*e - b*d))/(4*a*(4*a*c - b^2)^(
1/2))
```

3.49 $\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 392

$$\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx = -\frac{d}{ax} - \frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

```
[Out] -d/a/x-1/4*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*
(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+
1/4*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(d+(2*
a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/4*c^
(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(d+(-2*a*e+b*
d)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/4*c^(1/4)*
```

```
arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(d+(-2*a*e+b*d)/(-
4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1518, 1524, 304, 211, 214}

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx = -\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2\ 2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}} - \frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{2\ 2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}} + \frac{\sqrt[4]{c}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2\ 2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}} + \frac{\sqrt[4]{c}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{2\ 2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}} - \frac{d}{ax}$$

```
[In] Int[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(d/(a*x)) - (c^(1/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)
*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 -
4*a*c])^(1/4)) - (c^(1/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(
1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^
2 - 4*a*c])^(1/4)) + (c^(1/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Ar
cTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b
 + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{d}{ax}-\frac{\int \frac{x^2(bd-ae+cdx^4)}{a+bx^4+cx^8}\,dx}{a}\\ &= -\frac{d}{ax}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}-\frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}\end{aligned}$$

$$= -\frac{d}{ax} + \frac{\left(\sqrt{c}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}a} - \frac{\left(\sqrt{c}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}a} + \frac{\left(\sqrt{c}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}a} - \frac{\left(\sqrt{c}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}a}$$

$$= -\frac{d}{ax} - \frac{\sqrt[4]{c}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.22

$$\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx = -\frac{d}{ax} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{4a}$$

```
[In] Integrate[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
```

[Out] -(d/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) &]/(4*a)

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.19

method	result	size
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-cd\_R^{6}+(ae-bd)\_R^{2}\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}}{4a}-\frac{d}{ax}$	73
risch	Expression too large to display	1333

[In] int((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)

[Out] 1/4/a*sum((-c*d*_R^6+(a*e-b*d)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))-d/a/x

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21400 vs. 2(312) = 624.

Time = 137.79 (sec) , antiderivative size = 21400, normalized size of antiderivative = 54.59

$$\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx = \text{Too large to display}$$

[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx = \text{Timed out}$$

[In] integrate((e*x**4+d)/x**2/(c*x**8+b*x**4+a),x)

[Out] Timed out

Maxima [F]

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx=\int \frac{ex^4+d}{(cx^8+bx^4+a)x^2}\,dx$$

```
[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] -integrate((c*d*x^6 + (b*d - a*e)*x^2)/(c*x^8 + b*x^4 + a), x)/a - d/(a*x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx=\text{Timed out}$$

```
[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 13.32 (sec) , antiderivative size = 39028, normalized size of antiderivative = 99.56

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx=\text{Too large to display}$$

```
[In] int((d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x)
[Out] atan((((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*
c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*
b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) +
6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4
*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b
^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*
a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4
*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d
^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16
*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*d^4 + a^4*
b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/
2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^
3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^
3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a
```

*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 2
40*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d
^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48
*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*
c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2
*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*
c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*
c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1
024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81
920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 327
68*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*
a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a^15*c^8*d^3 + 4096*a^16*
b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^5
*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4*e
^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^7*c^4*d^2*
e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14*b^3*c^6*d
^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2) + x*(4*a^11*b*c^
8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*
c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3
*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6
*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a
*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 -
8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e +
128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3
*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 -
3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^
4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*
c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*
(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))
^(1/4)*1i + ((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) +
b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*
a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 6
1*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1
/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^
2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3
*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a
^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b
*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8
d^3 + x(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*

d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*
b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^
2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2)
+ 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c -
b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(
4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b
^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3
*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
+ 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 -
16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^
2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 5
1200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10
240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048
*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096
*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^12*b
^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14*b^5
*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12*b^7*c^
4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14*b^3
*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2) + x*(4*a^1
1*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32
*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^
12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13
*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*
(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*
d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^
3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^
2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b
^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2
*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^
2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e +
40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^
4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))
/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*
c^3)))^(1/4)*1i)/(((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1
/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4
+ 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e
^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)
^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b
*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 -
200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288
*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2

)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*
a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^
9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c -
b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*
a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4
- 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^
2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5
)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e -
66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128
*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d
^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c +
96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17
*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*
c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c
^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*
d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a^15*c^8*d^3
+ 4096*a^16*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*
a^12*b^6*c^5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^
14*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*
b^7*c^4*d^2*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^
14*b^3*c^6*d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2) + x*
(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^
5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3
+ 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 4
4*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^
4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*
b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*
c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 +
a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 -
4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c
^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*
c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d
^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 +
320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a
^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^
(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^
8*b^2*c^3)))^(1/4) - ((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)
^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*
e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*
d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b
^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d
^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^

3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^
3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e -
288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c -
b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 2
56*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096
*a^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/
2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4
+ 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^
3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^
5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(
-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*
d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 -
200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*
a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)
^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a
^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^
16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^
5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4
*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d
*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*
e) - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 28
16*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256
*a^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^
12*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576
*a^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2) +
x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d
*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e
^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2
+ 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 +
a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a
^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a
^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4
+ a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^
4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^
3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4
*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*
c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2
+ 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 -
6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^
5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256
*a^8*b^2*c^3)))^(1/4) + 2*a^14*c^5*e^7 + 2*a^11*c^8*d^6*e + 6*a^12*c^7*d^4*
e^3 + 6*a^13*c^6*d^2*e^5 + 6*a^11*b^2*c^6*d^4*e^3 - 2*a^11*b^3*c^5*d^3*e^4
+ 6*a^12*b^2*c^5*d^2*e^5 - 6*a^13*b*c^5*d*e^6 - 6*a^11*b*c^7*d^5*e^2 - 12*a
^12*b*c^6*d^3*e^4))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(

1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*2i + atan((((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a^15*c^8*d^3 + 4096*a^16*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^7*c^4*d^2*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14*b^3*c^6*d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2) + x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^

7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c
^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d
^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c
 - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*
a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 12
8*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-
(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e
 - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3
*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*
b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^
3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(
-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a
^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(
1/4)*1i + ((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^
4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^
6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*
a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2
) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*
(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c
 - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(
-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3
*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c
^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4
 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8*d
^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*
c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*
b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) +
6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4
*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b
^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*
a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4
*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d
^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16
*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2
 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 512
00*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 1024
0*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a
^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a
^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^12*b^6
*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14*b^5*c
^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12*b^7*c^4*

d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14*b^3*c
^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2) + x*(4*a^11*
b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a
^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12
*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b
*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-
(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^
4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*
e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*
d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8
*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e
^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)
^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 4
0*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*
b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2
e^2(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(
512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^
3)))^(1/4)*1i)/(((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2
) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 +
16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3
+ 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^
2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-
(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d
e^3(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 2
00*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a
^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^
9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*
d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b
^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^
3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 -
120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*
e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 6
6*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a
^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3
e(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 9
6*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c
^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^
6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5
*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*
e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a^15*c^8*d^3 +

```
4096*a^16*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^
12*b^6*c^5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14
*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^
7*c^4*d^2*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14
*b^3*c^6*d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2) + x*(4
*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5
 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 +
4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*
a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*
e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*
c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^
4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^
2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4
*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2
*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3
*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 32
0*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3
*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1
/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*
b^2*c^3)))^(1/4) - ((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(
1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^
4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*
e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2
)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4
*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*
b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3
 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 28
8*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256
*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a
^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2)
 - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 +
16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3
+ 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2
*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*
e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 20
0*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^
5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9
*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16
*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*
```

d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e
^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e
 - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)
 - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816
*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a
^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12
*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a
^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2) + x
*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e
^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3
 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 +
44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a
^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4
*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5
*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 -
 a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4
 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*
c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a
*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*
d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 +
 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*
a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)
^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a
^8*b^2*c^3)))^(1/4) + 2*a^14*c^5*e^7 + 2*a^11*c^8*d^6*e + 6*a^12*c^7*d^4*e^
3 + 6*a^13*c^6*d^2*e^5 + 6*a^11*b^2*c^6*d^4*e^3 - 2*a^11*b^3*c^5*d^3*e^4 +
6*a^12*b^2*c^5*d^2*e^5 - 6*a^13*b*c^5*d*e^6 - 6*a^11*b*c^7*d^5*e^2 - 12*a^1
2*b*c^6*d^3*e^4))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/
2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4
+ 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^
3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^
5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(
-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*
d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 -
200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*
a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)
^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a
^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*2i - 2*at
an((((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^
2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^
5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*
a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2
)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*

c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i - 4096*a^15*c^8*d^3 + 4096*a^16*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^7*c^4*d^2*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14*b^3*c^6*d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) + ((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(

1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e
^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*
a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*
b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c
^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^
8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4
d^4(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6
*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a
^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2)
 + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(
-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-
(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*
b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^
3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 -
 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d
^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 +
51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 1
0240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 204
8*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i -
4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^
12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14
*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12*b^
7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14
*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2)*1i - x
*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e
^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3
 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 +
44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a
^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4
*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5
*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 +
 a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4
- 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*
c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a
*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*
d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 +
 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*
a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)
^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a
^8*b^2*c^3)))^(1/4))/(((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5

)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c *e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3 *d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2* b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c* d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a ^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e ^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(409 6*a^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1 /2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e ^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2) ^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2* d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4* (-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b *d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288 *a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256* a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a ^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c ^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^ 4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7* d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d *e)*1i - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 76 8*a^12*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 2 4576*a^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^ 2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^1 4*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^ 6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d ^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^ 5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3* c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b ^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240 *a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3 *e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a ^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c* d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d *e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i - ((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i - 4096*a^15*c^8*d^3 + 4096*a^16*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^7*c^4*d^2*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14*b^3*c^6*d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) -

4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 6
6*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a
^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3
e(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 9
6*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i + 2*a^14*c^5*e^7 + 2*a^11*c^8*d
^6*e + 6*a^12*c^7*d^4*e^3 + 6*a^13*c^6*d^2*e^5 + 6*a^11*b^2*c^6*d^4*e^3 - 2
*a^11*b^3*c^5*d^3*e^4 + 6*a^12*b^2*c^5*d^2*e^5 - 6*a^13*b*c^5*d*e^6 - 6*a^1
1*b*c^7*d^5*e^2 - 12*a^12*b*c^6*d^3*e^4))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^
4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^
4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*
d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*
c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a
*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d
^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e
 + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*
a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^
2*c^3)))^(1/4) - 2*atan((((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2
)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^
3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*
c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c
 - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a
^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2
*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) +
4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*
d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*
e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8
 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(
x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e
^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c
^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2
*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c -
 b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*
d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^
2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^
2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*
b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 327
68*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^

14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^1
5*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b
^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i - 4096*a^1
5*c^8*d^3 + 4096*a^16*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d
^3 + 2816*a^12*b^6*c^5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^
3 + 256*a^14*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e +
768*a^12*b^7*c^4*d^2*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2
+ 24576*a^14*b^3*c^6*d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d
*e^2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*
a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2
*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^
7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4
*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1
/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e
^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b
^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*
a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 4
8*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5
*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^
2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a
c - b^2)^5)^(1/2))/(512(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4
*c^2 - 256*a^8*b^2*c^3)))^(1/4) + ((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*
a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 -
8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e +
128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4
*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^
3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2
+ 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a
^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2
*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^
2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512
*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))
)^(3/4)*(4096*a^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c
- b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a
^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128
*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(
4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e
- 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*
a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b
^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3
*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-

(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^
5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1
/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 1228
8*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024
*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304
*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^
15*b^3*c^6*d*e)*1i - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*
b^8*c^4*d^3 - 2816*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b
^2*c^7*d^3 - 256*a^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^
7*d^2*e - 768*a^12*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c
^4*d*e^2 - 24576*a^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*
b^2*c^6*d*e^2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^
5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32
*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*
a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*
d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b
^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^
3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 -
120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*
e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) +
4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 6
6*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a
^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3
e(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 9
6*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4))/(((-(b^9*d^4 + a^4*b^5*e^4 - a^4*
e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*
c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^
4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^
2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4
*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2
*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3
*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 32
0*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3
*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1
/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*
b^2*c^3)))^(3/4)*(4096*a^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*
(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*
d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^
3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^
2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b
^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2
*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^

2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i - ((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-

(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e
- 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3
*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*
b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^
3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(
-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a
^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(
1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 122
88*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 102
4*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 9830
4*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a
^15*b^3*c^6*d*e)*1i - 4096*a^15*c^8*d^3 + 4096*a^16*b*c^6*e^3 + 12288*a^16*
c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^5*d^3 - 10496*a^13*b^4*c
^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*
e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^7*c^4*d^2*e - 7680*a^13*b^5*c^5*d
^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14*b^3*c^6*d^2*e + 6912*a^14*b^4*c^
5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2)*1i - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5
*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11
*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^1
1*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^
2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) -
b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16
*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 +
61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(
1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e
^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*
a*c - b^2)^5)^(1/2) + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*
a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*
b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c
^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i + 2*a^14*c
^5*e^7 + 2*a^11*c^8*d^6*e + 6*a^12*c^7*d^4*e^3 + 6*a^13*c^6*d^2*e^5 + 6*a^1
1*b^2*c^6*d^4*e^3 - 2*a^11*b^3*c^5*d^3*e^4 + 6*a^12*b^2*c^5*d^2*e^5 - 6*a^1
3*b*c^5*d*e^6 - 6*a^11*b*c^7*d^5*e^2 - 12*a^12*b*c^6*d^3*e^4))*(-(b^9*d^4 +
a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) - b^4*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6
*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a
^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 -
13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*
b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
+ 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3
*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^

```
2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^2*b*c*d^3*e*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7
*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - d/(a*x)
```

3.50 $\int \frac{d+ex^4}{x^3(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 199

$$\int \frac{d+ex^4}{x^3(a+bx^4+cx^8)}\,dx=-\frac{d}{2ax^2}-\frac{\sqrt{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/2*d/a/x^2-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c
^(1/2)*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))
^(1/2)-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)
*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1504, 1295, 1180, 211}

$$\int \frac{d+ex^4}{x^3(a+bx^4+cx^8)}\,dx=-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{2ax^2}$$

```
[In] Int[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/2*d/(a*x^2) - (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b + Sqrt[
b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1504

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex^2}{x^2\left(a+bx^2+cx^4\right)}\,dx, x, x^2\right)$$

$$= -\frac{d}{2ax^2} - \frac{\text{Subst}\left(\int \frac{bd-ae+cdx^2}{a+bx^2+cx^4}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{d}{2ax^2} - \frac{\left(c\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)}{4a} - \frac{\left(c\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)}{4a}$$

$$= -\frac{d}{2ax^2} - \frac{\sqrt{c}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.45

$$\int \frac{d+ex^4}{x^3(a+bx^4+cx^8)}\,dx$$

$$= -\frac{d}{2ax^2} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^4}{b\#1^2+2c\#1^6}\&\right]}{4a}$$

```
[In] Integrate[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/2*d/(a*x^2) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log
[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/(4*a)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.89

method	result
default	$\frac{2c\left(\frac{\left(bd-2ae-d\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\left(-d\sqrt{-4ac+b^2}+2ae-bd\right)\sqrt{2}\,\text{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{a}-\frac{d}{2a\,x^2}$
risch	$-\frac{d}{2a\,x^2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(16a^5c^2-8a^4b^2c+b^4a^3\right)\_Z^4+\left(-4a^3b\,e^2c-16a^3de\,c^2+a^2b^3e^2+12a^2b^2dec+12a^2b\,c^2d^2-2a\,b^4de-7a\,b^3c\,d^2+b^5d^2\right)\_\right.}\right.}{}$

```
[In] int((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a*c*(1/8*(b*d-2*a*e-d*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))-1/8*(-d*(-4*a*c+b^2)^(1/2)+2*a*e-b*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
```

```
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)))-1/2*d/a/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2772 vs. $2(157) = 314$.

Time = 1.06 (sec) , antiderivative size = 2772, normalized size of antiderivative = 13.93

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*
a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*
a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*
d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((3*a*b^2*c*d^2*e^2
- 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d
^3*e)*x^2 + 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 -
 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2
 - 4*a^4*c)*e^3 - ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5
*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 +
 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^
7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (
a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*
c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*
b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) - sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 +
 (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-
(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2
*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2
 - 4*a^4*c))*log((3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^
2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^2 - 1/2*sqrt(1/2)*((b^5 - 5*a*b
^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a
^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 - 4*a^4*c)*e^3 - ((a^3*b^4 - 6*a^4*b^2
*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4
 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b
^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*
c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3
 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e -
2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))
+ sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2
*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b
^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2
*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((3*a*b^2*c*d^2*e^2 - 3
*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*
```

```
e)*x^2 + 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13
*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 -
4*a^4*c)*e^3 + ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*
c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*
(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c
)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3
*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2
)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2
 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b
^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*
a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*
c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 -
4*a^4*c))*log((3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 -
 a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^2 - 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*
c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*
b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 - 4*a^4*c)*e^3 + ((a^3*b^4 - 6*a^4*b^2*c
+ 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 -
(b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2
- a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*
d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 -
a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(
3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - 2
*d)/(a*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx=\text{Timed out}$$

```
[In] integrate((e*x**4+d)/x**3/(c*x**8+b*x**4+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx=\int \frac{ex^4+d}{(cx^8+bx^4+a)x^3}\,dx$$

```
[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] -integrate((c*d*x^4 + b*d - a*e)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2*d/(a*x^2
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3003 vs. 2(157) = 314.

Time = 2.11 (sec) , antiderivative size = 3003, normalized size of antiderivative = 15.09

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/8*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^3*c^2 - 2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 +
 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^2*c^3 + 16*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*c^4 - 32*a^2*c^4 + 2*(b^2 - 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a*c)*a*
c^3)*d*x^4*abs(a) - (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*d*x^4 + (sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c + 16*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*
a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2)*d*abs(a) - (sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
- 2*a*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^2 + 16*a^2*b^2*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c^3 - 32*a^3*c^3 + 2*(b^2 - 4*a*c)*a*b^2*c - 8*(b^2 - 4*a*c)*a^2
*c^2)*e*abs(a) - (2*a*b^4*c^2 - 8*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c^2)*d + (2*a^2*b^3*c^2 - 8
*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2
- 4*a*c)*a^2*b*c^2)*e)*arctan(2*sqrt(1/2)*x^2/sqrt((a*b + sqrt(a^2*b^2 - 4*
a^3*c))/(a*c)))/((a^2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*
b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*abs(a)*abs(c)) - 1/8*((sqrt(2)*sqrt(b*c -
```

```
sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 2*b^4*c^2 + 16*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3
- 16*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 + 32*a^2*c
^4 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*d*x^4*abs(a) - (2*a*b
^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3
 - 2*(b^2 - 4*a*c)*a*b*c^3)*d*x^4 + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^
2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*c
)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*d*abs(a) - (sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*a*b^4*c + 16*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 16*
a^2*b^2*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 32*a^3*c^
3 - 2*(b^2 - 4*a*c)*a*b^2*c + 8*(b^2 - 4*a*c)*a^2*c^2)*e*abs(a) - (2*a*b^4*
c^2 - 8*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 -
2*(b^2 - 4*a*c)*a*b^2*c^2)*d + (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*e)*arc
tan(2*sqrt(1/2)*x^2/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^2*b^4
- 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a^
3*c^3)*abs(a)*abs(c)) - 1/2*d/(a*x^2)
```

Mupad [B] (verification not implemented)

Time = 12.12 (sec) , antiderivative size = 15013, normalized size of antiderivative = 75.44

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx=\text{Too large to display}$$

```
[In] int((d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x)
```

```
[Out] - atan((((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d
^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12
*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*
c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c -
b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b
^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 -
 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(3
2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 +
 a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/
2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c
 - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^
12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(9216*a^11*b^5*
c^5*d - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e -
8192*a^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^
2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2
*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)
))^(1/2) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2
 - 3072*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e +
4096*a^12*b^2*c^6*d*e) + x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a
^12*c^7*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7
*d^3 + 192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e
- 960*a^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2
))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a
*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b
^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d
^4 - 64*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 +
 128*a^10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) + x^2*
(8*a^11*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*
d^2*e^3 - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*
e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3
)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(
4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i
- ((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a
*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b
^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e
```

- 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(9216*a^11*b^5*c^5*d - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^12*b^2*c^6*d*e) - x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 + 192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) - x^2*(8*a^11*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(

1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(9216*a^11*b^5*c^5*d - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^12*b^2*c^6*d*e) + x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 + 192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) + x^2*(8*a^11*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + ((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(9216*a^11*b^5*c^5*d - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -

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4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b
^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*
b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072*a^11*b^3*c^6*
d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^12*b^2*c^6*d*e
) - x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7*d*e^2 - 64*a^
8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 + 192*a^11*b^3*c
^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a^10*b^3*c^6*d^
2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b^5*d^2 + a^2*b
^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)
+ 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64*a^9*b^2*c^7*d
^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^10*b*c^7*d^3*e
 - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) - x^2*(8*a^11*c^6*e^5 - 8*a
^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 - 16*a^10*b*c
^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*
c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2
*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e
^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2)
)/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5*d^2 + a^2*b^3*e
^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i -
 atan((((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^
2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*
a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c
^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^
4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 -
16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32
*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 -
a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c
- b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^1
2*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(9216*a^11*b^5*c
^5*d - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8
192*a^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2
 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*
a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))
```

)^(1/2) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2
- 3072*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4
096*a^12*b^2*c^6*d*e) + x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^
12*c^7*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*
d^3 + 192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e -
960*a^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2)
)*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^
2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8
*a^4*b^2*c)))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^
4 - 64*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 +
128*a^10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) + x^2*(
8*a^11*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d
^2*e^3 - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e
^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i -
((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^
2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8
*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)
^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e
- 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3
*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*
b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^
2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*
a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)
^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*
c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(9216*a^11*b^5*c^5*d -
1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^
12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2
*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*
e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2
) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072
*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^
12*b^2*c^6*d*e) - x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7
*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 +
192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a
^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b

^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*
e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b
^2*c)))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64
*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^
10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) - x^2*(8*a^11
*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3
- 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/(((-(b
^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*
e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b
^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
- b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*
b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d
*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 +
16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4
*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2
- 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*
c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1
/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 -
32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(9216*a^11*b^5*c^5*d - 1024*
a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^12*b^4
*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2*b^3*e
^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4
*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 40
96*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072*a^11*
b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^12*b^2
*c^6*d*e) + x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7*d*e^2
- 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 + 192*a^
11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a^10*b^
3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b^5*d^2
+ a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*
a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))
)^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64*a^9*b
^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^10*b*c

^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) + x^2*(8*a^11*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + ((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(9216*a^11*b^5*c^5*d - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^12*b^2*c^6*d*e) - x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 + 192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) - x^2*(8*a^11*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))

```
/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5*d^2 + a^2*b^3*e^
2 - a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^
(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*
a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i -
d/(2*a*x^2)
```

3.51 $\int \frac{d+ex^4}{x^4(a+bx^4+cx^8)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 394

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx = -\frac{d}{3ax^3}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

```
[Out] -1/3*d/a/x^3+1/4*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(
1/4))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^
(3/4)+1/4*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*
(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+
1/4*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(d+(-2*
a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^
(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(d+(-2*a*e+b
*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1518, 1436, 218, 214, 211}

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx = \frac{c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{c^{3/4}\arctan\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} + \frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{d}{3ax^3}$$

```
[In] Int[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/3*d/(a*x^3) + (c^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(
1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^
2 - 4*a*c])^(3/4)) + (c^(3/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqr
t[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Arc
Tanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b
- Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c]
)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a
*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d}{3ax^3} - \frac{\int \frac{3(bd-ae)+3cdx^4}{a+bx^4+cx^8}\,dx}{3a}\\
&= -\frac{d}{3ax^3} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}\\
&= -\frac{d}{3ax^3} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2a\sqrt{-b-\sqrt{b^2-4ac}}}\\
&\quad + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2a\sqrt{-b-\sqrt{b^2-4ac}}}\\
&\quad + \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2a\sqrt{-b+\sqrt{b^2-4ac}}}\\
&\quad + \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2a\sqrt{-b+\sqrt{b^2-4ac}}}
\end{aligned}
$$

$$= -\frac{d}{3ax^3} + \frac{c^{3/4}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}$$
$$+ \frac{c^{3/4}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$
$$+ \frac{c^{3/4}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}$$
$$+ \frac{c^{3/4}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.22

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx$$
$$= -\frac{\frac{4d}{x^3} + 3\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{12a}$$

```
[In] Integrate[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/12*((4*d)/x^3 + 3*RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e
*Log[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/a
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.17

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)} \frac{\left(-cd\_R^4+ae-bd\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4a}-\frac{d}{3a\,x^3}$	68
risch	Expression too large to display	1633

```
[In] int((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*sum((-_R^4*c*d+a*e-b*d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_
Z^4*b+a))-1/3*d/a/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20184 vs. 2(312) = 624.

Time = 52.64 (sec) , antiderivative size = 20184, normalized size of antiderivative = 51.23

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx = \text{Timed out}$$

```
[In] integrate((e*x**4+d)/x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx=\int \frac{ex^4+d}{(cx^8+bx^4+a)x^4}\,dx$$

```
[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] -integrate((c*d*x^4 + b*d - a*e)/(c*x^8 + b*x^4 + a), x)/a - 1/3*d/(a*x^3)
```

Giac [F(-1)]

Timed out.

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx=\text{Timed out}$$

```
[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 13.45 (sec) , antiderivative size = 65350, normalized size of antiderivative = 165.86

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx=\text{Too large to display}$$

```
[In] int((d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x)
[Out] atan((((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a
^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b
^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86
*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*
(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^
3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2
*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1
/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^
5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3
- 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3
*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^
4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2
) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*
c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4
```

*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4
*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*
e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^
3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^
4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c
 - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4
 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3
*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c
*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 29
2*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^
5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b
^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d
^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4
 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c
^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d -
 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 19660
8*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) + x*(81920*a^15*b*c^8*d^2 - 49152*a
^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13
*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14
*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*
c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^
2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*
c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3
 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3
d^4(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a
^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^
2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^
2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4
*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d
*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 7
8*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a
^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b
^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^
9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^
5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13
*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^
3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^
13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^

2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) + x*(8*a^13*c
^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*
c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3
*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7
*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^
7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c
*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*
e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*
b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) +
a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e
^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e
^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c
- b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^
4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4
*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^
7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^
(1/4)*1i - ((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*
c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3
 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3
d^4(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a
^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^
2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^
2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4
*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d
*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 7
8*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a
^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b
^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^
9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b
*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c
^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 2
80*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(
4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c
*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*
b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^

3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*
b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
 - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 6
40*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*
a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*
c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^1
1*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a
^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^
6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e -
196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) - x*(81920*a^15*b*c^8*d^2 - 49
152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464
*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264
*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12
*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^
15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/
2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-
(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*
d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^
3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) +
 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e +
 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c -
b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2
) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^
6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*
e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e -
200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*
a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^
2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e
^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c +
96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c
^9*d^5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16
*a^13*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^
11*b^3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 2
56*a^13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^
11*b^2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) - x*(8*a
^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*
a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^1
0*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*
b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a
^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*
b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b
^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231

*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/
2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*
d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^
2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*
d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1
/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2
)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4
*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a
^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*
b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^
5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e
^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(51
2*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^
3)))^(1/4)*1i)/(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/
2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-
(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*
d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^
3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) +
40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e +
6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c -
b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2
) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^
6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*
e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e -
200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*
a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^
2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e
^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c +
96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^
6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*
a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*
a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^
4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^
4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*
b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6
*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a
^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c
d^4(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56
*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^
(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*
e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 +
320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a
^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 25

6*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262
144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b
^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5
*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) + x*(81920*a^15*b*c^8*d^2
 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 +
62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 -
11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048
*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 1638
40*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5
)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7
*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4
 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1
/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^
3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*
a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a
^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2
*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3
*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3
+ 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*
c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6
*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^1
1*b*c^9*d^5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5
 + 16*a^13*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 2
88*a^11*b^3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^
3 + 256*a^13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 1
12*a^11*b^2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) + x
*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2
 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 1
6*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*
a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^
4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11
*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*
a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4
- 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5
)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2
*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c
 - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5
*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3
*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 +

680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480
*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b
^2*c^3)))^(1/4) + ((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(
1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*
(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^
4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 -
a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2)
 + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e
 + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c
- b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4
*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1
/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*
b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^
3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e
- 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 1
6*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d
e^3(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c
+ 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 +
b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 4
8*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 12
8*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*
d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*
e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*
a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) +
 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720
*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4
*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) +
56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5
)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^
3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2
 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12
*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 +
256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(2
62144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15
*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c
^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) - x*(81920*a^15*b*c^8*d
^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2
+ 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2
- 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 20
48*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 16

```
3840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)
^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c
*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a
^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d
^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^
(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*
d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4
*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 +
6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^
5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48
*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c
^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d
^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^
3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a
*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*
b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b
^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a
^11*b*c^9*d^5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d
^5 + 16*a^13*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 +
 288*a^11*b^3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*
e^3 + 256*a^13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e -
 112*a^11*b^2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) -
 x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e
^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 -
 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 2
4*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*
d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 -
11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^
4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)
^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a
^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a
*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b
^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2
)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*
c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e
^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2
+ 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 4
80*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*
c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/
2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10
*b^2*c^3)))^(1/4)))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^
(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4
*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c
```

^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*2i + atan((((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(

4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9
*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4
*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d -
 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 26214
4*a^17*b*c^7*e) + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^1
1*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^
14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^1
5*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6
*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4
+ a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a
^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^
3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 -
231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^
(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b
^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c -
 b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c
^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(
-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 68
0*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a
^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^
2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^
2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/
(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2
*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^5 + 192*a^12*c^9*d^4*e -
 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5 + 128*a^13*
c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^2 + 96*a^11*
b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e^4 + 16*a^9*
b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e - 128*a^12*b
*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) + x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6
 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*
c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b
^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d
^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c
 - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4
+ a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e
- 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^
3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b
^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*
a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e
^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2
*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c
- b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3
*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^

3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i - ((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^1

3*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11
*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 -
11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d
^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2
)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*
a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*
b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a
*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*
e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2
+ 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 +
480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2
*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1
/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^1
0*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^5 + 192*a^12*c^9*d^
4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5 + 128*
a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^2 + 96*
a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e^4 + 16
*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e - 128*a
^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) - x*(8*a^13*c^7*e^6 - 8*a^10*c^1
0*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9
*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a
^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*
c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(
4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3
*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d
^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a
^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*
c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4
- 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^
3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4
*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*
c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 2
92*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a
^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*
b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1
/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^
4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i)/(((-(b^11
*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 -
11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -

4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d
^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2
)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*
a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*
b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^
2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a
*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*
e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2
 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 +
480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2
*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1
/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^1
0*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^
4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*
c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4
+ a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/
2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3
*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*
c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a
^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^
(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^
4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*
d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*
e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 -
 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c
*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*
c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^
13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b
^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6
*e + 262144*a^17*b*c^7*e) + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2
+ 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 -
 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 +
 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 2457
6*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-
(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*
d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1
/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*
c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c
- b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4
 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*
(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366

*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) + x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + ((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*

d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e
^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3
+ 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a
*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b
^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^
(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a
^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^
5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*
e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^
7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^
4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(
1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d
^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*
a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6
*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5
)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*
a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^
2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^
3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3
 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*
c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b
*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^
6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*
a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16
*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c
^6*e + 262144*a^17*b*c^7*e) - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^
2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2
 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2
 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24
576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*
(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^
5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^
7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*
c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e
^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3
66*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e
*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3
*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*
d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d
*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*
a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2

)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) - x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*2i + 2*atan((((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3

*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d
^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a
^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*
c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4
 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^
3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4
*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*
c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 2
92*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a
^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*
b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1
/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^
4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4
 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*
a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a
^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 -
 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)
^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*
b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c
- b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*
c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c -
 b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*
(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 6
80*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*
a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b
^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))
/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^
2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^
5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e +
 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e)*1i + x
*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 133
12*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 10
24*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 655
36*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a
^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6
d^4(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a
^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a
^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4
 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4
*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b
^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*

a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 16*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i - x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - ((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) +

56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5
)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^
3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2
 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12
*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 +
256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((
(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^
5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^
7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*
c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e
^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^
4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3
66*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e
*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3
*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*
d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d
*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*
a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*
a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*
b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^
7*e)*1i - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^
4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c
^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^
6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e
 - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^
7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c
*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*
e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*
b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) +
a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e
^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e
^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c
- b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^
4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4
*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-

(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^
7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^
(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d^4*e + 16*a
^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128*a^13*c^8*d
^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96*a^11*b^4*c
^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 16*a^9*b^6*c
^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*a^12*b*c^8*
d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i + x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 +
4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^
7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2
*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5
*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c -
b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 -
a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e -
128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*
c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2
)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*
b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e
^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c -
b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e
+ 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*
b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*
c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3
*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8
*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*
a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4))/(((-(b^11*d^4 + a^4
*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^
5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8
*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a
^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2)
+ a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^
2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)
^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^
2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2
) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^
5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a
*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4
*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*
c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*
(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)
))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 1

12*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e)*1i + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 16*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i - x*(8*a^13*c^7*e^6 - 8*a^10 *c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4 *a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9* b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4 *(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b *c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c ^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 2 80*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c *d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2* b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^ 3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4* (-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2* b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 6 40*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320* a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b* c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^1 1*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i + ((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d ^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/ 2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c ^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366* a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(- (4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^ 3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2 *e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^ 3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4 *a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3 *b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256 *a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5* c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128* a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4* d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5) ^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10 *d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)

^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e)*1i - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 16*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i + x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a

```
^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^
2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^
(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^
2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) -
5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(
1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c -
 b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4
*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*
d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/
2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7
*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(
1/4)*1i))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 11
2*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c
- b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 +
 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d
^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6
*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*
b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*
e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a
*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e
^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*
a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5
*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3
*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(
1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4
*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*
b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + 2*atan((((-(b^11*d^4 + a^4*b^7*e^4 -
b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 4
8*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 12
8*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*
d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*
e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*
a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720
*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4
*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) +
56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5
)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^
3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2
 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12
*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c -
 b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 +
256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((
```

(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^
5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^
(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^
7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*
c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e
^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^
4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 3
66*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e
*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3
*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*
d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d
*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(
-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*
a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*
a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*
b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^
7*e)*1i + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^
4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c
^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^
6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e
 - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^
7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c
*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*
e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*
b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) -
a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e
^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e
^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c
- b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^
4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4
*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1
/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^
7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^
(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d^4*e + 16*a
^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128*a^13*c^8*d
^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96*a^11*b^4*c
^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 16*a^9*b^6*c
^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*a^12*b*c^8*

d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i - x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) - ((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2

e^2(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*
a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*
e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78
*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^
5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^
3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(
4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9
*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4
*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d -
 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 26214
4*a^17*b*c^7*e)*1i - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*
a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880
*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*
a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*
b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d
^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 1
1*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4
*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4
 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^
5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^
2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*
c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^
5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)
^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^
3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 +
 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 48
0*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c -
 b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c
*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*
b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d
^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128
*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96
*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 1
6*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*
a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i + x*(8*a^13*c^7*e^6 - 8*a^10
*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4
*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 +
28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*
b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b
*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c
^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 2

80*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(
4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c
*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*
b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^
3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*
b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
 - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 6
40*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*
a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*
c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5
)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^1
1*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4))/(((-(b^1
1*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4
- 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2)
- 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*
d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^
2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6
*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4
*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4
*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*
a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d
e^3(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^
2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 +
 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*
c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^
2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(
1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^
10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5
)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e
^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7
*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4
 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1
/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^
3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*
a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)
^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a
^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2
*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3
*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3
- 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c
 - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*
c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6

*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a
^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*
b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^
6*e + 262144*a^17*b*c^7*e)*1i + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*
e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d
^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e
^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e +
24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e)
)*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*
c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5
)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*
b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*
a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2
*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*
d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) +
366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4
*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3
e(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a
^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*
c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2
*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e
*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 1
8*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b
^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2
- 256*a^10*b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192
*a^12*c^9*d^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^
6*e^5 - 128*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d
^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^
7*d*e^4 - 16*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d
^4*e + 128*a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i - x*(8*a^13*c^7*e
^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*
d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7
*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e
^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^
4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4
- 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3
+ 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*
c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*
b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 -
15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/
2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 -
720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a
*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2
) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^
2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^

3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i + ((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e)*1i - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^1

5*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2
) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(
4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d
*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3
*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) +
40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e -
6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b
^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^
2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2)
 + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6
*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e
 - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 2
00*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a
^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2
)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^
3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 9
6*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 64*a^14*c^7*e^5 + 128*a^11*b
*c^9*d^5 - 192*a^12*c^9*d^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 -
16*a^13*b^2*c^6*e^5 - 128*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*
a^11*b^3*c^7*d^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 -
 256*a^13*b*c^7*d*e^4 - 16*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*
a^11*b^2*c^8*d^4*e + 128*a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i + x
*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2
 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 1
6*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*
a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^
4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11
*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*
a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4
- 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5
)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2
*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c
 - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5
*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c
- b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3
*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 +
680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480
*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c -
b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*
d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b
^2*c^3)))^(1/4)*1i))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^
4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*

c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3))))^(1/4) - d/(3*a*x^3)

3.52 $\int \frac{x^4(1-x^4)}{1-x^4+x^8} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 278

$$\begin{aligned}\int \frac{x^4(1-x^4)}{1-x^4+x^8} \, dx = &-x-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} \\ &+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} \\ &-\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} \\ &-\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}\end{aligned}$$

```
[Out] -x-1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^
(1/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*
6^(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)
))*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/
2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x
^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^
(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1516, 1360, 1183, 648, 632, 210, 642}

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-x$$

```
[In] Int[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -x - ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcT
an[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[
2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[
3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x
^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 -
Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]
/(4*Sqrt[6])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1360

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\text{integral} = -x + \int \frac{1}{1 - x^4 + x^8}\,dx$$

$$= -x + \frac{\int \frac{\sqrt{3}-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -x + \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-1+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-1+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}$$
$$+ \frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}+\left(1+\sqrt{3}\right)x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= -x - \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}$$
$$+ \frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2-\sqrt{3}\right)}} + \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2-\sqrt{3}\right)}} + \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2+\sqrt{3}\right)}} + \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2+\sqrt{3}\right)}}$$

$$= -x - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$
$$- \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$
$$- \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, -\sqrt{2-\sqrt{3}}+2x\right)}{2\sqrt{6\left(2-\sqrt{3}\right)}}$$
$$- \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, \sqrt{2-\sqrt{3}}+2x\right)}{2\sqrt{6\left(2-\sqrt{3}\right)}}$$
$$- \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, -\sqrt{2+\sqrt{3}}+2x\right)}{2\sqrt{6\left(2+\sqrt{3}\right)}}$$
$$- \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, \sqrt{2+\sqrt{3}}+2x\right)}{2\sqrt{6\left(2+\sqrt{3}\right)}}$$

$$= -x - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}$$
$$+ \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$
$$- \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.17

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = -x+\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)}{-\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
[Out] -x + RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.12

method	result	size
default	$-x+\frac{\left(\sum_{\_R=\text{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)\right)}{4}$	34
risch	$-x+\frac{\left(\sum_{\_R=\text{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)\right)}{4}$	34

```
[In] int(x^4*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] -x+1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.37

$$\begin{aligned}\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx =& \left(\frac{1}{24}i+\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left((3i+3)\sqrt{3}\sqrt{2}x+6x^2+6i\right)\\ &-\left(\frac{1}{24}i-\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left(-(3i-3)\sqrt{3}\sqrt{2}x+6x^2-6i\right)\\ &+\left(\frac{1}{24}i-\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left((3i-3)\sqrt{3}\sqrt{2}x+6x^2-6i\right)\\ &-\left(\frac{1}{24}i+\frac{1}{24}\right)\sqrt{3}\sqrt{2}\log\left(-(3i+3)\sqrt{3}\sqrt{2}x+6x^2+6i\right)-x\end{aligned}$$

```
[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] (1/24*I + 1/24)*sqrt(3)*sqrt(2)*log((3*I + 3)*sqrt(3)*sqrt(2)*x + 6*x^2 + 6
*I) - (1/24*I - 1/24)*sqrt(3)*sqrt(2)*log(-(3*I - 3)*sqrt(3)*sqrt(2)*x + 6*
x^2 - 6*I) + (1/24*I - 1/24)*sqrt(3)*sqrt(2)*log((3*I - 3)*sqrt(3)*sqrt(2)*
x + 6*x^2 - 6*I) - (1/24*I + 1/24)*sqrt(3)*sqrt(2)*log(-(3*I + 3)*sqrt(3)*s
qrt(2)*x + 6*x^2 + 6*I) - x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.61

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = -x - \frac{\sqrt{6}\left(-2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)-2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24} - \frac{\sqrt{6}\left(-2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)-2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24} - \frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24} + \frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

```
[In] integrate(x**4*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -x - sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**2 + 2
*sqrt(6)*x - 3))/24 - sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) - 2*atan(sqrt(6)*
x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*
x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(
6)*x + 1)/24
```

Maxima [F]

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = \int -\frac{(x^4-1)x^4}{x^8-x^4+1}\,dx$$

```
[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -x + integrate(1/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.75

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = \frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-x$$

```
[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.20

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = -x+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

```
[In] int(-(x^4*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

```
[Out] - x - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/
12) - 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/
12)
```

3.53 $\int \frac{x^3(1-x^4)}{1-x^4+x^8} dx$

Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8} dx = -\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1-x^4+x^8\right)$$

```
[Out] -1/8*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1482, 648, 632, 210, 642}

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8} dx = -\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(x^8-x^4+1\right)$$

```
[In] Int[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3] - Log[1 - x^4 + x^8]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right) - \frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)\\
&= -\frac{1}{8}\log\left(1-x^4+x^8\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = \frac{\arctan\left(\frac{-1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1-x^4+x^8\right)$$

```
[In] Integrate[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x^4 + x^8]/8
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{\ln\left(x^{8}-x^{4}+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{4}-1\right)\sqrt{3}}{3}\right)}{12}$	33
risch	$-\frac{\ln\left(4x^{8}-4x^{4}+4\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{4}-1\right)\sqrt{3}}{3}\right)}{12}$	35

```
[In] int(x^3*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx=\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right)-\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

```
[In] integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx=-\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

```
[In] integrate(x**3*(-x**4+1)/(x**8-x**4+1),x)
[Out] -log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) - \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

```
[In] integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^4-1)\right) - \frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

```
[In] integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx = -\frac{\ln\left(x^8-x^4+1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}$$

```
[In] int(-(x^3*(x^4 - 1))/(x^8 - x^4 + 1),x)
[Out] - log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12
```

3.54 $\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 355

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right) + \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)$$

```
[Out] 1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(
1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((-2
*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(
1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(
3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(
1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/
6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/
2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/
2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1520, 1293, 1183, 648, 632, 210, 642}

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = \frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right) + \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)$$

```
[In] Int[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])
]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqr
t[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
- Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^
2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqr
t[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[
3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1520

```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Dist[c/(2*q*r), Int[(f*x)^m*(Simp[d*r - (c*d - e*q)*x^(n/2), x]/
(q - r*x^(n/2) + c*x^n)), x], x] + Dist[c/(2*q*r), Int[(f*x)^m*(Simp[d*r +
(c*d - e*q)*x^(n/2), x]/(q + r*x^(n/2) + c*x^n)), x], x]] /;  !LtQ[2*c*q -
b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c
, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{x^2\left(\sqrt{3}-2x^2\right)}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{x^2\left(\sqrt{3}+2x^2\right)}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{-2+\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{2+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{2\sqrt{2-\sqrt{3}}-\left(2-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{2\sqrt{2-\sqrt{3}}+\left(2-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}$$

$$- \frac{\int \frac{-2\sqrt{2+\sqrt{3}}-\left(-2-\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\int \frac{-2\sqrt{2+\sqrt{3}}+\left(-2-\sqrt{3}\right)x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx$$

$$+ \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx$$

$$- \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx$$

$$- \frac{\left(-2+\sqrt{3}\right)\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx$$

$$+ \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx$$

$$- \frac{1}{8}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx$$

$$- \frac{1}{8}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx$$

$$= \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)$$

$$- \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)$$

$$+ \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)$$

$$- \frac{1}{4}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\mathrm{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, -\sqrt{2+\sqrt{3}}+2x\right)$$

$$- \frac{1}{4}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\mathrm{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, \sqrt{2+\sqrt{3}}+2x\right)$$

$$+ \frac{1}{4}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\mathrm{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, -\sqrt{2-\sqrt{3}}+2x\right)$$

$$+ \frac{1}{4}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\mathrm{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, \sqrt{2-\sqrt{3}}+2x\right)$$

$$= \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right) + \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.15

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = -\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&, \frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1+2\#1^5}\&\right]$$

```
[In] Integrate[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
[Out] -1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1 + 2
*#1^5) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.13

method	result	size
default	$-\frac{\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(\_R^6-\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}}{4}$	46
risch	$\frac{\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-\_R^6+\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}}{4}$	46

```
[In] int(x^2*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4*sum((_R^6-_R^2)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.54

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\left(i\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\left(-i\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\left(i\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)$$
$$-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\left(-i\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\sqrt{i\sqrt{3}+1}+24x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\left(i\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\sqrt{-i\sqrt{3}+1}+24x\right)$$
$$-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\left(-i\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\sqrt{-i\sqrt{3}+1}+24x\right)$$
$$-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\left(i\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\sqrt{-i\sqrt{3}+1}+24x\right)$$
$$+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\left(-i\sqrt{3}\sqrt{2}-3\sqrt{2}\right)\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\sqrt{-i\sqrt{3}+1}+24x\right)$$

```
[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(I*sqrt(3)*sqrt
(2) - 3*sqrt(2))*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*sqrt(3) + 1) + 24
*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(-I*sqrt(3
)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*sqrt(3) + 1
) + 24*x) + 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*(I*
sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*sqrt(I*sqrt
(3) + 1) + 24*x) - 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt
(6)*(-I*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*sqr
t(I*sqrt(3) + 1) + 24*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*
log(sqrt(6)*(I*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(sqrt(2)*sqrt(-I*sqrt(3) +
1))*sqrt(-I*sqrt(3) + 1) + 24*x) - 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3
) + 1))*log(sqrt(6)*(-I*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(2)*sqrt(-I*s
qrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x) - 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt
(-I*sqrt(3) + 1))*log(sqrt(6)*(I*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-sqrt(2)
*sqrt(-I*sqrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x) + 1/24*sqrt(6)*sqrt(-sq
rt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*(-I*sqrt(3)*sqrt(2) - 3*sqrt(2))*sq
rt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*sqrt(-I*sqrt(3) + 1) + 24*x)
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.08

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = -\operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(442368t^7-384t^3+x\right)\right)\right)$$

```
[In] integrate(x**2*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(442368*_t**7 - 3
84*_t**3 + x)))
```

Maxima [F]

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = \int -\frac{(x^4-1)x^2}{x^8-x^4+1}\,dx$$

```
[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)*x^2/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.71

$$\begin{aligned}\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = &-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\ &-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\end{aligned}$$

```
[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.70

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}-\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(-1+\sqrt{3}\,1\mathrm{i}\right)}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12} + \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}-\frac{2^{3/4}\sqrt{3}\,x\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} - \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}+\frac{2^{3/4}\sqrt{3}\,x}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{3/4}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

```
[In] int(-(x^2*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)) + (3^(1/2)*x*
(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/4)*1i)
/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)) - (3
^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/
4))/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x)/(2*(3^(1/2)*1i + 1)^(3/4)) - (2^
(3/4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/
12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4)) + (2^(
3/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.55 $\int \frac{x(1-x^4)}{1-x^4+x^8} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \frac{x(1-x^4)}{1-x^4+x^8} \, dx = -\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}$$

```
[Out] -1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/12*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1504, 1178, 642}

$$\int \frac{x(1-x^4)}{1-x^4+x^8} \, dx = \frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}} - \frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

```
[In] Int[(x*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*Log[1 - Sqrt[3]*x^2 + x^4]/Sqrt[3] + Log[1 + Sqrt[3]*x^2 + x^4]/(4*Sqr
t[3])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
```

```
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1504

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx, x, x^2\right)\\ &= -\frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx, x, x^2\right)}{4\sqrt{3}} - \frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx, x, x^2\right)}{4\sqrt{3}}\\ &= -\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx = \frac{-\log\left(-1+\sqrt{3}x^2-x^4\right)+\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}$$

```
[In] Integrate[(x*(1 - x^4))/(1 - x^4 + x^8),x]
[Out] (-Log[-1 + Sqrt[3]*x^2 - x^4] + Log[1 + Sqrt[3]*x^2 + x^4])/(4*Sqrt[3])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	39
risch	$-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	39

```
[In] int(x*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/12*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx=\frac{1}{12}\sqrt{3}\log\left(\frac{x^8+5\,x^4+2\,\sqrt{3}(x^6+x^2)+1}{x^8-x^4+1}\right)$$

```
[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*log((x^8 + 5*x^4 + 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx=-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}$$

```
[In] integrate(x*(-x**4+1)/(x**8-x**4+1),x)
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/12
```

Maxima [F]

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx=\int -\frac{(x^4-1)x}{x^8-x^4+1}\,dx$$

```
[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
[Out] -integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.62

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx = -\frac{1}{12}\sqrt{3}\log\left(\frac{x^2-\sqrt{3}+\frac{1}{x^2}}{x^2+\sqrt{3}+\frac{1}{x^2}}\right)$$

```
[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
[Out] -1/12*sqrt(3)*log((x^2 - sqrt(3) + 1/x^2)/(x^2 + sqrt(3) + 1/x^2))
```

Mupad [B] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.40

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x^2}{x^4+1}\right)}{6}$$

```
[In] int(-(x*(x^4 - 1))/(x^8 - x^4 + 1),x)
[Out] (3^(1/2)*atanh((3^(1/2)*x^2)/(x^4 + 1)))/6
```

3.56 $\int \frac{1-x^4}{1-x^4+x^8} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 355

$$\int \frac{1-x^4}{1-x^4+x^8} \, dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)$$

```
[Out] 1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(
1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/4*arctan((-2
*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(
1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(
3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(
1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/
6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/
2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/
2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1435, 1183, 648, 632, 210, 642}

$$\int \frac{1-x^4}{1-x^4+x^8} dx = -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)$$

```
[In] Int[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[3*(2 - Sqrt[3
])] + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sq
rt[3])]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt
[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sq
rt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt
[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1435

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-2+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-2+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}$$

$$+ \frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}-\left(2+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2+\sqrt{3}\right)}+\left(2+\sqrt{3}\right)x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= -\left(\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx\right)$$
$$-\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx$$
$$+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx+\frac{\left(-2+\sqrt{3}\right)\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3\left(2-\sqrt{3}\right)}}$$
$$-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx$$
$$+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx$$
$$+\frac{1}{8}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx$$
$$+\frac{1}{8}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx$$

$$=\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)-\frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)$$
$$-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)$$
$$+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx,x,-\sqrt{2+\sqrt{3}}+2x\right)$$
$$+\frac{1}{4}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx,x,\sqrt{2+\sqrt{3}}+2x\right)$$
$$-\frac{1}{4}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx,x,-\sqrt{2-\sqrt{3}}+2x\right)$$
$$-\frac{1}{4}\sqrt{\frac{1}{3}\left(7+4\sqrt{3}\right)}\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx,x,\sqrt{2-\sqrt{3}}+2x\right)$$

$$= -\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)-\frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}x + x^2\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1 + \sqrt{2+\sqrt{3}}x+x^2\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.16

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

```
[In] Integrate[(1 - x^4)/(1 - x^4 + x^8),x]
[Out] -1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 +
 2*#1^7) & ]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.12

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	44
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	44

```
[In] int((-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.17

$$\begin{aligned}\int \frac{1-x^4}{1-x^4+x^8}\,dx =& \frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(i\sqrt{3}+3\right)+12\,x\right)\\ &+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(i\sqrt{3}+3\right)\right.\\ &\left.+12\,x\right)\\ &-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(i\sqrt{3}-3\right)+12\,x\right)\\ &-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(i\sqrt{3}-3\right)\right.\\ &\left.+12\,x\right)\\ &+\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(-i\sqrt{3}+3\right)+12\,x\right)\\ &+\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{i\sqrt{3}+1}}\left(-i\sqrt{3}+3\right)\right.\\ &\left.+12\,x\right)\\ &-\frac{1}{24}\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(-i\sqrt{3}-3\right)\right.\\ &\left.+12\,x\right)\\ &-\frac{1}{24}\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\log\left(\sqrt{6}\sqrt{-\sqrt{2}\sqrt{-i\sqrt{3}+1}}\left(-i\sqrt{3}-3\right)\right.\\ &\left.+12\,x\right)\end{aligned}$$

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*sq
rt(-I*sqrt(3) + 1))*(I*sqrt(3) + 3) + 12*x) + 1/24*sqrt(6)*sqrt(-sqrt(2)*sq
```

```
rt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*(I*sqrt
(3) + 3) + 12*x) - 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(
6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*(I*sqrt(3) - 3) + 12*x) - 1/24*sqrt(6)
*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3
) + 1))*(I*sqrt(3) - 3) + 12*x) + 1/24*sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3)
+ 1))*log(sqrt(6)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*(-I*sqrt(3) + 3) + 12*x
) + 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(sqrt(6)*sqrt(-sqrt(
2)*sqrt(I*sqrt(3) + 1))*(-I*sqrt(3) + 3) + 12*x) - 1/24*sqrt(6)*sqrt(sqrt(2
)*sqrt(-I*sqrt(3) + 1))*log(sqrt(6)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*(-I*
sqrt(3) - 3) + 12*x) - 1/24*sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*log
(sqrt(6)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*(-I*sqrt(3) - 3) + 12*x)
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.07

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(9216t^5 - 8t + x\right)\right)\right)$$

```
[In] integrate((-x**4+1)/(x**8-x**4+1),x)
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 8*_
t + x)))
```

Maxima [F]

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = \int -\frac{x^4-1}{x^8-x^4+1}\,dx$$

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
[Out] -integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.71

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.59

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}-\frac{\sqrt{3}\,x}{\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}\right)\left(8-\sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12} + \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}-\frac{2^{1/4}\,\sqrt{3}\,x\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}\,1\mathrm{i}}{12} + \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1\mathrm{i}}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}+\frac{2^{1/4}\,\sqrt{3}\,x}{2\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

```
[In] int(-(x^4 - 1)/(x^8 - x^4 + 1),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^(
1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3^(
1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4
))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3
^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (2^(3/
4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*3^(1/2
)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.57 $\int \frac{1-x^4}{x(1-x^4+x^8)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 41

$$\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx=\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)$$

```
[Out] ln(x)-1/8*ln(x^8-x^4+1)+1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1488, 814, 648, 632, 210, 642}

$$\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx=\frac{\arctan\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

```
[In] Int[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
```

```
[Out] ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4 + x^8]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1-x}{x\left(1-x+x^2\right)}\,dx, x, x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)\,dx, x, x^4\right)\\
&= \log(x)-\frac{1}{4}\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \log(x)-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)-\frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\\
&= \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx = \log(x)-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^4}{-1+2\#1^4}\&\right]$$

```
[In] Integrate[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
[Out] Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 + 2*#1^4) & ]/4
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln(x)-\frac{\ln(x^8-x^4+1)}{8}-\frac{\sqrt{3}\arctan\left(\frac{2\left(x^4-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$	33
default	$\ln(x)-\frac{\ln(x^8-x^4+1)}{8}-\frac{\sqrt{3}\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)}{12}$	35

```
[In] int((-x^4+1)/x/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] ln(x)-1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(2/3*(x^4-1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx = -\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4-1)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

```
[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="fricas")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + lo
g(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx = \log(x) - \frac{\log\left(x^8-x^4+1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

```
[In] integrate((-x**4+1)/x/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4-1\right)\right) - \frac{1}{8}\,\log\left(x^8-x^4+1\right) + \frac{1}{4}\,\log\left(x^4\right)$$

```
[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/
4*log(x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx = -\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^4-1\right)\right) - \frac{1}{8}\,\log\left(x^8-x^4+1\right) + \frac{1}{4}\,\log\left(x^4\right)$$

```
[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/
4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 8.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1-x^4}{x\,(1-x^4+x^8)}\,dx = \ln(x) - \frac{\ln(x^8-x^4+1)}{8} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}\,x^4}{3}\right)}{12}$$

```
[In] int(-(x^4 - 1)/(x*(x^8 - x^4 + 1)),x)
```

```
[Out] log(x) - log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3
))/12
```

3.58 $\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 280

$$\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx=-\frac{1}{x}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}-\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$

```
[Out] -1/x+1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*
6^(1/2)-1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2))
)*6^(1/2)+1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/
2)))*6^(1/2)-1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(
1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1
+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/2*
2^(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1518, 1386, 1183, 648, 632, 210, 642}

$$\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx=\frac{\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{1}{x}$$

```
[In] Int[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) +
 ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(Sqrt[2 +
Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*
x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log
[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x +
 x^2]/(4*Sqrt[6])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1386

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[x^(
m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Dist[1/(2*c*
r), Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{x}-\int \frac{x^6}{1-x^4+x^8}\,dx\\
&= -\frac{1}{x}+\frac{\int \frac{1-\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}-\frac{\int \frac{1+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}\\
&= -\frac{1}{x}-\frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{2-\sqrt{3}}+\left(1-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}\\
&\quad+\frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}+\left(1+\sqrt{3}\right)x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}
\end{aligned}$$

$$= -\frac{1}{x} - \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}$$

$$- \frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2-\sqrt{3}\right)}} - \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2-\sqrt{3}\right)}} - \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2+\sqrt{3}\right)}} - \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6\left(2+\sqrt{3}\right)}}$$

$$= -\frac{1}{x} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$

$$- \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, -\sqrt{2-\sqrt{3}}+2x\right)}{2\sqrt{6\left(2-\sqrt{3}\right)}}$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, \sqrt{2-\sqrt{3}}+2x\right)}{2\sqrt{6\left(2-\sqrt{3}\right)}}$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, -\sqrt{2+\sqrt{3}}+2x\right)}{2\sqrt{6\left(2+\sqrt{3}\right)}}$$

$$+ \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, \sqrt{2+\sqrt{3}}+2x\right)}{2\sqrt{6\left(2+\sqrt{3}\right)}}$$

$$= -\frac{1}{x} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}$$

$$- \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$

$$- \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.17

$$\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx = -\frac{1}{x}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^3}{-1+2\#1^4}\&\right]$$

```
[In] Integrate[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
[Out] -x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]/
4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.14

method	result	size
default	$-\frac{\left(\sum_{\_R=\text{RootOf}(9\_Z^4+1)}\_R\ln\left(9x\_R^3-3\_R^2+x^2\right)\right)}{4}-\frac{1}{x}$	38
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\text{RootOf}(9\_Z^4+1)}\_R\ln\left(-9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	38

```
[In] int((-x^4+1)/x^2/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))-1/x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.40

$$\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx$$
$$= \frac{-(i-1)\sqrt{3}\sqrt{2}x\log\left((3i+3)\sqrt{3}\sqrt{2}x+6x^2+6i\right)+(i+1)\sqrt{3}\sqrt{2}x\log\left(-(3i-3)\sqrt{3}\sqrt{2}x+6x^2-6i\right)}{}$$

```
[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/24*(-(I - 1)*sqrt(3)*sqrt(2)*x*log((3*I + 3)*sqrt(3)*sqrt(2)*x + 6*x^2 +
6*I) + (I + 1)*sqrt(3)*sqrt(2)*x*log(-(3*I - 3)*sqrt(3)*sqrt(2)*x + 6*x^2 -
```

```
6*I) - (I + 1)*sqrt(3)*sqrt(2)*x*log((3*I - 3)*sqrt(3)*sqrt(2)*x + 6*x^2 -
 6*I) + (I - 1)*sqrt(3)*sqrt(2)*x*log(-(3*I + 3)*sqrt(3)*sqrt(2)*x + 6*x^2
+ 6*I) - 24)/x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.60

$$\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx = -\frac{\sqrt{6}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24} - \frac{\sqrt{6}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24} - \frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24} + \frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24} - \frac{1}{x}$$

```
[In] integrate((-x**4+1)/x**2/(x**8-x**4+1),x)
```

```
[Out] -sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt
(6)*x - 3))/24 - sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
 sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
 1)/24 - 1/x
```

Maxima [F]

$$\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx = \int -\frac{x^4-1}{(x^8-x^4+1)x^2}\,dx$$

```
[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate(x^6/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.75

$$\int \frac{1-x^4}{x^2\,(1-x^4+x^8)}\,dx = -\frac{1}{12}\,\sqrt{6}\arctan\left(\frac{4\,x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{12}\,\sqrt{6}\arctan\left(\frac{4\,x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right) - \frac{1}{12}\,\sqrt{6}\arctan\left(\frac{4\,x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) - \frac{1}{12}\,\sqrt{6}\arctan\left(\frac{4\,x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) + \frac{1}{24}\,\sqrt{6}\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right) - \frac{1}{24}\,\sqrt{6}\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}+\sqrt{2}\right)+1\right) + \frac{1}{24}\,\sqrt{6}\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{24}\,\sqrt{6}\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{6}-\sqrt{2}\right)+1\right) - \frac{1}{x}$$

```
[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="giac")
[Out] -1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*sqrt(6
)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/12*sqrt(6)*arct
an((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sq
rt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*
sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/x
```

Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.21

$$\int \frac{1-x^4}{x^2\,(1-x^4+x^8)}\,dx = -\frac{1}{x} + \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right) + \sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

```
[In] int(-(x^4 - 1)/(x^2*(x^8 - x^4 + 1)),x)
```

```
[Out] 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12) +
6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12) -
1/x
```

3.59 $\int \frac{1-x^4}{x^3(1-x^4+x^8)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = -\frac{1}{2x^2}+\frac{1}{4}\arctan\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\arctan\left(\sqrt{3}+2x^2\right)-\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}$$

```
[Out] -1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24*ln(1+x^4-
x^2*3^(1/2))*3^(1/2)+1/24*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1504, 1295, 1141, 1175, 632, 210, 1178, 642}

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = \frac{1}{4}\arctan\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\arctan\left(2x^2+\sqrt{3}\right)-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

```
[In] Int[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] -1/2*1/x^2 + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[3] + 2*x^2]/4 - Log[1
- Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1504

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{x^2(1-x^2+x^4)}\,dx, x, x^2\right) \\
&= -\frac{1}{2x^2} - \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1-x^2+x^4}\,dx, x, x^2\right) \\
&= -\frac{1}{2x^2} + \frac{1}{4}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx, x, x^2\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{1-x^2+x^4}\,dx, x, x^2\right) \\
&= -\frac{1}{2x^2} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, x^2\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, x^2\right) \\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx, x, x^2\right)}{8\sqrt{3}} - \frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx, x, x^2\right)}{8\sqrt{3}} \\
&= -\frac{1}{2x^2} - \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} \\
&\quad + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2x^2\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{3}+2x^2\right) \\
&= -\frac{1}{2x^2} + \frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right) - \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right) \\
&\quad - \frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.55

$$\int \frac{1-x^4}{x^3(1-x^4+x^8)}\,dx = -\frac{1}{2x^2} - \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&, \frac{\log(x-\#1)\#1^2}{-1+2\#1^4}\&\right]$$

```
[In] Integrate[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
[Out] -1/2*1/x^2 - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^4) &
 ]/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

method	result	size
risch	$-\frac{1}{2x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(-6\_R^3+x^2-\_R\right)\right)}{4}$	40
default	$-\frac{1}{2x^2}+\frac{\sqrt{3}\left(-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)}{2}-\sqrt{3}\,\arctan\left(2x^2-\sqrt{3}\right)\right)}{12}+\frac{\sqrt{3}\left(\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)}{2}-\sqrt{3}\,\arctan\left(2x^2+\sqrt{3}\right)\right)}{12}$	82

```
[In] int((-x^4+1)/x^3/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/2/x^2+1/4*sum(_R*ln(-6*_R^3+x^2-_R),_R=RootOf(9*_Z^4+3*_Z^2+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.90

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = -\frac{\sqrt{6}x^2\sqrt{i\sqrt{3}-1}\log\left(6\,x^2+i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}\right)-\sqrt{6}x^2\sqrt{i\sqrt{3}-1}\log\left(6\,x^2-i\sqrt{6}\sqrt{3}\sqrt{i\sqrt{3}-1}\right)-}{}$$

```
[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="fricas")
[Out] -1/24*(sqrt(6)*x^2*sqrt(I*sqrt(3) - 1)*log(6*x^2 + I*sqrt(6)*sqrt(3)*sqrt(I
*sqrt(3) - 1)) - sqrt(6)*x^2*sqrt(I*sqrt(3) - 1)*log(6*x^2 - I*sqrt(6)*sqrt
(3)*sqrt(I*sqrt(3) - 1)) - sqrt(6)*x^2*sqrt(-I*sqrt(3) - 1)*log(6*x^2 + I*s
qrt(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1)) + sqrt(6)*x^2*sqrt(-I*sqrt(3) - 1)*log
(6*x^2 - I*sqrt(6)*sqrt(3)*sqrt(-I*sqrt(3) - 1)) + 12)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = -\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}-\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}-\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}-\frac{1}{2x^2}$$

```
[In] integrate((-x**4+1)/x**3/(x**8-x**4+1),x)
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt(3))/4 - 1/(2*x**2)
```

Maxima [F]

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = \int -\frac{x^4-1}{(x^8-x^4+1)x^3}\,dx$$

```
[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="maxima")
[Out] -1/2/x^2 - integrate(x^5/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = -\frac{1}{24}\sqrt{3}x^4\log\left(x^4+\sqrt{3}x^2+1\right)+\frac{1}{24}\sqrt{3}x^4\log\left(x^4-\sqrt{3}x^2+1\right) - \frac{1}{4}x^4\arctan\left(2x^2+\sqrt{3}\right)-\frac{1}{4}x^4\arctan\left(2x^2-\sqrt{3}\right)-\frac{1}{2x^2}$$

```
[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="giac")
[Out] -1/24*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) + 1/24*sqrt(3)*x^4*log(x^4 - s
qrt(3)*x^2 + 1) - 1/4*x^4*arctan(2*x^2 + sqrt(3)) - 1/4*x^4*arctan(2*x^2 -
sqrt(3)) - 1/2/x^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.63

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = \mathrm{atan}\left(\frac{2x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \mathrm{atan}\left(\frac{2x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\frac{1}{2x^2}$$

```
[In] int(-(x^4 - 1)/(x^3*(x^8 - x^4 + 1)),x)
[Out] atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x^2)/(3^(1
/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - 1/(2*x^2)
```

3.60 $\int \frac{1-x^4}{x^4(1-x^4+x^8)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 370

$$\begin{aligned}
\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx =& -\frac{1}{3x^3}-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)\\
&+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\
&+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)\\
&-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)\\
&+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)\\
&-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)\\
&-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)\\
&+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)
\end{aligned}$$

```
[Out] -1/3/x^3-1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)
))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*
6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)
+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^
(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))
/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/4*arctan((2*x+1/2*6
```

```
^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/
8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/8*ln(1+
x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1518, 12, 1387, 1141, 1175, 632, 210, 1178, 642}

$$\begin{aligned}\int \frac{1-x^4}{x^4(1-x^4+x^8)}\,dx = &-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)\\ &+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)\\ &+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)\\ &-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{3x^3}\\ &+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)\\ &-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)\\ &-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)\\ &+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)\end{aligned}$$

```
[In] Int[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2
 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/S
qrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2
*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]
] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sq
rt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1387

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
```

```
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*(n
/2)] && NegQ[b^2 - 4*a*c]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{3x^3} - \frac{1}{3}\int \frac{3x^4}{1-x^4+x^8}\,dx \\
&= -\frac{1}{3x^3} - \int \frac{x^4}{1-x^4+x^8}\,dx \\
&= -\frac{1}{3x^3} - \frac{\int \frac{x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} \\
&= -\frac{1}{3x^3} + \frac{\int \frac{1-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} - \frac{\int \frac{1+x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} - \frac{\int \frac{1-x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} + \frac{\int \frac{1+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}} \\
&= -\frac{1}{3x^3} + \frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} - \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}} \\
&\quad + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{-1-\sqrt{2-\sqrt{3}}x-x^2}\,dx}{8\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}-2x}{-1+\sqrt{2-\sqrt{3}}x-x^2}\,dx}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{-1-\sqrt{2+\sqrt{3}}x-x^2}\,dx}{8\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2+\sqrt{3}}-2x}{-1+\sqrt{2+\sqrt{3}}x-x^2}\,dx}{8\sqrt{3\left(2+\sqrt{3}\right)}}
\end{aligned}$$

$$= -\frac{1}{3x^3} + \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, -\sqrt{2-\sqrt{3}}+2x\right)}{4\sqrt{3}} - \frac{\text{Subst}\left(\int \frac{1}{-2-\sqrt{3}-x^2}\,dx, x, \sqrt{2-\sqrt{3}}+2x\right)}{4\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, -\sqrt{2+\sqrt{3}}+2x\right)}{4\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{1}{-2+\sqrt{3}-x^2}\,dx, x, \sqrt{2+\sqrt{3}}+2x\right)}{4\sqrt{3}}$$

$$= -\frac{1}{3x^3} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.13

$$\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx = -\frac{1}{3x^3} - \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&, \frac{\log(x-\#1)\#1}{-1+2\#1^4}\&\right]$$

```
[In] Integrate[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) & ]
/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.10

method	result	size
risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(81\_Z^8-9\_Z^4+1\right)}\_R\ln\left(18\_R^5-\_R+x\right)\right)}{4}$	38
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}-\frac{1}{3x^3}$	46

```
[In] int((-x^4+1)/x^4/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/3/x^3+1/4*sum(_R*ln(18*_R^5-_R+x),_R=RootOf(81*_Z^8-9*_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.13

$$\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx = \frac{\sqrt{6}x^3\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(i\sqrt{6}\sqrt{3}\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}+6x\right)-\sqrt{6}x^3\sqrt{\sqrt{2}\sqrt{i\sqrt{3}+1}}\log\left(-i\sqrt{6}\sqrt{3}\sqrt{\sqrt{2}\sqrt{}}\right.}{}$$

```
[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/24*(sqrt(6)*x^3*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1))*log(I*sqrt(6)*sqrt(3)*s
qrt(sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6*x) - sqrt(6)*x^3*sqrt(sqrt(2)*sqrt(I*s
qrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6*x
) + sqrt(6)*x^3*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1))*log(I*sqrt(6)*sqrt(3)*sq
rt(-sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6*x) - sqrt(6)*x^3*sqrt(-sqrt(2)*sqrt(I*
sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(-sqrt(2)*sqrt(I*sqrt(3) + 1)) + 6
*x) - sqrt(6)*x^3*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(I*sqrt(6)*sqrt(3)*
sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1)) + 6*x) + sqrt(6)*x^3*sqrt(sqrt(2)*sqrt(-
I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(sqrt(2)*sqrt(-I*sqrt(3) + 1)) +
 6*x) - sqrt(6)*x^3*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1))*log(I*sqrt(6)*sqrt(
3)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) + 1)) + 6*x) + sqrt(6)*x^3*sqrt(-sqrt(2)*s
qrt(-I*sqrt(3) + 1))*log(-I*sqrt(6)*sqrt(3)*sqrt(-sqrt(2)*sqrt(-I*sqrt(3) +
 1)) + 6*x) - 8)/x^3
```

Sympy [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.09

$$\int \frac{1-x^4}{x^4(1-x^4+x^8)}\,dx$$
$$= -\operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(-18432t^5+4t+x\right)\right)\right)-\frac{1}{3x^3}$$

```
[In] integrate((-x**4+1)/x**4/(x**8-x**4+1),x)
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*_t**5 + 4
*_t + x))) - 1/(3*x**3)
```

Maxima [F]

$$\int \frac{1-x^4}{x^4(1-x^4+x^8)}\,dx = \int -\frac{x^4-1}{(x^8-x^4+1)x^4}\,dx$$

```
[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="maxima")
[Out] -1/3/x^3 - integrate(x^4/(x^8 - x^4 + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.70

$$\begin{aligned}\int \frac{1-x^4}{x^4(1-x^4+x^8)}\,dx = &-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)\\ &-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)\\ &-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)\\ &-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)\\ &+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{3x^3}\end{aligned}$$

```
[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/3/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.29

$$\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx$$
$$= -\frac{1}{3\,x^3} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}}{2\,\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}+\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}\,1\mathrm{i}}{2\,\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}\right)\,{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}\,1\mathrm{i}}{12}$$
$$+ \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}\,1\mathrm{i}}{2\,\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}-\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}}{2\,\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8\mathrm{i}}}{4}\right)}\right)\,{\left(8-\sqrt{3}\,8\mathrm{i}\right)}^{1/4}}{12}$$
$$- \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}}{2\,\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}-\frac{2^{3/4}\,\sqrt{3}\,x\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}\,1\mathrm{i}}{2\,\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\right)\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}\,1\mathrm{i}}{12}$$
$$- \frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}\,1\mathrm{i}}{2\,\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}+\frac{2^{3/4}\,\sqrt{3}\,x\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}}{2\,\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\right)\,{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^{1/4}}{12}$$

```
[In] int(-(x^4 - 1)/(x^4*(x^8 - x^4 + 1)),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2
)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i
)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(
8 - 3^(1/2)*8i)^(1/4)*1i)/12 - 1/(3*x^3) + (3^(1/2)*atan((x*(8 - 3^(1/2)*8i
)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1
/2)/4)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^
(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/12 - (2^(
3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1/2)*
1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) - (2^(3/
4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2)
```

```
)/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)^(1/
4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*
((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/
2)*1i)/2)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1/
2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(
1/2)*1i + 1)^(1/4))/12
```

3.61 $\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 280

$$\begin{aligned}\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx &= \frac{(a^2d^2+b^2e^2+ae(bd-ce))\,x}{a^3e^3}-\frac{(ad+be)x^2}{2a^2e^2}+\frac{x^3}{3ae}\\ &\quad+\frac{(b^5d-5ab^3cd+5a^2bc^2d-b^4ce+4ab^2c^2e-2a^2c^3e)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}\\ &\quad-\frac{d^5\log(d+ex)}{e^4\,(ad^2-e(bd-ce))}+\frac{(b^4d-3ab^2cd+a^2c^2d-b^3ce+2abc^2e)\log\left(c+bx+ax^2\right)}{2a^4\,(ad^2-e(bd-ce))}\end{aligned}$$

```
[Out] (a^2*d^2+b^2*e^2+a*e*(b*d-c*e))*x/a^3/e^3-1/2*(a*d+b*e)*x^2/a^2/e^2+1/3*x^3
/a/e-d^5*ln(e*x+d)/e^4/(a*d^2-e*(b*d-c*e))+1/2*(a^2*c^2*d-3*a*b^2*c*d+2*a*b
*c^2*e+b^4*d-b^3*c*e)*ln(a*x^2+b*x+c)/a^4/(a*d^2-e*(b*d-c*e))+(5*a^2*b*c^2*
d-2*a^2*c^3*e-5*a*b^3*c*d+4*a*b^2*c^2*e+b^5*d-b^4*c*e)*arctanh((2*a*x+b)/(-
4*a*c+b^2)^(1/2))/a^4/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used

= {1583, 1642, 648, 632, 212, 642}

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$$
$$= -\frac{x^2(ad+be)}{2a^2e^2} + \frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)(5a^2bc^2d-2a^2c^3e-5ab^3cd+4ab^2c^2e+b^5d-b^4ce)}{a^4\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}$$
$$+ \frac{(a^2c^2d-3ab^2cd+2abc^2e+b^4d-b^3ce)\log(ax^2+bx+c)}{2a^4(ad^2-e(bd-ce))}$$
$$+ \frac{x(a^2d^2+ae(bd-ce)+b^2e^2)}{a^3e^3} - \frac{d^5\log(d+ex)}{e^4(ad^2-e(bd-ce))} + \frac{x^3}{3ae}$$

```
[In] Int[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] ((a^2*d^2 + b^2*e^2 + a*e*(b*d - c*e))*x)/(a^3*e^3) - ((a*d + b*e)*x^2)/(2*
a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e +
4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^4*S
qrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d^5*Log[d + e*x])/(e^4*(a*d^2
- e*(b*d - c*e))) + ((b^4*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2
*e)*Log[c + b*x + a*x^2])/(2*a^4*(a*d^2 - e*(b*d - c*e)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^5}{(d+ex)\left(c+bx+ax^2\right)}\,dx\\
&= \int \left(\frac{a^2d^2+b^2e^2+ae(bd-ce)}{a^3e^3}-\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{ae}+\frac{d^5}{e^3\left(-ad^2+e(bd-ce)\right)(d+ex)}\right.\\
&\quad \left.+\frac{c\left(b^3d-2abcd-b^2ce+ac^2e\right)+\left(b^4d-3ab^2cd+a^2c^2d-b^3ce+2abc^2e\right)x}{a^3\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx\\
&= \frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3}-\frac{(ad+be)x^2}{2a^2e^2}+\frac{x^3}{3ae}-\frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)}\\
&\quad +\frac{\int \frac{c\left(b^3d-2abcd-b^2ce+ac^2e\right)+\left(b^4d-3ab^2cd+a^2c^2d-b^3ce+2abc^2e\right)x}{c+bx+ax^2}\,dx}{a^3\left(ad^2-e(bd-ce)\right)}\\
&= \frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3}-\frac{(ad+be)x^2}{2a^2e^2}+\frac{x^3}{3ae}-\frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)}\\
&\quad +\frac{\left(b^4d-3ab^2cd+a^2c^2d-b^3ce+2abc^2e\right)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2a^4\left(ad^2-e(bd-ce)\right)}\\
&\quad -\frac{\left(b^5d-5ab^3cd+5a^2bc^2d-b^4ce+4ab^2c^2e-2a^2c^3e\right)\int \frac{1}{c+bx+ax^2}\,dx}{2a^4\left(ad^2-e(bd-ce)\right)}\\
&= \frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3}-\frac{(ad+be)x^2}{2a^2e^2}+\frac{x^3}{3ae}-\frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)}\\
&\quad +\frac{\left(b^4d-3ab^2cd+a^2c^2d-b^3ce+2abc^2e\right)\log\left(c+bx+ax^2\right)}{2a^4\left(ad^2-e(bd-ce)\right)}\\
&\quad +\frac{\left(b^5d-5ab^3cd+5a^2bc^2d-b^4ce+4ab^2c^2e-2a^2c^3e\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2ax\right)}{a^4\left(ad^2-e(bd-ce)\right)}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(a^2d^2+b^2e^2+ae(bd-ce))\,x}{a^3e^3} - \frac{(ad+be)x^2}{2a^2e^2} + \frac{x^3}{3ae} \\
&\quad + \frac{(b^5d-5ab^3cd+5a^2bc^2d-b^4ce+4ab^2c^2e-2a^2c^3e)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} \\
&\quad - \frac{d^5\log(d+ex)}{e^4\,(ad^2-e(bd-ce))} \\
&\quad + \frac{(b^4d-3ab^2cd+a^2c^2d-b^3ce+2abc^2e)\log\left(c+bx+ax^2\right)}{2a^4\,(ad^2-e(bd-ce))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.01

$$
\begin{aligned}
&\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx \\
&= \frac{(a^2d^2+abde+b^2e^2-ace^2)\,x}{a^3e^3} - \frac{(ad+be)x^2}{2a^2e^2} + \frac{x^3}{3ae} \\
&\quad + \frac{(b^5d-5ab^3cd+5a^2bc^2d-b^4ce+4ab^2c^2e-2a^2c^3e)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a^4\sqrt{-b^2+4ac}\,(-ad^2+bde-ce^2)} \\
&\quad - \frac{d^5\log(d+ex)}{e^4\,(ad^2-bde+ce^2)} + \frac{(b^4d-3ab^2cd+a^2c^2d-b^3ce+2abc^2e)\log\left(c+bx+ax^2\right)}{2a^4\,(ad^2-bde+ce^2)}
\end{aligned}
$$

```
[In] Integrate[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] ((a^2*d^2 + a*b*d*e + b^2*e^2 - a*c*e^2)*x)/(a^3*e^3) - ((a*d + b*e)*x^2)/(
2*a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e
+ 4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^4
*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + b*d*e - c*e^2)) - (d^5*Log[d + e*x])/(e^4*(
a*d^2 - b*d*e + c*e^2)) + ((b^4*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a
*b*c^2*e)*Log[c + b*x + a*x^2])/(2*a^4*(a*d^2 - b*d*e + c*e^2))
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.02

method	result
default	$\frac{\frac{1}{3}a^2e^2x^3-\frac{1}{2}a^2de\,x^2-\frac{1}{2}ab\,e^2x^2+a^2d^2x+abdex-e^2acx+b^2e^2x}{a^3e^3}+\frac{\frac{\left(a^2c^2d-3a\,b^2cd+2ab\,c^2e+d\,b^4-b^3ce\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(-2ab\,c^2d+a\right.}{}}{(a\,d^2-}$
risch	$\frac{x^3}{3ae}-\frac{d\,x^2}{2a\,e^2}-\frac{b\,x^2}{2a^2e}+\frac{d^2x}{a\,e^3}+\frac{bdx}{a^2e^2}-\frac{cx}{a^2e}+\frac{b^2x}{a^3e}-\frac{d^5\ln(ex+d)}{e^4(a\,d^2-bde+c\,e^2)}+\frac{\_R=\text{RootOf}\left(\left(4a^3c\,d^2-b^2d^2a^2-4a^2bcde+4a^2c^2e^2\right.\right.}{}$

```
[In] int(x^3/(a+c/x^2+b/x)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3/e^3*(1/3*a^2*e^2*x^3-1/2*a^2*d*e*x^2-1/2*a*b*e^2*x^2+a^2*d^2*x+a*b*d*
e*x-e^2*a*c*x+b^2*e^2*x)+1/(a*d^2-b*d*e+c*e^2)/a^3*(1/2*(a^2*c^2*d-3*a*b^2*
c*d+2*a*b*c^2*e+b^4*d-b^3*c*e)/a*ln(a*x^2+b*x+c)+2*(-2*a*b*c^2*d+a*c^3*e+b^
3*c*d-b^2*c^2*e-1/2*(a^2*c^2*d-3*a*b^2*c*d+2*a*b*c^2*e+b^4*d-b^3*c*e)*b/a)/
(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))-1/e^4*d^5/(a*d^2-b*d
*e+c*e^2)*ln(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 18.16 (sec) , antiderivative size = 1027, normalized size of antiderivative = 3.67

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$$

$$= \left[-\frac{6\,(a^4b^2-4\,a^5c)d^5\log\left(ex+d\right)-2\left((a^4b^2-4\,a^5c)d^2e^3-(a^3b^3-4\,a^4bc)de^4+(a^3b^2c-4\,a^4c^2)e^5\right)x^3+3\,(\ldots}{\ldots} \right.$$

$$\left. -\frac{6\,(a^4b^2-4\,a^5c)d^5\log\left(ex+d\right)-2\left((a^4b^2-4\,a^5c)d^2e^3-(a^3b^3-4\,a^4bc)de^4+(a^3b^2c-4\,a^4c^2)e^5\right)x^3+3\,(\ldots}{\ldots} \right.$$

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/6*(6*(a^4*b^2 - 4*a^5*c)*d^5*log(e*x + d) - 2*((a^4*b^2 - 4*a^5*c)*d^2*
e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c - 4*a^4*c^2)*e^5)*x^3 + 3*((
a^4*b^2 - 4*a^5*c)*d^3*e^2 - (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*d*e^4 + (a
^2*b^3*c - 4*a^3*b*c^2)*e^5)*x^2 + 3*((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d*e^4
 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^5)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2
+ 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c))
 - 6*((a^4*b^2 - 4*a^5*c)*d^4*e - (a*b^5 - 6*a^2*b^3*c + 8*a^3*b*c^2)*d*e^4
 + (a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*e^5)*x - 3*((b^6 - 7*a*b^4*c + 13*
a^2*b^2*c^2 - 4*a^3*c^3)*d*e^4 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^5)*l
og(a*x^2 + b*x + c))/((a^5*b^2 - 4*a^6*c)*d^2*e^4 - (a^4*b^3 - 4*a^5*b*c)*d
*e^5 + (a^4*b^2*c - 4*a^5*c^2)*e^6), -1/6*(6*(a^4*b^2 - 4*a^5*c)*d^5*log(e*
x + d) - 2*((a^4*b^2 - 4*a^5*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^
3*b^2*c - 4*a^4*c^2)*e^5)*x^3 + 3*((a^4*b^2 - 4*a^5*c)*d^3*e^2 - (a^2*b^4 -
 5*a^3*b^2*c + 4*a^4*c^2)*d*e^4 + (a^2*b^3*c - 4*a^3*b*c^2)*e^5)*x^2 - 6*((
b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d*e^4 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^
5)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c))
 - 6*((a^4*b^2 - 4*a^5*c)*d^4*e - (a*b^5 - 6*a^2*b^3*c + 8*a^3*b*c^2)*d*e^4
 + (a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*e^5)*x - 3*((b^6 - 7*a*b^4*c + 13*
a^2*b^2*c^2 - 4*a^3*c^3)*d*e^4 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^5)*l
og(a*x^2 + b*x + c))/((a^5*b^2 - 4*a^6*c)*d^2*e^4 - (a^4*b^3 - 4*a^5*b*c)*d
*e^5 + (a^4*b^2*c - 4*a^5*c^2)*e^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate(x**3/(a+c/x**2+b/x)/(e*x+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx &= -\frac{d^5\log\left(|ex+d|\right)}{ad^2e^4-bde^5+ce^6}+\frac{\left(b^4d-3\,ab^2cd+a^2c^2d-b^3ce+2\,abc^2e\right)\log\left(ax^2+bx+c\right)}{2\left(a^5d^2-a^4bde+a^4ce^2\right)}\\&\quad-\frac{\left(b^5d-5\,ab^3cd+5\,a^2bc^2d-b^4ce+4\,ab^2c^2e-2\,a^2c^3e\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^5d^2-a^4bde+a^4ce^2\right)\sqrt{-b^2+4\,ac}}\\&\quad+\frac{2\,a^2e^2x^3-3\,a^2dex^2-3\,abe^2x^2+6\,a^2d^2x+6\,abdex+6\,b^2e^2x-6\,ace^2x}{6\,a^3e^3}\end{aligned}$$

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
[Out] -d^5*log(abs(e*x + d))/(a*d^2*e^4 - b*d*e^5 + c*e^6) + 1/2*(b^4*d - 3*a*b^2
*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2*e)*log(a*x^2 + b*x + c)/(a^5*d^2 - a
^4*b*d*e + a^4*c*e^2) - (b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e + 4*
a*b^2*c^2*e - 2*a^2*c^3*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^5*d^2
 - a^4*b*d*e + a^4*c*e^2)*sqrt(-b^2 + 4*a*c)) + 1/6*(2*a^2*e^2*x^3 - 3*a^2*
d*e*x^2 - 3*a*b*e^2*x^2 + 6*a^2*d^2*x + 6*a*b*d*e*x + 6*b^2*e^2*x - 6*a*c*e
^2*x)/(a^3*e^3)
```

Mupad [B] (verification not implemented)

Time = 11.49 (sec) , antiderivative size = 2490, normalized size of antiderivative = 8.89

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Too large to display}$$

```
[In] int(x^3/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log(4*a^5*c*d^7 - a^4*b^2*d^7 + b^3*c^3*e^7 - b^6*d^3*e^4 - 6*a^2*c^4*d*e^
6 - 3*b^4*c^2*d*e^6 + 3*b^5*c*d^2*e^5 - 2*a^2*c^4*e^7*x - b^2*c^3*e^7*(b^2
- 4*a*c)^(1/2) + b^5*d^3*e^4*(b^2 - 4*a*c)^(1/2) + 2*a^3*c^3*d^3*e^4 - 4*a^
4*c^2*d^5*e^2 - 3*a*b*c^4*e^7 + a^4*b*d^7*(b^2 - 4*a*c)^(1/2) + a*c^4*e^7*(
b^2 - 4*a*c)^(1/2) + 2*a^5*d^7*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*c^3*d^2*e^5*(b
^2 - 4*a*c)^(1/2) + 8*a^5*c*d^6*e*x - 9*a^2*b^2*c^2*d^3*e^4 - 4*a^4*c*d^6*e
*(b^2 - 4*a*c)^(1/2) + 12*a*b^2*c^3*d*e^6 + 6*a*b^4*c*d^3*e^4 + a*b^2*c^3*e
^7*x - a*b^5*d^3*e^4*x - 2*a^4*b^2*d^6*e*x + 3*b^3*c^2*d*e^6*(b^2 - 4*a*c)^
(1/2) - 3*b^4*c*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 15*a*b^3*c^2*d^2*e^5 + 15*a^2
*b*c^3*d^2*e^5 + a^3*b^2*c*d^5*e^2 + a^3*b^3*d^5*e^2*x + 6*a^3*c^3*d^2*e^5*
x - 4*a*b^3*c*d^3*e^4*(b^2 - 4*a*c)^(1/2) + a^3*b*c*d^5*e^2*(b^2 - 4*a*c)^(
1/2) + a*b^4*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*c^3*d*e^6*x*(b^2 - 4*a*c
)^(1/2) - 2*a^4*c*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + 5*a^2*b^3*c*d^3*e^4*x - 5
*a^3*b*c^2*d^3*e^4*x + 9*a*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c^
2*d^3*e^4*(b^2 - 4*a*c)^(1/2) + a^3*b^2*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + a^3
*c^2*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) - 12*a^2*b^2*c^2*d^2*e^5*x - 6*a*b*c^3*d
*e^6*(b^2 - 4*a*c)^(1/2) - a*b*c^3*e^7*x*(b^2 - 4*a*c)^(1/2) - 2*a^4*b*d^6*
e*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^3*c^2*d*e^6*x + 3*a*b^4*c*d^2*e^5*x + 9*a^2
*b*c^3*d*e^6*x - 4*a^4*b*c*d^5*e^2*x + 3*a*b^2*c^2*d*e^6*x*(b^2 - 4*a*c)^(1
/2) - 3*a*b^3*c*d^2*e^5*x*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c^2*d^2*e^5*x*(b^2
- 4*a*c)^(1/2) - 3*a^2*b^2*c*d^3*e^4*x*(b^2 - 4*a*c)^(1/2))*(b^5*d*(b^2 - 4
*a*c)^(1/2) - b^6*d + 4*a^3*c^3*d + b^5*c*e - 13*a^2*b^2*c^2*d + 7*a*b^4*c*
d - b^4*c*e*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c^2*e + 8*a^2*b*c^3*e - 2*a^2*c^3
*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*d*(b^2 - 4*a*c)^(1/2) + 4*a*b^2*c^2*e*
(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*d*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^6*c*d^2 - a
^5*b^2*d^2 + 4*a^5*c^2*e^2 - a^4*b^2*c*e^2 + a^4*b^3*d*e - 4*a^5*b*c*d*e))
- (d^5*log(d + e*x))/(c*e^6 + a*d^2*e^4 - b*d*e^5) - x*((b*d + c*e)/(a^2*e^
2) - (a*d + b*e)^2/(a^3*e^3)) + (log(a^4*b^2*d^7 - 4*a^5*c*d^7 - b^3*c^3*e^
7 + b^6*d^3*e^4 + 6*a^2*c^4*d*e^6 + 3*b^4*c^2*d*e^6 - 3*b^5*c*d^2*e^5 + 2*a
^2*c^4*e^7*x - b^2*c^3*e^7*(b^2 - 4*a*c)^(1/2) + b^5*d^3*e^4*(b^2 - 4*a*c)^
(1/2) - 2*a^3*c^3*d^3*e^4 + 4*a^4*c^2*d^5*e^2 + 3*a*b*c^4*e^7 + a^4*b*d^7*(
b^2 - 4*a*c)^(1/2) + a*c^4*e^7*(b^2 - 4*a*c)^(1/2) + 2*a^5*d^7*x*(b^2 - 4*a
*c)^(1/2) - 3*a^2*c^3*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 8*a^5*c*d^6*e*x + 9*a^2
*b^2*c^2*d^3*e^4 - 4*a^4*c*d^6*e*(b^2 - 4*a*c)^(1/2) - 12*a*b^2*c^3*d*e^6 -
 6*a*b^4*c*d^3*e^4 - a*b^2*c^3*e^7*x + a*b^5*d^3*e^4*x + 2*a^4*b^2*d^6*e*x
+ 3*b^3*c^2*d*e^6*(b^2 - 4*a*c)^(1/2) - 3*b^4*c*d^2*e^5*(b^2 - 4*a*c)^(1/2)
```

```
 + 15*a*b^3*c^2*d^2*e^5 - 15*a^2*b*c^3*d^2*e^5 - a^3*b^2*c*d^5*e^2 - a^3*b^
3*d^5*e^2*x - 6*a^3*c^3*d^2*e^5*x - 4*a*b^3*c*d^3*e^4*(b^2 - 4*a*c)^(1/2) +
 a^3*b*c*d^5*e^2*(b^2 - 4*a*c)^(1/2) + a*b^4*d^3*e^4*x*(b^2 - 4*a*c)^(1/2)
- 3*a^2*c^3*d*e^6*x*(b^2 - 4*a*c)^(1/2) - 2*a^4*c*d^5*e^2*x*(b^2 - 4*a*c)^(
1/2) - 5*a^2*b^3*c*d^3*e^4*x + 5*a^3*b*c^2*d^3*e^4*x + 9*a*b^2*c^2*d^2*e^5*
(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c^2*d^3*e^4*(b^2 - 4*a*c)^(1/2) + a^3*b^2*d^5
*e^2*x*(b^2 - 4*a*c)^(1/2) + a^3*c^2*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) + 12*a^2
*b^2*c^2*d^2*e^5*x - 6*a*b*c^3*d*e^6*(b^2 - 4*a*c)^(1/2) - a*b*c^3*e^7*x*(b
^2 - 4*a*c)^(1/2) - 2*a^4*b*d^6*e*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^3*c^2*d*e^6
*x - 3*a*b^4*c*d^2*e^5*x - 9*a^2*b*c^3*d*e^6*x + 4*a^4*b*c*d^5*e^2*x + 3*a*
b^2*c^2*d*e^6*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^3*c*d^2*e^5*x*(b^2 - 4*a*c)^(1/
2) + 6*a^2*b*c^2*d^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*b^2*c*d^3*e^4*x*(b^2
 - 4*a*c)^(1/2))*(4*a^3*c^3*d - b^5*d*(b^2 - 4*a*c)^(1/2) - b^6*d + b^5*c*e
 - 13*a^2*b^2*c^2*d + 7*a*b^4*c*d + b^4*c*e*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c
^2*e + 8*a^2*b*c^3*e + 2*a^2*c^3*e*(b^2 - 4*a*c)^(1/2) - 5*a^2*b*c^2*d*(b^2
 - 4*a*c)^(1/2) - 4*a*b^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 5*a*b^3*c*d*(b^2 - 4*
a*c)^(1/2)))/(2*(4*a^6*c*d^2 - a^5*b^2*d^2 + 4*a^5*c^2*e^2 - a^4*b^2*c*e^2
+ a^4*b^3*d*e - 4*a^5*b*c*d*e)) + x^3/(3*a*e) - (x^2*(a*d + b*e))/(2*a^2*e^
2)
```

3.62 $\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 218

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae} - \frac{(b^4d-4ab^2cd+2a^2c^2d-b^3ce+3abc^2e)\,\text{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} + \frac{d^4\log(d+ex)}{e^3\,(ad^2-e(bd-ce))} - \frac{(b^3d-2abcd-b^2ce+ac^2e)\log\left(c+bx+ax^2\right)}{2a^3\,(ad^2-e(bd-ce))}$$

```
[Out] -(a*d+b*e)*x/a^2/e^2+1/2*x^2/a/e+d^4*ln(e*x+d)/e^3/(a*d^2-e*(b*d-c*e))-1/2*
(-2*a*b*c*d+a*c^2*e+b^3*d-b^2*c*e)*ln(a*x^2+b*x+c)/a^3/(a*d^2-e*(b*d-c*e))-
(2*a^2*c^2*d-4*a*b^2*c*d+3*a*b*c^2*e+b^4*d-b^3*c*e)*arctanh((2*a*x+b)/(-4*a
*c+b^2)^(1/2))/a^3/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used

$= \{1583, 1642, 648, 632, 212, 642\}$

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = -\frac{\left(-2abcd+ac^2e+b^3d-b^2ce\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)} - \frac{x(ad+be)}{a^2e^2} - \frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(2a^2c^2d-4ab^2cd+3abc^2e+b^4d-b^3ce\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)} + \frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)} + \frac{x^2}{2ae}$$

```
[In] Int[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) - ((b^4*d - 4*a*b^2*c*d + 2*a^2*
c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^3
*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + (d^4*Log[d + e*x])/(e^3*(a*d^
2 - e*(b*d - c*e))) - ((b^3*d - 2*a*b*c*d - b^2*c*e + a*c^2*e)*Log[c + b*x
+ a*x^2])/(2*a^3*(a*d^2 - e*(b*d - c*e)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^4}{(d+ex)\left(c+bx+ax^2\right)}\,dx\\
&= \int \left(\frac{-ad-be}{a^2e^2}+\frac{x}{ae}+\frac{d^4}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{-c(b^2d-acd-bce)-(b^3d-2abcd-b^2ce+ac^2e)\,x}{a^2\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx\\
&= -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{-c(b^2d-acd-bce)-(b^3d-2abcd-b^2ce+ac^2e)x}{c+bx+ax^2}\,dx}{a^2\left(ad^2-e(bd-ce)\right)}\\
&= -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)} - \frac{(b^3d-2abcd-b^2ce+ac^2e)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2a^3\left(ad^2-e(bd-ce)\right)} + \frac{(b^4d-4ab^2cd+2a^2c^2d-b^3ce+3abc^2e)\int \frac{1}{c+bx+ax^2}\,dx}{2a^3\left(ad^2-e(bd-ce)\right)}\\
&= -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}-\frac{(b^3d-2abcd-b^2ce+ac^2e)\log\left(c+bx+ax^2\right)}{2a^3\left(ad^2-e(bd-ce)\right)} - \frac{(b^4d-4ab^2cd+2a^2c^2d-b^3ce+3abc^2e)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2ax\right)}{a^3\left(ad^2-e(bd-ce)\right)}\\
&= -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}-\frac{(b^4d-4ab^2cd+2a^2c^2d-b^3ce+3abc^2e)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)} + \frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}-\frac{(b^3d-2abcd-b^2ce+ac^2e)\log\left(c+bx+ax^2\right)}{2a^3\left(ad^2-e(bd-ce)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae} + \frac{(b^4d-4ab^2cd+2a^2c^2d-b^3ce+3abc^2e)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a^3\sqrt{-b^2+4ac}\,(ad^2+e(-bd+ce))} + \frac{d^4\log(d+ex)}{e^3(ad^2+e(-bd+ce))} + \frac{(-b^3d+2abcd+b^2ce-ac^2e)\log(c+x(b+ax))}{2a^3(ad^2+e(-bd+ce))}$$

```
[In] Integrate[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) + ((b^4*d - 4*a*b^2*c*d + 2*a^2*
c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^3
*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))) + (d^4*Log[d + e*x])/(e^3*(
a*d^2 + e*(-(b*d) + c*e))) + ((-(b^3*d) + 2*a*b*c*d + b^2*c*e - a*c^2*e)*Lo
g[c + x*(b + a*x)])/(2*a^3*(a*d^2 + e*(-(b*d) + c*e)))
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.95

method	result
default	$-\frac{-\frac{1}{2}ae\,x^2+adx+bex}{e^2a^2}+\frac{\frac{\left(2abcd-a\,c^2e-b^3d+b^2ce\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(a\,c^2d-b^2cd+b\,c^2e-\frac{\left(2abcd-a\,c^2e-b^3d+b^2ce\right)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right.}{\sqrt{4ac-b^2}}}{(a\,d^2-bde+c\,e^2)a^2}$
risch	$\frac{x^2}{2ae}-\frac{dx}{e^2a}-\frac{bx}{e\,a^2}+\frac{d^4\ln(ex+d)}{e^3(a\,d^2-bde+c\,e^2)}+\frac{\sum_{\_R=\text{RootOf}\left(\left(4a^3c\,d^2-b^2d^2a^2-4a^2bcde+4a^2c^2e^2+a\,b^3de-a\,b^2c\,e^2\right)\_Z^2+\left(-8a^2b\,c^2d\,e^2+\right.\right.}}{}$

```
[In] int(x^2/(a+c/x^2+b/x)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^2/a^2*(-1/2*a*e*x^2+a*d*x+b*e*x)+1/(a*d^2-b*d*e+c*e^2)/a^2*(1/2*(2*a*b
*c*d-a*c^2*e-b^3*d+b^2*c*e)/a*ln(a*x^2+b*x+c)+2*(a*c^2*d-b^2*c*d+b*c^2*e-1/
2*(2*a*b*c*d-a*c^2*e-b^3*d+b^2*c*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)
/(4*a*c-b^2)^(1/2)))+1/e^3*d^4/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 9.55 (sec) , antiderivative size = 798, normalized size of antiderivative = 3.66

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$$
$$= \left[\frac{2\,(a^3b^2-4\,a^4c)d^4\log\left(ex+d\right)+((a^3b^2-4\,a^4c)d^2e^2-(a^2b^3-4\,a^3bc)de^3+(a^2b^2c-4\,a^3c^2)e^4)x^2+((b^4-\right.$$

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^3*b^2 - 4*a^4*c)*d^4*log(e*x + d) + ((a^3*b^2 - 4*a^4*c)*d^2*e^2
 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^4 -
 4*a*b^2*c + 2*a^2*c^2)*d*e^3 - (b^3*c - 3*a*b*c^2)*e^4)*sqrt(b^2 - 4*a*c)*
log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*
x^2 + b*x + c)) - 2*((a^3*b^2 - 4*a^4*c)*d^3*e - (a*b^4 - 5*a^2*b^2*c + 4*a
^3*c^2)*d*e^3 + (a*b^3*c - 4*a^2*b*c^2)*e^4)*x - ((b^5 - 6*a*b^3*c + 8*a^2*
b*c^2)*d*e^3 - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^4)*log(a*x^2 + b*x + c))
/((a^4*b^2 - 4*a^5*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c -
4*a^4*c^2)*e^5), 1/2*(2*(a^3*b^2 - 4*a^4*c)*d^4*log(e*x + d) + ((a^3*b^2 -
4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^
4)*x^2 - 2*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d*e^3 - (b^3*c - 3*a*b*c^2)*e^4)*
sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) -
2*((a^3*b^2 - 4*a^4*c)*d^3*e - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d*e^3 + (a
*b^3*c - 4*a^2*b*c^2)*e^4)*x - ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d*e^3 - (b^
4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^4)*log(a*x^2 + b*x + c))/((a^4*b^2 - 4*a^5
*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c - 4*a^4*c^2)*e^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.01

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \frac{d^4\log\left(|ex+d|\right)}{ad^2e^3-bde^4+ce^5} - \frac{\left(b^3d-2\,abcd-b^2ce+ac^2e\right)\log\left(ax^2+bx+c\right)}{2\left(a^4d^2-a^3bde+a^3ce^2\right)} + \frac{\left(b^4d-4\,ab^2cd+2\,a^2c^2d-b^3ce+3\,abc^2e\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^4d^2-a^3bde+a^3ce^2\right)\sqrt{-b^2+4\,ac}} + \frac{aex^2-2\,adx-2\,bex}{2\,a^2e^2}$$

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] d^4*log(abs(e*x + d))/(a*d^2*e^3 - b*d*e^4 + c*e^5) - 1/2*(b^3*d - 2*a*b*c*
d - b^2*c*e + a*c^2*e)*log(a*x^2 + b*x + c)/(a^4*d^2 - a^3*b*d*e + a^3*c*e^
2) + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - b^3*c*e + 3*a*b*c^2*e)*arctan((2*
a*x + b)/sqrt(-b^2 + 4*a*c))/((a^4*d^2 - a^3*b*d*e + a^3*c*e^2)*sqrt(-b^2 +
 4*a*c)) + 1/2*(a*e*x^2 - 2*a*d*x - 2*b*e*x)/(a^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 2051, normalized size of antiderivative = 9.41

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Too large to display}$$

```
[In] int(x^2/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (d^4*log(d + e*x))/(c*e^5 + a*d^2*e^3 - b*d*e^4) - (log(4*a^4*c*d^6 - 2*a*c
^4*e^6 - a^3*b^2*d^6 + b^2*c^3*e^6 - b^5*d^3*e^3 - 3*b^3*c^2*d*e^5 + 3*b^4*
c*d^2*e^4 + b^4*d^3*e^3*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^3*d^2*e^4 - 4*a^3*c^2
*d^4*e^2 + a^3*b*d^6*(b^2 - 4*a*c)^(1/2) - b*c^3*e^6*(b^2 - 4*a*c)^(1/2) +
2*a^4*d^6*x*(b^2 - 4*a*c)^(1/2) + 9*a*b*c^3*d*e^5 + a^2*c^2*d^3*e^3*(b^2 -
4*a*c)^(1/2) + a*b*c^3*e^6*x + 8*a^4*c*d^5*e*x - 3*a*c^3*d*e^5*(b^2 - 4*a*c
)^(1/2) - 4*a^3*c*d^5*e*(b^2 - 4*a*c)^(1/2) - a*c^3*e^6*x*(b^2 - 4*a*c)^(1/
2) + 5*a*b^3*c*d^3*e^3 - a*b^4*d^3*e^3*x - 2*a^3*b^2*d^5*e*x + 6*a^2*c^3*d*
e^5*x + 3*b^2*c^2*d*e^5*(b^2 - 4*a*c)^(1/2) - 3*b^3*c*d^2*e^4*(b^2 - 4*a*c)
^(1/2) - 12*a*b^2*c^2*d^2*e^4 - 5*a^2*b*c^2*d^3*e^3 + a^2*b^2*c*d^4*e^2 + a
^2*b^3*d^4*e^2*x - 2*a^3*c^2*d^3*e^3*x + 6*a*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1
/2) - 3*a*b^2*c*d^3*e^3*(b^2 - 4*a*c)^(1/2) + a^2*b*c*d^4*e^2*(b^2 - 4*a*c)
^(1/2) + a*b^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*c*d^4*e^2*x*(b^2 - 4*a
*c)^(1/2) - 9*a^2*b*c^2*d^2*e^4*x + 4*a^2*b^2*c*d^3*e^3*x + a^2*b^2*d^4*e^2
*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*b*
d^5*e*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c^2*d*e^5*x + 3*a*b^3*c*d^2*e^4*x - 4
*a^3*b*c*d^4*e^2*x + 3*a*b*c^2*d*e^5*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^2*
e^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d
*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^2*c^3*e + b^4*c*e + 6*a*b^3*c*d - b^3*c*
e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a*b^2*c^2*e + 2*a^2*c^2*d*(b^2 -
4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*e*(b^2 - 4*a*c)^
(1/2)))/(2*(4*a^5*c*d^2 - a^4*b^2*d^2 + 4*a^4*c^2*e^2 - a^3*b^2*c*e^2 + a^3
*b^3*d*e - 4*a^4*b*c*d*e)) + (log(2*a*c^4*e^6 - 4*a^4*c*d^6 + a^3*b^2*d^6 -
 b^2*c^3*e^6 + b^5*d^3*e^3 + 3*b^3*c^2*d*e^5 - 3*b^4*c*d^2*e^4 + b^4*d^3*e^
3*(b^2 - 4*a*c)^(1/2) - 6*a^2*c^3*d^2*e^4 + 4*a^3*c^2*d^4*e^2 + a^3*b*d^6*(
b^2 - 4*a*c)^(1/2) - b*c^3*e^6*(b^2 - 4*a*c)^(1/2) + 2*a^4*d^6*x*(b^2 - 4*a
*c)^(1/2) - 9*a*b*c^3*d*e^5 + a^2*c^2*d^3*e^3*(b^2 - 4*a*c)^(1/2) - a*b*c^3
*e^6*x - 8*a^4*c*d^5*e*x - 3*a*c^3*d*e^5*(b^2 - 4*a*c)^(1/2) - 4*a^3*c*d^5*
e*(b^2 - 4*a*c)^(1/2) - a*c^3*e^6*x*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*d^3*e^3
 + a*b^4*d^3*e^3*x + 2*a^3*b^2*d^5*e*x - 6*a^2*c^3*d*e^5*x + 3*b^2*c^2*d*e^
5*(b^2 - 4*a*c)^(1/2) - 3*b^3*c*d^2*e^4*(b^2 - 4*a*c)^(1/2) + 12*a*b^2*c^2*
d^2*e^4 + 5*a^2*b*c^2*d^3*e^3 - a^2*b^2*c*d^4*e^2 - a^2*b^3*d^4*e^2*x + 2*a
^3*c^2*d^3*e^3*x + 6*a*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^3*e^
3*(b^2 - 4*a*c)^(1/2) + a^2*b*c*d^4*e^2*(b^2 - 4*a*c)^(1/2) + a*b^3*d^3*e^3
*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*c*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) + 9*a^2*b*c^
2*d^2*e^4*x - 4*a^2*b^2*c*d^3*e^3*x + a^2*b^2*d^4*e^2*x*(b^2 - 4*a*c)^(1/2)
 + 3*a^2*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*b*d^5*e*x*(b^2 - 4*a*c)^
(1/2) + 3*a*b^2*c^2*d*e^5*x - 3*a*b^3*c*d^2*e^4*x + 4*a^3*b*c*d^4*e^2*x + 3
*a*b*c^2*d*e^5*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^2*e^4*x*(b^2 - 4*a*c)^(1
/2) - 2*a^2*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^5*d + b^4*d*(b^2 - 4*a*c)
^(1/2) - 4*a^2*c^3*e - b^4*c*e - 6*a*b^3*c*d - b^3*c*e*(b^2 - 4*a*c)^(1/2)
+ 8*a^2*b*c^2*d + 5*a*b^2*c^2*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2
*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^5*c*d^
2 - a^4*b^2*d^2 + 4*a^4*c^2*e^2 - a^3*b^2*c*e^2 + a^3*b^3*d*e - 4*a^4*b*c*d
*e)) + x^2/(2*a*e) - (x*(a*d + b*e))/(a^2*e^2)
```

3.63 $\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 176

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx=\frac{x}{ae}+\frac{(b^3d-3abcd-b^2ce+2ac^2e)\,\text{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}-\frac{d^3\log(d+ex)}{e^2\,(ad^2-e(bd-ce))}+\frac{(b^2d-acd-bce)\log\left(c+bx+ax^2\right)}{2a^2\,(ad^2-e(bd-ce))}$$

```
[Out] x/a/e-d^3*ln(e*x+d)/e^2/(a*d^2-e*(b*d-c*e))+1/2*(-a*c*d+b^2*d-b*c*e)*ln(a*x
^2+b*x+c)/a^2/(a*d^2-e*(b*d-c*e))+(-3*a*b*c*d+2*a*c^2*e+b^3*d-b^2*c*e)*arct
anh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/a^2/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1583, 1642, 648, 632, 212, 642}

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx=\frac{\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)(-3abcd+2ac^2e+b^3d-b^2ce)}{a^2\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}+\frac{(-acd+b^2d-bce)\log\left(ax^2+bx+c\right)}{2a^2\,(ad^2-e(bd-ce))}-\frac{d^3\log(d+ex)}{e^2\,(ad^2-e(bd-ce))}+\frac{x}{ae}$$

```
[In] Int[x/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTanh[(b + 2*a*x)/Sq
rt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d^3*Lo
g[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))) + ((b^2*d - a*c*d - b*c*e)*Log[c
+ b*x + a*x^2])/(2*a^2*(a*d^2 - e*(b*d - c*e)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \frac{x^3}{(d+ex)\,(c+bx+ax^2)}\,dx$$

$$= \int \left(\frac{1}{ae}+\frac{d^3}{e\left(-ad^2+e(bd-ce)\right)(d+ex)}+\frac{c(bd-ce)+\left(b^2d-acd-bce\right)x}{a\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx$$

$$= \frac{x}{ae}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{c(bd-ce)+\left(b^2d-acd-bce\right)x}{c+bx+ax^2}\,dx}{a\left(ad^2-bde+ce^2\right)}$$

$$= \frac{x}{ae}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2d-acd-bce\right)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2a^2\left(ad^2-e(bd-ce)\right)} - \frac{\left(b^3d-3abcd-b^2ce+2ac^2e\right)\int \frac{1}{c+bx+ax^2}\,dx}{2a^2\left(ad^2-e(bd-ce)\right)}$$

$$= \frac{x}{ae}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2d-acd-bce\right)\log\left(c+bx+ax^2\right)}{2a^2\left(ad^2-e(bd-ce)\right)} + \frac{\left(b^3d-3abcd-b^2ce+2ac^2e\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2ax\right)}{a^2\left(ad^2-e(bd-ce)\right)}$$

$$= \frac{x}{ae}+\frac{\left(b^3d-3abcd-b^2ce+2ac^2e\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)} - \frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2d-acd-bce\right)\log\left(c+bx+ax^2\right)}{2a^2\left(ad^2-e(bd-ce)\right)}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.01

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \frac{x}{ae}+\frac{\left(b^3d-3abcd-b^2ce+2ac^2e\right)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a^2\sqrt{-b^2+4ac}\left(-ad^2+bde-ce^2\right)} - \frac{d^3\log(d+ex)}{e^2\left(ad^2-bde+ce^2\right)}+\frac{\left(b^2d-acd-bce\right)\log\left(c+bx+ax^2\right)}{2a^2\left(ad^2-bde+ce^2\right)}$$

```
[In] Integrate[x/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTan[(b + 2*a*x)/Sqr
t[-b^2 + 4*a*c]])/(a^2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + b*d*e - c*e^2)) - (d^
3*Log[d + e*x])/(e^2*(a*d^2 - b*d*e + c*e^2)) + ((b^2*d - a*c*d - b*c*e)*Lo
g[c + b*x + a*x^2])/(2*a^2*(a*d^2 - b*d*e + c*e^2))
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{x}{ae}+\frac{\frac{\left(-acd+b^{2}d-ebc\right)\ln\left(a\,x^{2}+bx+c\right)}{2a}+\frac{2\left(bcd-e\,c^{2}-\frac{\left(-acd+b^{2}d-ebc\right)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{\left(a\,d^{2}-bde+c\,e^{2}\right)a}-\frac{d^{3}\ln\left(ex+d\right)}{e^{2}\left(a\,d^{2}-bde+c\,e^{2}\right)}$	164
risch	Expression too large to display	15838

```
[In] int(x/(a+c/x^2+b/x)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] x/a/e+1/(a*d^2-b*d*e+c*e^2)/a*(1/2*(-a*c*d+b^2*d-b*c*e)/a*ln(a*x^2+b*x+c)+2
*(b*c*d-e*c^2-1/2*(-a*c*d+b^2*d-b*c*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x
+b)/(4*a*c-b^2)^(1/2)))-1/e^2*d^3/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 3.16 (sec) , antiderivative size = 596, normalized size of antiderivative = 3.39

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$$
$$= \Bigg[-\frac{2\,(a^2b^2-4\,a^3c)d^3\log\left(ex+d\right)-((b^3-3\,abc)de^2-(b^2c-2\,ac^2)e^3)\sqrt{b^2-4\,ac}\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac+\sqrt{\cdots}}{ax^2+bx+c\cdots}\right.\cdots}{2\,((a^3b^2-\cdots}$$
$$-\frac{2\,(a^2b^2-4\,a^3c)d^3\log\left(ex+d\right)-2\,((b^3-3\,abc)de^2-(b^2c-2\,ac^2)e^3)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,ax\cdots}{b^2-4\,ac}\right.\cdots}{2\,((a^3b^2-4\,a^4c)\cdots}$$

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2*b^2 - 4*a^3*c)*d^3*log(e*x + d) - ((b^3 - 3*a*b*c)*d*e^2 - (b
^2*c - 2*a*c^2)*e^3)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a
*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) - 2*((a^2*b^2 - 4*a^
3*c)*d^2*e - (a*b^3 - 4*a^2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)*x - ((b
^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*log(a*x^2 + b*
x + c))/((a^3*b^2 - 4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b
^2*c - 4*a^3*c^2)*e^4), -1/2*(2*(a^2*b^2 - 4*a^3*c)*d^3*log(e*x + d) - 2*((
b^3 - 3*a*b*c)*d*e^2 - (b^2*c - 2*a*c^2)*e^3)*sqrt(-b^2 + 4*a*c)*arctan(-sq
rt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - 2*((a^2*b^2 - 4*a^3*c)*d^2*e
- (a*b^3 - 4*a^2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)*x - ((b^4 - 5*a*b^
2*c + 4*a^2*c^2)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*log(a*x^2 + b*x + c))/((a
^3*b^2 - 4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^
3*c^2)*e^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate(x/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.05

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = -\frac{d^3\log\left(|ex+d|\right)}{ad^2e^2-bde^3+ce^4}+\frac{(b^2d-acd-bce)\log\left(ax^2+bx+c\right)}{2\left(a^3d^2-a^2bde+a^2ce^2\right)} - \frac{(b^3d-3\,abcd-b^2ce+2\,ac^2e)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3d^2-a^2bde+a^2ce^2)\sqrt{-b^2+4\,ac}}+\frac{x}{ae}$$

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] -d^3*log(abs(e*x + d))/(a*d^2*e^2 - b*d*e^3 + c*e^4) + 1/2*(b^2*d - a*c*d -
 b*c*e)*log(a*x^2 + b*x + c)/(a^3*d^2 - a^2*b*d*e + a^2*c*e^2) - (b^3*d - 3
*a*b*c*d - b^2*c*e + 2*a*c^2*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^
3*d^2 - a^2*b*d*e + a^2*c*e^2)*sqrt(-b^2 + 4*a*c)) + x/(a*e)
```

Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 1367, normalized size of antiderivative = 7.77

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \frac{x}{a\,e}$$
$$-\frac{\ln\left(c^3\,e^5\,\sqrt{b^2-4\,a\,c}-b\,c^3\,e^5-4\,a^3\,c\,d^5+a^2\,b^2\,d^5+b^4\,d^3\,e^2+3\,b^2\,c^2\,d\,e^4-3\,b^3\,c\,d^2\,e^3-b^3\,d^3\,e^2\,\sqrt{b^2-4\,a}\right.}{}$$
$$-\frac{\ln\left(a^2\,b^2\,d^5-b\,c^3\,e^5-c^3\,e^5\,\sqrt{b^2-4\,a\,c}-4\,a^3\,c\,d^5+b^4\,d^3\,e^2+3\,b^2\,c^2\,d\,e^4-3\,b^3\,c\,d^2\,e^3+b^3\,d^3\,e^2\,\sqrt{b^2-4\,a}\right.}{}$$
$$-\frac{d^3\,\ln\left(d+e\,x\right)}{a\,d^2\,e^2-b\,d\,e^3+c\,e^4}$$

```
[In] int(x/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] x/(a*e) - (log(c^3*e^5*(b^2 - 4*a*c)^(1/2) - b*c^3*e^5 - 4*a^3*c*d^5 + a^2*
b^2*d^5 + b^4*d^3*e^2 + 3*b^2*c^2*d*e^4 - 3*b^3*c*d^2*e^3 - b^3*d^3*e^2*(b^
2 - 4*a*c)^(1/2) + 6*a^2*c^2*d^3*e^2 - 6*a*c^3*d*e^4 - 2*a*c^3*e^5*x - a^2*
b*d^5*(b^2 - 4*a*c)^(1/2) - 2*a^3*d^5*x*(b^2 - 4*a*c)^(1/2) - 8*a^3*c*d^4*e
*x + 4*a^2*c*d^4*e*(b^2 - 4*a*c)^(1/2) - 3*b*c^2*d*e^4*(b^2 - 4*a*c)^(1/2)
+ 9*a*b*c^2*d^2*e^3 - 5*a*b^2*c*d^3*e^2 + 2*a^2*b^2*d^4*e*x - 3*a*c^2*d^2*e
^3*(b^2 - 4*a*c)^(1/2) + 3*b^2*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*d^
2*e^3*x - 2*a*b^2*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*c*d^3*e^2*x*(b^2 -
4*a*c)^(1/2) + 3*a*b*c^2*d*e^4*x + a*b*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 2*a^
2*b*d^4*e*x*(b^2 - 4*a*c)^(1/2) - 3*a*c^2*d*e^4*x*(b^2 - 4*a*c)^(1/2) - 3*a
*b^2*c*d^2*e^3*x + a^2*b*c*d^3*e^2*x + 3*a*b*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2
))*(b^4*d - b^3*d*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d - b^3*c*e - 5*a*b^2*c*d
 + 4*a*b*c^2*e - 2*a*c^2*e*(b^2 - 4*a*c)^(1/2) + b^2*c*e*(b^2 - 4*a*c)^(1/2
) + 3*a*b*c*d*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^4*c*d^2 - a^3*b^2*d^2 + 4*a^3*c
^2*e^2 - a^2*b^2*c*e^2 + a^2*b^3*d*e - 4*a^3*b*c*d*e)) - (log(a^2*b^2*d^5 -
 b*c^3*e^5 - c^3*e^5*(b^2 - 4*a*c)^(1/2) - 4*a^3*c*d^5 + b^4*d^3*e^2 + 3*b^
2*c^2*d*e^4 - 3*b^3*c*d^2*e^3 + b^3*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2
*d^3*e^2 - 6*a*c^3*d*e^4 - 2*a*c^3*e^5*x + a^2*b*d^5*(b^2 - 4*a*c)^(1/2) +
2*a^3*d^5*x*(b^2 - 4*a*c)^(1/2) - 8*a^3*c*d^4*e*x - 4*a^2*c*d^4*e*(b^2 - 4*
a*c)^(1/2) + 3*b*c^2*d*e^4*(b^2 - 4*a*c)^(1/2) + 9*a*b*c^2*d^2*e^3 - 5*a*b^
2*c*d^3*e^2 + 2*a^2*b^2*d^4*e*x + 3*a*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2) - 3*b
^2*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*d^2*e^3*x + 2*a*b^2*d^3*e^2*x*
(b^2 - 4*a*c)^(1/2) - 3*a^2*c*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*e
^4*x - a*b*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*d^4*e*x*(b^2 - 4*a*c)^(1
/2) + 3*a*c^2*d*e^4*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^2*e^3*x + a^2*b*c*d
^3*e^2*x - 3*a*b*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d + b^3*d*(b^2 - 4*a
*c)^(1/2) + 4*a^2*c^2*d - b^3*c*e - 5*a*b^2*c*d + 4*a*b*c^2*e + 2*a*c^2*e*(
b^2 - 4*a*c)^(1/2) - b^2*c*e*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*d*(b^2 - 4*a*c)^
(1/2)))/(2*(4*a^4*c*d^2 - a^3*b^2*d^2 + 4*a^3*c^2*e^2 - a^2*b^2*c*e^2 + a^2
```

```
*b^3*d*e - 4*a^3*b*c*d*e)) - (d^3*log(d + e*x))/(c*e^4 + a*d^2*e^2 - b*d*e^
3)
```

3.64 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = -\frac{(b^2d-2acd-bce)\,\text{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} + \frac{d^2\log(d+ex)}{e\,(ad^2-bde+ce^2)} - \frac{(bd-ce)\log\left(c+bx+ax^2\right)}{2a\,(ad^2-e(bd-ce))}$$

```
[Out] d^2*ln(e*x+d)/e/(a*d^2-b*d*e+c*e^2)-1/2*(b*d-c*e)*ln(a*x^2+b*x+c)/a/(a*d^2-
e*(b*d-c*e))-(-2*a*c*d+b^2*d-b*c*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/a
/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1459, 1642, 648, 632, 212, 642}

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = -\frac{\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)(-2acd+b^2d-bce)}{a\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} + \frac{d^2\log(d+ex)}{e\,(ad^2-bde+ce^2)} - \frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2a\,(ad^2-e(bd-ce))}$$

```
[In] Int[1/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((b^2*d - 2*a*c*d - b*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqr
t[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e)))) + (d^2*Log[d + e*x])/(e*(a*d^2 - b
```

```
*d*e + c*e^2)) - ((b*d - c*e)*Log[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d -
c*e)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1459

```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x
_)^(n_.))^(q_.), x_Symbol] :> Int[((d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p)
/x^(2*n*p), x] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[mn, -n] && EqQ[mn2
, 2*mn] && IntegerQ[p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{(d+ex)\,(c+bx+ax^2)}\,dx$$

$$= \int \left(\frac{d^2}{(ad^2-e(bd-ce))\,(d+ex)} + \frac{-cd-(bd-ce)x}{(ad^2-e(bd-ce))\,(c+bx+ax^2)}\right)dx$$

$$= \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}+\frac{\int \frac{-cd-(bd-ce)x}{c+bx+ax^2}\,dx}{ad^2-e(bd-ce)}$$

$$= \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2a\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2d-2acd-bce\right)\int \frac{1}{c+bx+ax^2}\,dx}{2a\left(ad^2-e(bd-ce)\right)}$$

$$= \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\log\left(c+bx+ax^2\right)}{2a\left(ad^2-e(bd-ce)\right)} - \frac{\left(b^2d-2acd-bce\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2ax\right)}{a\left(ad^2-e(bd-ce)\right)}$$

$$= -\frac{\left(b^2d-2acd-bce\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\log\left(c+bx+ax^2\right)}{2a\left(ad^2-e(bd-ce)\right)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = -\frac{2e\left(-b^2d+2acd+bce\right)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\left(-2ad^2\log(d+ex)+e(bd-ce)\log(c+x(b+\right.}{2a\sqrt{-b^2+4ac}\,e\left(ad^2+e(-bd+ce)\right)}$$

```
[In] Integrate[1/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -1/2*(2*e*(-(b^2*d) + 2*a*c*d + b*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c
]] + Sqrt[-b^2 + 4*a*c]*(-2*a*d^2*Log[d + e*x] + e*(b*d - c*e)*Log[c + x*(b
 + a*x)]))/(a*Sqrt[-b^2 + 4*a*c]*e*(a*d^2 + e*(-(b*d) + c*e)))
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\frac{(-bd+ec)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(-cd-\frac{(-bd+ec)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a\,d^2-bde+c\,e^2}+\frac{d^2\ln(ex+d)}{e\left(a\,d^2-bde+c\,e^2\right)}$	130
risch	Expression too large to display	7752

```
[In] int(1/(a+c/x^2+b/x)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*d^2-b*d*e+c*e^2)*(1/2*(-b*d+c*e)/a*ln(a*x^2+b*x+c)+2*(-c*d-1/2*(-b*d+c
*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))+d^2*ln(e*x+
d)/e/(a*d^2-b*d*e+c*e^2)
```

Fricas [A] (verification not implemented)

none

Time = 1.07 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.72

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$$
$$=\left[\frac{2\,(ab^2-4\,a^2c)d^2\log(ex+d)+(bce^2-(b^2-2\,ac)de)\sqrt{b^2-4\,ac}\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,ax+b)}{ax^2+bx+c}\right)}{2\,((a^2b^2-4\,a^3c)d^2e-(ab^3-4\,a^2bc)de^2+(ab^2c-4\,a^2c}\right.$$

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*b^2 - 4*a^2*c)*d^2*log(e*x + d) + (b*c*e^2 - (b^2 - 2*a*c)*d*e)*
sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c
)*(2*a*x + b))/(a*x^2 + b*x + c)) - ((b^3 - 4*a*b*c)*d*e - (b^2*c - 4*a*c^2
)*e^2)*log(a*x^2 + b*x + c))/((a^2*b^2 - 4*a^3*c)*d^2*e - (a*b^3 - 4*a^2*b*
c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), 1/2*(2*(a*b^2 - 4*a^2*c)*d^2*log(e*x
 + d) + 2*(b*c*e^2 - (b^2 - 2*a*c)*d*e)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^
2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*c)*d*e - (b^2*c - 4*a
*c^2)*e^2)*log(a*x^2 + b*x + c))/((a^2*b^2 - 4*a^3*c)*d^2*e - (a*b^3 - 4*a^
2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \frac{d^2\log\left(|ex+d|\right)}{ad^2e-bde^2+ce^3} - \frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2\left(a^2d^2-abde+ace^2\right)} + \frac{(b^2d-2\,acd-bce)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^2d^2-abde+ace^2)\sqrt{-b^2+4\,ac}}$$

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] d^2*log(abs(e*x + d))/(a*d^2*e - b*d*e^2 + c*e^3) - 1/2*(b*d - c*e)*log(a*x
^2 + b*x + c)/(a^2*d^2 - a*b*d*e + a*c*e^2) + (b^2*d - 2*a*c*d - b*c*e)*arc
tan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*d^2 - a*b*d*e + a*c*e^2)*sqrt(-b^
2 + 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 966, normalized size of antiderivative = 6.48

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \frac{d^2\,\ln\left(d+e\,x\right)}{a\,d^2\,e-b\,d\,e^2+c\,e^3} - \frac{\ln\left(a\,b^2\,d^4-2\,c^3\,e^4-4\,a^2\,c\,d^4+b^3\,d^3\,e+c^2\,e^4\,x\,\sqrt{b^2-4\,a\,c}+10\,a\,c^2\,d^2\,e^2-4\,b^2\,c\,d^2\,e^2-b^3\,d^2\,e^2\,x+a\,b\ldots\right.}{\ldots} + \frac{\ln\left(2\,c^3\,e^4-a\,b^2\,d^4+4\,a^2\,c\,d^4-b^3\,d^3\,e+c^2\,e^4\,x\,\sqrt{b^2-4\,a\,c}-10\,a\,c^2\,d^2\,e^2+4\,b^2\,c\,d^2\,e^2+b^3\,d^2\,e^2\,x+a\,b\ldots\right.}{\ldots}$$

```
[In] int(1/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (d^2*log(d + e*x))/(c*e^3 + a*d^2*e - b*d*e^2) - (log(a*b^2*d^4 - 2*c^3*e^4
 - 4*a^2*c*d^4 + b^3*d^3*e + c^2*e^4*x*(b^2 - 4*a*c)^(1/2) + 10*a*c^2*d^2*e
^2 - 4*b^2*c*d^2*e^2 - b^3*d^2*e^2*x + a*b*d^4*(b^2 - 4*a*c)^(1/2) + 3*b*c^
2*d*e^3 - b*c^2*e^4*x + b^2*d^3*e*(b^2 - 4*a*c)^(1/2) + 3*c^2*d*e^3*(b^2 -
4*a*c)^(1/2) + 2*a^2*d^4*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^2*d^3*e*x + 6*a*c^2*
d*e^3*x - 10*a^2*c*d^3*e*x - 2*b*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*d^
3*e + b^2*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) - 5*a*c*d^3*e*(b^2 - 4*a*c)^(1/2) -
 a*b*d^3*e*x*(b^2 - 4*a*c)^(1/2) + a*b*c*d^2*e^2*x - 5*a*c*d^2*e^2*x*(b^2 -
 4*a*c)^(1/2))*(e*((b^2*c)/2 - 2*a*c^2 + (b*c*(b^2 - 4*a*c)^(1/2))/2) - (b^
3*d)/2 - (b^2*d*(b^2 - 4*a*c)^(1/2))/2 + a*c*d*(b^2 - 4*a*c)^(1/2) + 2*a*b*
c*d))/(4*a^3*c*d^2 - a^2*b^2*d^2 + 4*a^2*c^2*e^2 + a*b^3*d*e - a*b^2*c*e^2
- 4*a^2*b*c*d*e) + (log(2*c^3*e^4 - a*b^2*d^4 + 4*a^2*c*d^4 - b^3*d^3*e + c
```

```
^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 10*a*c^2*d^2*e^2 + 4*b^2*c*d^2*e^2 + b^3*d^2
*e^2*x + a*b*d^4*(b^2 - 4*a*c)^(1/2) - 3*b*c^2*d*e^3 + b*c^2*e^4*x + b^2*d^
3*e*(b^2 - 4*a*c)^(1/2) + 3*c^2*d*e^3*(b^2 - 4*a*c)^(1/2) + 2*a^2*d^4*x*(b^
2 - 4*a*c)^(1/2) - 3*a*b^2*d^3*e*x - 6*a*c^2*d*e^3*x + 10*a^2*c*d^3*e*x - 2
*b*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d^3*e + b^2*d^2*e^2*x*(b^2 - 4*a
*c)^(1/2) - 5*a*c*d^3*e*(b^2 - 4*a*c)^(1/2) - a*b*d^3*e*x*(b^2 - 4*a*c)^(1/
2) - a*b*c*d^2*e^2*x - 5*a*c*d^2*e^2*x*(b^2 - 4*a*c)^(1/2))*((b^3*d)/2 + e*
(2*a*c^2 - (b^2*c)/2 + (b*c*(b^2 - 4*a*c)^(1/2))/2) - (b^2*d*(b^2 - 4*a*c)^
(1/2))/2 + a*c*d*(b^2 - 4*a*c)^(1/2) - 2*a*b*c*d))/(4*a^3*c*d^2 - a^2*b^2*d
^2 + 4*a^2*c^2*e^2 + a*b^3*d*e - a*b^2*c*e^2 - 4*a^2*b*c*d*e)
```

3.65 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx = \frac{(bd-2ce)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} - \frac{d\log(d+ex)}{ad^2-e(bd-ce)} + \frac{d\log\left(c+bx+ax^2\right)}{2\left(ad^2-e(bd-ce)\right)}$$

```
[Out] -d*ln(e*x+d)/(a*d^2-e*(b*d-c*e))+1/2*d*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))+
(b*d-2*c*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c*e))/(-4*a
*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1583, 814, 648, 632, 212, 642}

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx = \frac{(bd-2ce)\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} + \frac{d\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)} - \frac{d\log(d+ex)}{ad^2-e(bd-ce)}$$

```
[In] Int[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]
```

```
[Out] ((b*d - 2*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(
a*d^2 - e*(b*d - c*e))) - (d*Log[d + e*x])/(a*d^2 - e*(b*d - c*e)) + (d*Log
[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x}{(d+ex)\left(c+bx+ax^2\right)}\,dx\\ &= \int \left(\frac{de}{\left(-ad^2+e(bd-ce)\right)(d+ex)}+\frac{ce+adx}{\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx\end{aligned}$$

$$= -\frac{d\log(d+ex)}{ad^2-bde+ce^2} + \frac{\int \frac{ce+adx}{c+bx+ax^2}\,dx}{ad^2-e(bd-ce)}$$

$$= -\frac{d\log(d+ex)}{ad^2-bde+ce^2} + \frac{d\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2\left(ad^2-bde+ce^2\right)} + \frac{(-bd+2ce)\int \frac{1}{c+bx+ax^2}\,dx}{2\left(ad^2-e(bd-ce)\right)}$$

$$= -\frac{d\log(d+ex)}{ad^2-bde+ce^2} + \frac{d\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)} + \frac{(bd-2ce)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2ax\right)}{ad^2-e(bd-ce)}$$

$$= \frac{(bd-2ce)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)} - \frac{d\log(d+ex)}{ad^2-bde+ce^2} + \frac{d\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx = \frac{2(bd-2ce)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\,d(2\log(d+ex)-\log(c+x(b+ax)))}{2\sqrt{-b^2+4ac}\left(-ad^2+e(bd-ce)\right)}$$

```
[In] Integrate[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]
```

```
[Out] (2*(b*d - 2*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c
]*d*(2*Log[d + e*x] - Log[c + x*(b + a*x)]))/(2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2
) + e*(b*d - c*e)))
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.85

method	result
default	$\frac{\frac{d\ln\left(a\,x^2+bx+c\right)}{2}+\frac{2\left(-\frac{bd}{2}+ec\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a\,d^2-bde+c\,e^2}-\frac{d\ln(ex+d)}{a\,d^2-bde+c\,e^2}$
risch	$-\frac{d\ln(ex+d)}{a\,d^2-bde+c\,e^2}+\left(\sum_{\_R=\text{RootOf}\left(\left(4a^2c\,d^2-a\,b^2d^2-4abcde+4a\,c^2e^2+b^3de-b^2c\,e^2\right)\_Z^2+\left(-4acd+b^2d\right)\_Z+c\right)}\_R\ln\left(\left(\left(-2a\right.\right.\right.\right.$

```
[In] int(1/(a+c/x^2+b/x)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*d^2-b*d*e+c*e^2)*(1/2*d*ln(a*x^2+b*x+c)+2*(-1/2*b*d+e*c)/(4*a*c-b^2)^(
1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))-d/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.46

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx$$
$$=\left[\frac{(b^2-4\,ac)d\log\left(ax^2+bx+c\right)-2\left(b^2-4\,ac\right)d\log\left(ex+d\right)-\sqrt{b^2-4\,ac}(bd-2\,ce)\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2}{ax^2}\right.}{2\left((ab^2-4\,a^2c)d^2-(b^3-4\,abc)de+(b^2c-4\,ac^2)e^2\right)}\right.$$

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2 - 4*a*c)*d*log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*d*log(e*x + d)
 - sqrt(b^2 - 4*a*c)*(b*d - 2*c*e)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c -
 sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)))/((a*b^2 - 4*a^2*c)*d^2
- (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2), 1/2*((b^2 - 4*a*c)*d*log(a*
x^2 + b*x + c) - 2*(b^2 - 4*a*c)*d*log(e*x + d) + 2*sqrt(-b^2 + 4*a*c)*(b*d
 - 2*c*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)))/((a*b^2 -
4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+c/x**2+b/x)/x/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx=-\frac{de\log\left(|ex+d|\right)}{ad^2e-bde^2+ce^3}+\frac{d\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}-\frac{(bd-2\,ce)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(ad^2-bde+ce^2\right)\sqrt{-b^2+4\,ac}}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] -d*e*log(abs(e*x + d))/(a*d^2*e - b*d*e^2 + c*e^3) + 1/2*d*log(a*x^2 + b*x
+ c)/(a*d^2 - b*d*e + c*e^2) - (b*d - 2*c*e)*arctan((2*a*x + b)/sqrt(-b^2 +
 4*a*c))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 801, normalized size of antiderivative = 6.46

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx$$

$$=\frac{\ln\left(a\,e\,x-\frac{\left(d\left(\frac{b\sqrt{b^2-4\,a\,c}}{2}-2\,a\,c+\frac{b^2}{2}\right)-c\,e\sqrt{b^2-4\,a\,c}\right)\left(x\left(d\,a^2\,e+b\,a\,e^2\right)+\frac{\left(d\left(\frac{b\sqrt{b^2-4\,a\,c}}{2}-2\,a\,c+\frac{b^2}{2}\right)-c\,e\sqrt{b^2-4\,a\,c}\right)\left(x\left(2\,a^3\,d^2\,e-2\,a^2\,b\,d\,e^2\cdots\right)\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-\cdots}\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e\cdots}$$

$$-\frac{\ln\left(\frac{\left(d\left(2\,a\,c+\frac{b\sqrt{b^2-4\,a\,c}}{2}-\frac{b^2}{2}\right)-c\,e\sqrt{b^2-4\,a\,c}\right)\left(x\left(d\,a^2\,e+b\,a\,e^2\right)-\frac{\left(d\left(2\,a\,c+\frac{b\sqrt{b^2-4\,a\,c}}{2}-\frac{b^2}{2}\right)-c\,e\sqrt{b^2-4\,a\,c}\right)\left(x\left(2\,a^3\,d^2\,e-2\,a^2\,b\,d\,e^2-6\,c\,a^2\cdots\right)\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2\cdots}\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\cdots}$$

$$-\frac{d\,\ln\left(d+e\,x\right)}{a\,d^2-b\,d\,e+c\,e^2}$$

```
[In] int(1/(x*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log(a*e*x - ((d*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*c + b^2/2) - c*e*(b^2 - 4
*a*c)^(1/2))*(x*(a*b*e^2 + a^2*d*e) + ((d*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*
c + b^2/2) - c*e*(b^2 - 4*a*c)^(1/2))*(x*(2*a*b^2*e^3 - 6*a^2*c*e^3 + 2*a^3
*d^2*e - 2*a^2*b*d*e^2) + a*b*c*e^3 + a*b^2*d*e^2 + a^2*b*d^2*e - 8*a^2*c*d
*e^2))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b
*c*d*e) + a*c*e^2 + a*b*d*e))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*
```

```
c*e^2 - b^3*d*e + 4*a*b*c*d*e))*(d*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*c + b^2
/2) - c*e*(b^2 - 4*a*c)^(1/2)))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^
2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) - (log(((d*(2*a*c + (b*(b^2 - 4*a*c)^(1/2)
)/2 - b^2/2) - c*e*(b^2 - 4*a*c)^(1/2))*(x*(a*b*e^2 + a^2*d*e) - ((d*(2*a*c
 + (b*(b^2 - 4*a*c)^(1/2))/2 - b^2/2) - c*e*(b^2 - 4*a*c)^(1/2))*(x*(2*a*b^
2*e^3 - 6*a^2*c*e^3 + 2*a^3*d^2*e - 2*a^2*b*d*e^2) + a*b*c*e^3 + a*b^2*d*e^
2 + a^2*b*d^2*e - 8*a^2*c*d*e^2))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 +
b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) + a*c*e^2 + a*b*d*e))/(a*b^2*d^2 - 4*a^2
*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) + a*e*x)*(d*(2*a*
c + (b*(b^2 - 4*a*c)^(1/2))/2 - b^2/2) - c*e*(b^2 - 4*a*c)^(1/2)))/(a*b^2*d
^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) - (d*lo
g(d + e*x))/(a*d^2 + c*e^2 - b*d*e)
```

3.66 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx = -\frac{(2ad-be)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} + \frac{e\log(d+ex)}{ad^2-bde+ce^2} - \frac{e\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}$$

```
[Out] e*ln(e*x+d)/(a*d^2-b*d*e+c*e^2)-1/2*e*ln(a*x^2+b*x+c)/(a*d^2-b*d*e+c*e^2)-(
2*a*d-b*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c*e))/(-4*a*
c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1583, 719, 31, 648, 632, 212, 642}

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx = -\frac{(2ad-be)\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} - \frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)} + \frac{e\log(d+ex)}{ad^2-bde+ce^2}$$

```
[In] Int[1/((a + c/x^2 + b/x)*x^2*(d + e*x)),x]
```

```
[Out] -(((2*a*d - b*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]
*(a*d^2 - e*(b*d - c*e)))) + (e*Log[d + e*x])/(a*d^2 - b*d*e + c*e^2) - (e*
Log[c + b*x + a*x^2])/(2*(a*d^2 - b*d*e + c*e^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{(d+ex)\left(c+bx+ax^2\right)}\,dx \\
&= \frac{e^2\int \frac{1}{d+ex}\,dx}{ad^2-bde+ce^2}+\frac{\int \frac{ad-be-aex}{c+bx+ax^2}\,dx}{ad^2-e(bd-ce)} \\
&= \frac{e\log(d+ex)}{ad^2-bde+ce^2}-\frac{e\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2\left(ad^2-bde+ce^2\right)}+\frac{(2ad-be)\int \frac{1}{c+bx+ax^2}\,dx}{2\left(ad^2-e(bd-ce)\right)} \\
&= \frac{e\log(d+ex)}{ad^2-bde+ce^2}-\frac{e\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}-\frac{(2ad-be)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2ax\right)}{ad^2-e(bd-ce)} \\
&= -\frac{(2ad-be)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}-\frac{e\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.85

$$
\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx = \frac{(-4ad+2be)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\,e(-2\log(d+ex)+\log(c+x(b+ax)))}{2\sqrt{-b^2+4ac}\left(-ad^2+e(bd-ce)\right)}
$$

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^2*(d + e*x)),x]
```

```
[Out] ((-4*a*d + 2*b*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*
c]*e*(-2*Log[d + e*x] + Log[c + x*(b + a*x)]))/(2*Sqrt[-b^2 + 4*a*c]*(-(a*d
^2) + e*(b*d - c*e)))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

method	result
default	$\frac{-\frac{e\ln\left(a\,x^2+bx+c\right)}{2}+\frac{2\left(da-\frac{be}{2}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a\,d^2-bde+c\,e^2}+\frac{e\ln(ex+d)}{a\,d^2-bde+c\,e^2}$
risch	$\frac{e\ln(ex+d)}{a\,d^2-bde+c\,e^2}+\left(\sum_{\_R=\text{RootOf}\left(\left(4a^2c\,d^2-a\,b^2d^2-4abcde+4a\,c^2e^2+b^3de-b^2c\,e^2\right)\_Z^2+\left(4ace-b^2e\right)\_Z+a\right)}\_R\ln\left(\left(\left(-2a^2d^2\right.\right.\right.\right.$

```
[In] int(1/(a+c/x^2+b/x)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*d^2-b*d*e+c*e^2)*(-1/2*e*ln(a*x^2+b*x+c)+2*(d*a-1/2*b*e)/(4*a*c-b^2)^(
1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))+e*ln(e*x+d)/(a*d^2-b*d*e+c*e^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.48

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx$$
$$= \left[-\frac{(b^2-4\,ac)e\log\left(ax^2+bx+c\right)-2\,(b^2-4\,ac)e\log\left(ex+d\right)+\sqrt{b^2-4\,ac}(2\,ad-be)\log\left(\frac{2\,a^2x^2+2\,abx+b^2\cdots}{a\cdots}\right.}{2\left((ab^2-4\,a^2c)d^2-(b^3-4\,abc)de+(b^2c-4\,ac^2)e^2\right)}\right.$$
$$-\frac{(b^2-4\,ac)e\log\left(ax^2+bx+c\right)-2\,(b^2-4\,ac)e\log\left(ex+d\right)+2\,\sqrt{-b^2+4\,ac}(2\,ad-be)\arctan\left(-\frac{\sqrt{-b^2\cdots}}{\cdots}\right.}{2\left((ab^2-4\,a^2c)d^2-(b^3-4\,abc)de+(b^2c-4\,ac^2)e^2\right)}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 4*a*c)*e*log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*e*log(e*x + d
) + sqrt(b^2 - 4*a*c)*(2*a*d - b*e)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c
+ sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)))/((a*b^2 - 4*a^2*c)*d^2
 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2), -1/2*((b^2 - 4*a*c)*e*log(
a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*e*log(e*x + d) + 2*sqrt(-b^2 + 4*a*c)*(2
*a*d - b*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)))/((a*b^2
- 4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx = \frac{e^2\log\left(|ex+d|\right)}{ad^2e-bde^2+ce^3} - \frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)} + \frac{(2\,ad-be)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(ad^2-bde+ce^2)\sqrt{-b^2+4\,ac}}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] e^2*log(abs(e*x + d))/(a*d^2*e - b*d*e^2 + c*e^3) - 1/2*e*log(a*x^2 + b*x +
 c)/(a*d^2 - b*d*e + c*e^2) + (2*a*d - b*e)*arctan((2*a*x + b)/sqrt(-b^2 +
4*a*c))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 10.16 (sec) , antiderivative size = 521, normalized size of antiderivative = 4.24

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx$$

$$= \frac{\ln\left(3\,a^2\,e^2\,x+a\,b\,e^2+a^2\,d\,e-\frac{a\,e\,\left(\frac{b^2\,e}{2}-2\,a\,c\,e+a\,d\,\sqrt{b^2-4\,a\,c}-\frac{b\,e\,\sqrt{b^2-4\,a\,c}}{2}\right)\,\left(2\,x\,a^2\,d^2+a\,b\,d^2-2\,x\,a\,b\,d\,e-8\,c\,a\,d\,e-6\,c\,x\,a\,e^2+b^2\,d\,e-\right.}{(4\,a\,c-b^2)\,(a\,d^2-b\,d\,e+c\,e^2)}\right.}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+}$$

$$- \frac{\ln\left(3\,a^2\,e^2\,x+a\,b\,e^2+a^2\,d\,e-\frac{a\,e\,\left(\frac{b^2\,e}{2}-2\,a\,c\,e-a\,d\,\sqrt{b^2-4\,a\,c}+\frac{b\,e\,\sqrt{b^2-4\,a\,c}}{2}\right)\,\left(2\,x\,a^2\,d^2+a\,b\,d^2-2\,x\,a\,b\,d\,e-8\,c\,a\,d\,e-6\,c\,x\,a\,e^2+b^2\right.}{(4\,a\,c-b^2)\,(a\,d^2-b\,d\,e+c\,e^2)}\right.}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e}$$

$$+ \frac{e\,\ln\left(d+e\,x\right)}{a\,d^2-b\,d\,e+c\,e^2}$$

```
[In] int(1/(x^2*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log(3*a^2*e^2*x + a*b*e^2 + a^2*d*e - (a*e*((b^2*e)/2 - 2*a*c*e + a*d*(b^2
 - 4*a*c)^(1/2) - (b*e*(b^2 - 4*a*c)^(1/2))/2)*(2*a^2*d^2*x + 2*b^2*e^2*x +
 a*b*d^2 + b*c*e^2 + b^2*d*e - 6*a*c*e^2*x - 8*a*c*d*e - 2*a*b*d*e*x))/((4*
a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(e*(2*a*c + (b*(b^2 - 4*a*c)^(1/2))/2
- b^2/2) - a*d*(b^2 - 4*a*c)^(1/2)))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2
 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) - (log(3*a^2*e^2*x + a*b*e^2 + a^2*d*
e - (a*e*((b^2*e)/2 - 2*a*c*e - a*d*(b^2 - 4*a*c)^(1/2) + (b*e*(b^2 - 4*a*c
)^(1/2))/2)*(2*a^2*d^2*x + 2*b^2*e^2*x + a*b*d^2 + b*c*e^2 + b^2*d*e - 6*a*
c*e^2*x - 8*a*c*d*e - 2*a*b*d*e*x))/((4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e))
)*(e*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*c + b^2/2) - a*d*(b^2 - 4*a*c)^(1/2))
)/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*
e) + (e*log(d + e*x))/(a*d^2 + c*e^2 - b*d*e)
```

3.67 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 158

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = \frac{(abd-b^2e+2ace)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}+\frac{\log(x)}{cd} - \frac{e^2\log(d+ex)}{d\,(ad^2-bde+ce^2)}-\frac{(ad-be)\log\left(c+bx+ax^2\right)}{2c\,(ad^2-e(bd-ce))}$$

```
[Out] ln(x)/c/d-e^2*ln(e*x+d)/d/(a*d^2-b*d*e+c*e^2)-1/2*(a*d-b*e)*ln(a*x^2+b*x+c)
/c/(a*d^2-e*(b*d-c*e))+(a*b*d+2*a*c*e-b^2*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)
^(1/2))/c/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1583, 907, 648, 632, 212, 642}

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = \frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)(abd+2ace+b^2(-e))}{c\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} - \frac{e^2\log(d+ex)}{d\,(ad^2-e(bd-ce))} - \frac{(ad-be)\log\left(ax^2+bx+c\right)}{2c\,(ad^2-e(bd-ce))}+\frac{\log(x)}{cd}$$

```
[In] Int[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]
```

```
[Out] ((a*b*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[
b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + Log[x]/(c*d) - (e^2*Log[d + e*x])/(
d*(a*d^2 - e*(b*d - c*e))) - ((a*d - b*e)*Log[c + b*x + a*x^2])/(2*c*(a*d^2
 - e*(b*d - c*e)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cdx}+\frac{e^3}{d\left(-ad^2+e(bd-ce)\right)(d+ex)}+\frac{b^2e-a(bd+ce)-a(ad-be)x}{c\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx$$

$$= \frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{b^2e-a(bd+ce)-a(ad-be)x}{c+bx+ax^2}\,dx}{c\left(ad^2-bde+ce^2\right)}$$

$$= \frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}+\frac{\left(-abd+b^2e-2ace\right)\int \frac{1}{c+bx+ax^2}\,dx}{2c\left(ad^2-bde+ce^2\right)}-\frac{(ad-be)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2c\left(ad^2-e(bd-ce)\right)}$$

$$= \frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}-\frac{(ad-be)\log\left(c+bx+ax^2\right)}{2c\left(ad^2-e(bd-ce)\right)}-\frac{\left(-abd+b^2e-2ace\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2ax\right)}{c\left(ad^2-bde+ce^2\right)}$$

$$= \frac{\left(abd-b^2e+2ace\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-bde+ce^2\right)}+\frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}-\frac{(ad-be)\log\left(c+bx+ax^2\right)}{2c\left(ad^2-e(bd-ce)\right)}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = -\frac{2d\left(abd-b^2e+2ace\right)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\left(-2\left(ad^2+e(-bd+ce)\right)\log(x)+2ce^2\log(d+ex\right.}{2c\sqrt{-b^2+4ac}\,d\left(ad^2+e(-bd+ce)\right)}$$

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]
```

```
[Out] -1/2*(2*d*(a*b*d - b^2*e + 2*a*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]]
+ Sqrt[-b^2 + 4*a*c]*(-2*(a*d^2 + e*(-(b*d) + c*e))*Log[x] + 2*c*e^2*Log[d
+ e*x] + d*(a*d - b*e)*Log[c + x*(b + a*x)]))/(c*Sqrt[-b^2 + 4*a*c]*d*(a*d^
2 + e*(-(b*d) + c*e)))
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{\ln(x)}{cd}+\frac{\frac{\left(-d\,a^{2}+abe\right)\ln\left(a\,x^{2}+bx+c\right)}{2a}+\frac{2\left(-dab-ace+b^{2}e-\frac{\left(-d\,a^{2}+abe\right)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{\left(a\,d^{2}-bde+c\,e^{2}\right)c}-\frac{e^{2}\ln(ex+d)}{d\left(a\,d^{2}-bde+c\,e^{2}\right)}$	160
risch	Expression too large to display	19172

```
[In] int(1/(a+c/x^2+b/x)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/c/d+1/(a*d^2-b*d*e+c*e^2)/c*(1/2*(-a^2*d+a*b*e)/a*ln(a*x^2+b*x+c)+2*(
-d*a*b-a*c*e+b^2*e-1/2*(-a^2*d+a*b*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+
b)/(4*a*c-b^2)^(1/2)))-e^2*ln(e*x+d)/d/(a*d^2-b*d*e+c*e^2)
```

Fricas [A] (verification not implemented)

none

Time = 184.15 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.19

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = \left[-\frac{2\left(b^2c-4\,ac^2\right)e^2\log\left(ex+d\right)-\left(abd^2-\left(b^2-2\,ac\right)de\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac+\sqrt{b^2-4\,ac}\left(2\,ax+b\right)}{ax^2+bx+c}\right.}{2\left(\left(ab^2c-4\,a^2c^2\right)d^3-\right.}\right.$$
$$-\frac{2\left(b^2c-4\,ac^2\right)e^2\log\left(ex+d\right)-2\left(abd^2-\left(b^2-2\,ac\right)de\right)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\left(2\,ax+b\right)}{b^2-4\,ac}\right)+\left(\left(ab\right.\right.}{2\left(\left(ab^2c-4\,a^2c^2\right)d^3-\left(b^3c\right.\right.}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c - 4*a*c^2)*e^2*log(e*x + d) - (a*b*d^2 - (b^2 - 2*a*c)*d*e)
*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*
c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((a*b^2 - 4*a^2*c)*d^2 - (b^3 - 4*a*b*
c)*d*e)*log(a*x^2 + b*x + c) - 2*((a*b^2 - 4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d
*e + (b^2*c - 4*a*c^2)*e^2)*log(x))/((a*b^2*c - 4*a^2*c^2)*d^3 - (b^3*c - 4
*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2), -1/2*(2*(b^2*c - 4*a*c^2)*e^2
*log(e*x + d) - 2*(a*b*d^2 - (b^2 - 2*a*c)*d*e)*sqrt(-b^2 + 4*a*c)*arctan(-
sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + ((a*b^2 - 4*a^2*c)*d^2 - (b
^3 - 4*a*b*c)*d*e)*log(a*x^2 + b*x + c) - 2*((a*b^2 - 4*a^2*c)*d^2 - (b^3 -
 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)*log(x))/((a*b^2*c - 4*a^2*c^2)*d^3 -
 (b^3*c - 4*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = -\frac{e^3\log(|ex+d|)}{ad^3e-bd^2e^2+cde^3}-\frac{(ad-be)\log(ax^2+bx+c)}{2\,(acd^2-bcde+c^2e^2)} - \frac{(abd-b^2e+2\,ace)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(acd^2-bcde+c^2e^2)\sqrt{-b^2+4\,ac}}+\frac{\log(|x|)}{cd}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] -e^3*log(abs(e*x + d))/(a*d^3*e - b*d^2*e^2 + c*d*e^3) - 1/2*(a*d - b*e)*lo
g(a*x^2 + b*x + c)/(a*c*d^2 - b*c*d*e + c^2*e^2) - (a*b*d - b^2*e + 2*a*c*e
)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a*c*d^2 - b*c*d*e + c^2*e^2)*sqr
t(-b^2 + 4*a*c)) + log(abs(x))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 11.09 (sec) , antiderivative size = 2399, normalized size of antiderivative = 15.18

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^3*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log(b^3*c^3*e^5 - 6*a^4*c^2*d^5 + 2*a^3*b^2*c*d^5 + 8*a^2*c^4*d*e^4 - b^4*
c^2*d*e^4 - 2*b^5*c*d^2*e^3 + 2*a^3*b^3*d^5*x + 8*a^2*c^4*e^5*x + b^4*c^2*e
^5*x - 2*b^6*d^2*e^3*x + b^2*c^3*e^5*(b^2 - 4*a*c)^(1/2) + 18*a^3*c^3*d^3*e
^2 - 4*a*b*c^4*e^5 - 4*a*c^4*e^5*(b^2 - 4*a*c)^(1/2) - 5*a^2*c^3*d^2*e^3*(b
^2 - 4*a*c)^(1/2) - 7*a^4*b*c*d^5*x - b^5*c*d*e^4*x - 27*a^2*b^2*c^2*d^3*e^
2 + 2*a^3*b*c*d^5*(b^2 - 4*a*c)^(1/2) - 3*a^4*c*d^5*x*(b^2 - 4*a*c)^(1/2) +
 2*a*b^2*c^3*d*e^4 + 6*a*b^4*c*d^3*e^2 - 6*a^2*b^3*c*d^4*e + 21*a^3*b*c^2*d
^4*e - 6*a*b^2*c^3*e^5*x + 6*a*b^5*d^3*e^2*x - 6*a^2*b^4*d^4*e*x - 14*a^4*c
^2*d^4*e*x + 7*a^3*c^2*d^4*e*(b^2 - 4*a*c)^(1/2) - b^3*c^2*d*e^4*(b^2 - 4*a
*c)^(1/2) - 2*b^4*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 2*a^3*b^2*d^5*x*(b^2 - 4*
a*c)^(1/2) + b^3*c^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^5*d^2*e^3*x*(b^2 - 4*a
*c)^(1/2) + 13*a*b^3*c^2*d^2*e^3 - 21*a^2*b*c^3*d^2*e^3 + 10*a^3*c^3*d^2*e^
3*x + 6*a*b^3*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^2*c*d^4*e*(b^2 - 4*a*
c)^(1/2) + 6*a*b^4*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^3*d^4*e*x*(b^2 -
 4*a*c)^(1/2) + 4*a^2*c^3*d*e^4*x*(b^2 - 4*a*c)^(1/2) - 32*a^2*b^3*c*d^3*e^
2*x + 35*a^3*b*c^2*d^3*e^2*x + 7*a*b^2*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2) - 13
*a^2*b*c^2*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 9*a^3*c^2*d^3*e^2*x*(b^2 - 4*a*c)^
(1/2) - 27*a^2*b^2*c^2*d^2*e^3*x + 4*a*b*c^3*d*e^4*(b^2 - 4*a*c)^(1/2) - 4*
a*b*c^3*e^5*x*(b^2 - 4*a*c)^(1/2) - b^4*c*d*e^4*x*(b^2 - 4*a*c)^(1/2) + 5*a
*b^3*c^2*d*e^4*x + 14*a*b^4*c*d^2*e^3*x - 4*a^2*b*c^3*d*e^4*x + 26*a^3*b^2*
c*d^4*e*x + 14*a^3*b*c*d^4*e*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^2*c^2*d*e^4*x*(b
^2 - 4*a*c)^(1/2) + 10*a*b^3*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2
*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 20*a^2*b^2*c*d^3*e^2*x*(b^2 - 4*a*c)^(1/2)
)*(d*((a*b^2)/2 - 2*a^2*c + (a*b*(b^2 - 4*a*c)^(1/2))/2) - (b^3*e)/2 - (b^2
*e*(b^2 - 4*a*c)^(1/2))/2 + a*c*e*(b^2 - 4*a*c)^(1/2) + 2*a*b*c*e))/(4*a*c^
3*e^2 + 4*a^2*c^2*d^2 - b^2*c^2*e^2 + b^3*c*d*e - a*b^2*c*d^2 - 4*a*b*c^2*d
*e) - (log(6*a^4*c^2*d^5 - b^3*c^3*e^5 - 2*a^3*b^2*c*d^5 - 8*a^2*c^4*d*e^4
+ b^4*c^2*d*e^4 + 2*b^5*c*d^2*e^3 - 2*a^3*b^3*d^5*x - 8*a^2*c^4*e^5*x - b^4
*c^2*e^5*x + 2*b^6*d^2*e^3*x + b^2*c^3*e^5*(b^2 - 4*a*c)^(1/2) - 18*a^3*c^3
*d^3*e^2 + 4*a*b*c^4*e^5 - 4*a*c^4*e^5*(b^2 - 4*a*c)^(1/2) - 5*a^2*c^3*d^2*
e^3*(b^2 - 4*a*c)^(1/2) + 7*a^4*b*c*d^5*x + b^5*c*d*e^4*x + 27*a^2*b^2*c^2*
d^3*e^2 + 2*a^3*b*c*d^5*(b^2 - 4*a*c)^(1/2) - 3*a^4*c*d^5*x*(b^2 - 4*a*c)^(
1/2) - 2*a*b^2*c^3*d*e^4 - 6*a*b^4*c*d^3*e^2 + 6*a^2*b^3*c*d^4*e - 21*a^3*b
*c^2*d^4*e + 6*a*b^2*c^3*e^5*x - 6*a*b^5*d^3*e^2*x + 6*a^2*b^4*d^4*e*x + 14
*a^4*c^2*d^4*e*x + 7*a^3*c^2*d^4*e*(b^2 - 4*a*c)^(1/2) - b^3*c^2*d*e^4*(b^2
 - 4*a*c)^(1/2) - 2*b^4*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 2*a^3*b^2*d^5*x*(b^
```

```
2 - 4*a*c)^(1/2) + b^3*c^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^5*d^2*e^3*x*(b^2
 - 4*a*c)^(1/2) - 13*a*b^3*c^2*d^2*e^3 + 21*a^2*b*c^3*d^2*e^3 - 10*a^3*c^3*
d^2*e^3*x + 6*a*b^3*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^2*c*d^4*e*(b^2
- 4*a*c)^(1/2) + 6*a*b^4*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^3*d^4*e*x*
(b^2 - 4*a*c)^(1/2) + 4*a^2*c^3*d*e^4*x*(b^2 - 4*a*c)^(1/2) + 32*a^2*b^3*c*
d^3*e^2*x - 35*a^3*b*c^2*d^3*e^2*x + 7*a*b^2*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2
) - 13*a^2*b*c^2*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 9*a^3*c^2*d^3*e^2*x*(b^2 - 4
*a*c)^(1/2) + 27*a^2*b^2*c^2*d^2*e^3*x + 4*a*b*c^3*d*e^4*(b^2 - 4*a*c)^(1/2
) - 4*a*b*c^3*e^5*x*(b^2 - 4*a*c)^(1/2) - b^4*c*d*e^4*x*(b^2 - 4*a*c)^(1/2)
 - 5*a*b^3*c^2*d*e^4*x - 14*a*b^4*c*d^2*e^3*x + 4*a^2*b*c^3*d*e^4*x - 26*a^
3*b^2*c*d^4*e*x + 14*a^3*b*c*d^4*e*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^2*c^2*d*e^
4*x*(b^2 - 4*a*c)^(1/2) + 10*a*b^3*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 13*a^2
*b*c^2*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 20*a^2*b^2*c*d^3*e^2*x*(b^2 - 4*a*c)
^(1/2))*((b^3*e)/2 + d*(2*a^2*c - (a*b^2)/2 + (a*b*(b^2 - 4*a*c)^(1/2))/2)
- (b^2*e*(b^2 - 4*a*c)^(1/2))/2 + a*c*e*(b^2 - 4*a*c)^(1/2) - 2*a*b*c*e))/(
4*a*c^3*e^2 + 4*a^2*c^2*d^2 - b^2*c^2*e^2 + b^3*c*d*e - a*b^2*c*d^2 - 4*a*b
*c^2*d*e) - (e^2*log(d + e*x))/(a*d^3 - b*d^2*e + c*d*e^2) + log(x)/(c*d)
```

3.68 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx = -\frac{1}{cdx}+\frac{(2a^2cd+b^3e-ab(bd+3ce))\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} - \frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)} + \frac{(abd-b^2e+ace)\log\left(c+bx+ax^2\right)}{2c^2\left(ad^2-e(bd-ce)\right)}$$

```
[Out] -1/c/d/x-(b*d+c*e)*ln(x)/c^2/d^2+e^3*ln(e*x+d)/d^2/(a*d^2-e*(b*d-c*e))+1/2*
(a*b*d+a*c*e-b^2*e)*ln(a*x^2+b*x+c)/c^2/(a*d^2-e*(b*d-c*e))+(2*a^2*c*d+b^3*
e-a*b*(b*d+3*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(a*d^2-e*(b*d-
c*e))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1583, 907, 648, 632, 212, 642}

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx = \frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)(2a^2cd-ab(bd+3ce)+b^3e)}{c^2\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} + \frac{(abd+ace+b^2(-e))\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2-e(bd-ce)\right)} + \frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}-\frac{\log(x)(bd+ce)}{c^2d^2}-\frac{1}{cdx}$$

```
[In] Int[1/((a + c/x^2 + b/x)*x^4*(d + e*x)),x]
```

```
[Out] -(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTanh[(b + 2*a*x)
/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - ((b*
d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d + e*x])/(d^2*(a*d^2 - e*(b*d - c*e)
)) + ((a*b*d - b^2*e + a*c*e)*Log[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d
- c*e)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
```

```
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^2(d+ex)\left(c+bx+ax^2\right)}\,dx \\
&= \int \left(\frac{1}{cdx^2}+\frac{-bd-ce}{c^2d^2x}+\frac{e^4}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{-a^2cd-b^3e+ab(bd+2ce)+a\left(abd-b^2e+ace\right)x}{c^2\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx \\
&= -\frac{1}{cdx}-\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{-a^2cd-b^3e+ab(bd+2ce)+a\left(abd-b^2e+ace\right)x}{c+bx+ax^2}\,dx}{c^2\left(ad^2-e(bd-ce)\right)} \\
&= -\frac{1}{cdx}-\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)} + \frac{\left(abd-b^2e+ace\right)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2c^2\left(ad^2-e(bd-ce)\right)}-\frac{\left(2a^2cd+b^3e-ab(bd+3ce)\right)\int \frac{1}{c+bx+ax^2}\,dx}{2c^2\left(ad^2-e(bd-ce)\right)} \\
&= -\frac{1}{cdx}-\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)} + \frac{\left(abd-b^2e+ace\right)\log\left(c+bx+ax^2\right)}{2c^2\left(ad^2-e(bd-ce)\right)} + \frac{\left(2a^2cd+b^3e-ab(bd+3ce)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2ax\right)}{c^2\left(ad^2-e(bd-ce)\right)} \\
&= -\frac{1}{cdx}+\frac{\left(2a^2cd+b^3e-ab(bd+3ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{(bd+ce)\log(x)}{c^2d^2} + \frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(abd-b^2e+ace\right)\log\left(c+bx+ax^2\right)}{2c^2\left(ad^2-e(bd-ce)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx = -\frac{1}{cdx}+\frac{\left(2a^2cd+b^3e-ab(bd+3ce)\right)\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}\left(-ad^2+e(bd-ce)\right)} - \frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{ad^4+d^2e(-bd+ce)} + \frac{\left(abd-b^2e+ace\right)\log(c+x(b+ax))}{2c^2\left(ad^2+e(-bd+ce)\right)}$$

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^4*(d + e*x)),x]
```

```
[Out] -(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTan[(b + 2*a*x)/
Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e))) -
((b*d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d + e*x])/(a*d^4 + d^2*e*(-(b*d)
+ c*e)) + ((a*b*d - b^2*e + a*c*e)*Log[c + x*(b + a*x)])/(2*c^2*(a*d^2 + e*
(-(b*d) + c*e)))
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.07

method	result
default	$-\frac{1}{cdx}+\frac{(-bd-ec)\ln(x)}{c^2d^2}+\frac{\frac{\left(a^2bd+a^2ce-a\,b^2e\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(-a^2cd+a\,b^2d+2abce-b^3e-\frac{\left(a^2bd+a^2ce-a\,b^2e\right)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{(a\,d^2-bde+c\,e^2)c^2}$
risch	Expression too large to display

```
[In] int(1/(a+c/x^2+b/x)/x^4/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c/d/x+1/c^2/d^2*(-b*d-c*e)*ln(x)+1/(a*d^2-b*d*e+c*e^2)/c^2*(1/2*(a^2*b*d
+a^2*c*e-a*b^2*e)/a*ln(a*x^2+b*x+c)+2*(-a^2*c*d+a*b^2*d+2*a*b*c*e-b^3*e-1/2
*(a^2*b*d+a^2*c*e-a*b^2*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b
^2)^(1/2)))+e^3/d^2/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx =& \frac{e^4\log\left(|ex+d|\right)}{ad^4e-bd^3e^2+cd^2e^3}+\frac{(abd-b^2e+ace)\log\left(ax^2+bx+c\right)}{2\left(ac^2d^2-bc^2de+c^3e^2\right)}\\&+\frac{(ab^2d-2\,a^2cd-b^3e+3\,abce)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(ac^2d^2-bc^2de+c^3e^2)\sqrt{-b^2+4\,ac}}\\&-\frac{(bd+ce)\log\left(|x|\right)}{c^2d^2}-\frac{1}{cdx}\end{aligned}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] e^4*log(abs(e*x + d))/(a*d^4*e - b*d^3*e^2 + c*d^2*e^3) + 1/2*(a*b*d - b^2*
e + a*c*e)*log(a*x^2 + b*x + c)/(a*c^2*d^2 - b*c^2*d*e + c^3*e^2) + (a*b^2*
d - 2*a^2*c*d - b^3*e + 3*a*b*c*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/(
(a*c^2*d^2 - b*c^2*d*e + c^3*e^2)*sqrt(-b^2 + 4*a*c)) - (b*d + c*e)*log(abs
(x))/(c^2*d^2) - 1/(c*d*x)
```

Mupad [B] (verification not implemented)

Time = 22.98 (sec) , antiderivative size = 2388, normalized size of antiderivative = 12.37

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^4*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (e^3*log(d + e*x))/(a*d^4 + c*d^2*e^2 - b*d^3*e) + (log((a^4*e^4*x)/(c^2*d^
2) - (((a*e*x*(a^4*d^4 + b^4*e^4 + 2*a^2*c^2*e^4 + 2*a^3*c*d^2*e^2 - 4*a*b^
2*c*e^4 + 2*a^2*b*c*d*e^3))/(c^2*d^2) - (((a*e*(a^2*b^2*d^4 - 4*a*c^3*e^4 -
 a^3*c*d^4 + b^2*c^2*e^4 + b^4*d^2*e^2 + 4*a^2*c^2*d^2*e^2 - 2*a*b^3*d^3*e
+ b^3*c*d*e^3 - 4*a*b*c^2*d*e^3 + 5*a^2*b*c*d^3*e - 5*a*b^2*c*d^2*e^2))/(c*
d) + (a*e*x*(2*a^3*b*d^4 + 2*b^3*c*e^4 + 2*b^4*d*e^3 - 2*a*b^3*d^2*e^2 - 2*
a^2*b^2*d^3*e + 12*a^2*c^2*d*e^3 - 8*a*b*c^2*e^4 + a^3*c*d^3*e - 11*a*b^2*c
*d*e^3 + 8*a^2*b*c*d^2*e^2))/(c*d) + (a*e*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2
) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*e - a*b^2*d*(b^2 - 4*a*
c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2))*(
4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c
^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x -
8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b
*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x
 - 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^
4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a
*b^2*c*e - a*b^2*d*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*
a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e))
 + (a*e*(b*d + c*e)*(a^3*d^3 + b^3*e^3 - 3*a*b*c*e^3))/(c^2*d^2))*(b^4*e +
b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c
*e - a*b^2*d*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*
e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4
*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*
b^2*c*e - a*b^2*d*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a
*b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^4*e^2 + 4*a^2*c^3*d^2 - b^2*c^3*e^2
- a*b^2*c^2*d^2 + b^3*c^2*d*e - 4*a*b*c^3*d*e)) + (log((a^4*e^4*x)/(c^2*d^2
) - (((a*e*x*(a^4*d^4 + b^4*e^4 + 2*a^2*c^2*e^4 + 2*a^3*c*d^2*e^2 - 4*a*b^2
*c*e^4 + 2*a^2*b*c*d*e^3))/(c^2*d^2) - (((a*e*(a^2*b^2*d^4 - 4*a*c^3*e^4 -
a^3*c*d^4 + b^2*c^2*e^4 + b^4*d^2*e^2 + 4*a^2*c^2*d^2*e^2 - 2*a*b^3*d^3*e +
 b^3*c*d*e^3 - 4*a*b*c^2*d*e^3 + 5*a^2*b*c*d^3*e - 5*a*b^2*c*d^2*e^2))/(c*d
) + (a*e*x*(2*a^3*b*d^4 + 2*b^3*c*e^4 + 2*b^4*d*e^3 - 2*a*b^3*d^2*e^2 - 2*a
^2*b^2*d^3*e + 12*a^2*c^2*d*e^3 - 8*a*b*c^2*e^4 + a^3*c*d^3*e - 11*a*b^2*c*
d*e^3 + 8*a^2*b*c*d^2*e^2))/(c*d) + (a*e*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2)
 + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*e + a*b^2*d*(b^2 - 4*a*c
)^(1/2) - 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2))*(4
*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^
2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8
*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*
c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x
- 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4
*e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*
b^2*c*e + a*b^2*d*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a
*b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e))
+ (a*e*(b*d + c*e)*(a^3*d^3 + b^3*e^3 - 3*a*b*c*e^3))/(c^2*d^2))*(b^4*e - b
^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*
```

```
e + a*b^2*d*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e
*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4*
e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b
^2*c*e + a*b^2*d*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*
b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^4*e^2 + 4*a^2*c^3*d^2 - b^2*c^3*e^2 -
 a*b^2*c^2*d^2 + b^3*c^2*d*e - 4*a*b*c^3*d*e)) - 1/(c*d*x) - (log(x)*(b*d +
 c*e))/(c^2*d^2)
```

3.69 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 252

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx = -\frac{1}{2cdx^2}+\frac{bd+ce}{c^2d^2x} - \frac{(b^4e+a^2c(3bd+2ce)-ab^2(bd+4ce))\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} + \frac{(b^2d^2+bcde-c(ad^2-ce^2))\log(x)}{c^3d^3} - \frac{e^4\log(d+ex)}{d^3\,(ad^2-e(bd-ce))} + \frac{(a^2cd+b^3e-ab(bd+2ce))\log\left(c+bx+ax^2\right)}{2c^3\,(ad^2-e(bd-ce))}$$

```
[Out] -1/2/c/d/x^2+(b*d+c*e)/c^2/d^2/x+(b^2*d^2+b*c*d*e-c*(a*d^2-c*e^2))*ln(x)/c^
3/d^3-e^4*ln(e*x+d)/d^3/(a*d^2-e*(b*d-c*e))+1/2*(a^2*c*d+b^3*e-a*b*(b*d+2*c
*e))*ln(a*x^2+b*x+c)/c^3/(a*d^2-e*(b*d-c*e))-(b^4*e+a^2*c*(3*b*d+2*c*e)-a*b
^2*(b*d+4*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(a*d^2-e*(b*d-c*e
))/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1583, 907, 648, 632, 212, 642}

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(a^2c(3bd+2ce)-ab^2(bd+4ce)+b^4e\right)}{c^3\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} + \frac{\left(a^2cd-ab(bd+2ce)+b^3e\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)} + \frac{\log(x)\left(-c(ad^2-ce^2)+b^2d^2+bcde\right)}{c^3d^3} - \frac{e^4\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)} + \frac{bd+ce}{c^2d^2x} - \frac{1}{2cdx^2}$$

```
[In] Int[1/((a + c/x^2 + b/x)*x^5*(d + e*x)),x]
```

```
[Out] -1/2*1/(c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d + 2*c*e
) - a*b^2*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[
b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + ((b^2*d^2 + b*c*d*e - c*(a*d^2 - c*
e^2))*Log[x])/(c^3*d^3) - (e^4*Log[d + e*x])/(d^3*(a*d^2 - e*(b*d - c*e)))
+ ((a^2*c*d + b^3*e - a*b*(b*d + 2*c*e))*Log[c + b*x + a*x^2])/(2*c^3*(a*d^
2 - e*(b*d - c*e)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^3(d+ex)\left(c+bx+ax^2\right)}\,dx \\
&= \int \left(\frac{1}{cdx^3}+\frac{-bd-ce}{c^2d^2x^2}+\frac{b^2d^2+bcde-c(ad^2-ce^2)}{c^3d^3x}+\frac{e^5}{d^3\left(-ad^2+e(bd-ce)\right)(d+ex)}\right. \\
&\qquad \left.+\frac{b^4e+a^2c(2bd+ce)-ab^2(bd+3ce)+a\left(a^2cd+b^3e-ab(bd+2ce)\right)x}{c^3\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx \\
&= -\frac{1}{2cdx^2}+\frac{bd+ce}{c^2d^2x}+\frac{\left(b^2d^2+bcde-c(ad^2-ce^2)\right)\log(x)}{c^3d^3} \\
&\quad -\frac{e^4\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{b^4e+a^2c(2bd+ce)-ab^2(bd+3ce)+a\left(a^2cd+b^3e-ab(bd+2ce)\right)x}{c+bx+ax^2}\,dx}{c^3\left(ad^2-e(bd-ce)\right)} \\
&= -\frac{1}{2cdx^2}+\frac{bd+ce}{c^2d^2x}+\frac{\left(b^2d^2+bcde-c(ad^2-ce^2)\right)\log(x)}{c^3d^3} \\
&\quad -\frac{e^4\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)}+\frac{\left(a^2cd+b^3e-ab(bd+2ce)\right)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2c^3\left(ad^2-e(bd-ce)\right)} \\
&\quad +\frac{\left(b^4e+a^2c(3bd+2ce)-ab^2(bd+4ce)\right)\int \frac{1}{c+bx+ax^2}\,dx}{2c^3\left(ad^2-e(bd-ce)\right)}
\end{aligned}$$

$$= -\frac{1}{2cdx^2} + \frac{bd+ce}{c^2d^2x} + \frac{(b^2d^2+bcde-c(ad^2-ce^2))\log(x)}{c^3d^3} - \frac{e^4\log(d+ex)}{d^3(ad^2-e(bd-ce))} + \frac{(a^2cd+b^3e-ab(bd+2ce))\log(c+bx+ax^2)}{2c^3(ad^2-e(bd-ce))} - \frac{(b^4e+a^2c(3bd+2ce)-ab^2(bd+4ce))\,\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2ax\right)}{c^3(ad^2-e(bd-ce))}$$

$$= -\frac{1}{2cdx^2} + \frac{bd+ce}{c^2d^2x} - \frac{(b^4e+a^2c(3bd+2ce)-ab^2(bd+4ce))\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))} + \frac{(b^2d^2+bcde-c(ad^2-ce^2))\log(x)}{c^3d^3} - \frac{e^4\log(d+ex)}{d^3(ad^2-e(bd-ce))} + \frac{(a^2cd+b^3e-ab(bd+2ce))\log(c+bx+ax^2)}{2c^3(ad^2-e(bd-ce))}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx = -\frac{1}{2cdx^2} + \frac{bd+ce}{c^2d^2x} - \frac{(b^4e+a^2c(3bd+2ce)-ab^2(bd+4ce))\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{c^3\sqrt{-b^2+4ac}\,(-ad^2+e(bd-ce))} + \frac{(b^2d^2+bcde+c(-ad^2+ce^2))\log(x)}{c^3d^3} - \frac{e^4\log(d+ex)}{ad^5+d^3e(-bd+ce)} + \frac{(a^2cd+b^3e-ab(bd+2ce))\log(c+x(b+ax))}{2c^3(ad^2+e(-bd+ce))}$$

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^5*(d + e*x)),x]
```

```
[Out] -1/2*1/(c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d + 2*c*e
) - a*b^2*(b*d + 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c^3*Sqrt[
-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e))) + ((b^2*d^2 + b*c*d*e + c*(-(a*d^
2) + c*e^2))*Log[x])/(c^3*d^3) - (e^4*Log[d + e*x])/(a*d^5 + d^3*e*(-(b*d)
+ c*e)) + ((a^2*c*d + b^3*e - a*b*(b*d + 2*c*e))*Log[c + x*(b + a*x)])/(2*c
^3*(a*d^2 + e*(-(b*d) + c*e)))
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.10

method	result
default	$-\frac{1}{2cd\,x^{2}}-\frac{-bd-ec}{x\,c^{2}d^{2}}+\frac{\left(-d^{2}ac+b^{2}d^{2}+bcde+e^{2}c^{2}\right)\ln\left(x\right)}{d^{3}c^{3}}+\frac{\frac{\left(a^{3}cd-a^{2}b^{2}d-2a^{2}bce+a\,b^{3}e\right)\ln\left(a\,x^{2}+bx+c\right)}{2a}+\frac{2\left(2a^{2}bcd+a^{2}c^{2}e-a\,b^{3}d-3a\right.}{}}{\left(a\,d^{2}-bde+c\,e\right.}$
risch	Expression too large to display

```
[In] int(1/(a+c/x^2+b/x)/x^5/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c/d/x^2-(-b*d-c*e)/x/c^2/d^2+1/d^3/c^3*(-a*c*d^2+b^2*d^2+b*c*d*e+c^2*e
^2)*ln(x)+1/(a*d^2-b*d*e+c*e^2)/c^3*(1/2*(a^3*c*d-a^2*b^2*d-2*a^2*b*c*e+a*b
^3*e)/a*ln(a*x^2+b*x+c)+2*(2*a^2*b*c*d+a^2*c^2*e-a*b^3*d-3*a*b^2*c*e+b^4*e-
1/2*(a^3*c*d-a^2*b^2*d-2*a^2*b*c*e+a*b^3*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((
2*a*x+b)/(4*a*c-b^2)^(1/2)))-e^4/d^3/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.09

$$\begin{aligned}\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx &= -\frac{e^5\log\left(|ex+d|\right)}{ad^5e-bd^4e^2+cd^3e^3}-\frac{(ab^2d-a^2cd-b^3e+2\,abce)\log\left(ax^2+bx+c\right)}{2\left(ac^3d^2-bc^3de+c^4e^2\right)}\\&\quad-\frac{(ab^3d-3\,a^2bcd-b^4e+4\,ab^2ce-2\,a^2c^2e)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(ac^3d^2-bc^3de+c^4e^2)\sqrt{-b^2+4\,ac}}\\&\quad+\frac{(b^2d^2-acd^2+bcde+c^2e^2)\log\left(|x|\right)}{c^3d^3}-\frac{c^2d^2-2\left(bcd^2+c^2de\right)x}{2\,c^3d^3x^2}\end{aligned}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] -e^5*log(abs(e*x + d))/(a*d^5*e - b*d^4*e^2 + c*d^3*e^3) - 1/2*(a*b^2*d - a
^2*c*d - b^3*e + 2*a*b*c*e)*log(a*x^2 + b*x + c)/(a*c^3*d^2 - b*c^3*d*e + c
^4*e^2) - (a*b^3*d - 3*a^2*b*c*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*arcta
n((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a*c^3*d^2 - b*c^3*d*e + c^4*e^2)*sqrt(-
b^2 + 4*a*c)) + (b^2*d^2 - a*c*d^2 + b*c*d*e + c^2*e^2)*log(abs(x))/(c^3*d^
3) - 1/2*(c^2*d^2 - 2*(b*c*d^2 + c^2*d*e)*x)/(c^3*d^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 28.67 (sec) , antiderivative size = 3530, normalized size of antiderivative = 14.01

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^5*(d + e*x)*(a + b/x + c/x^2)),x)
[Out] (log((a^4*e^4*(b^2*d^2 + c^2*e^2 - a*c*d^2 + b*c*d*e))/(c^4*d^4) - (((((a*e
*(a^2*b^3*d^5 - 4*a*c^4*e^5 + b^2*c^3*e^5 + b^5*d^3*e^2 - 3*a^3*c^2*d^4*e +
 b^3*c^2*d*e^4 + b^4*c*d^2*e^3 + 4*a^2*c^3*d^2*e^3 - 2*a^3*b*c*d^5 - 2*a*b^
4*d^4*e - 4*a*b*c^3*d*e^4 - 6*a*b^3*c*d^3*e^2 + 7*a^2*b^2*c*d^4*e - 5*a*b^2
*c^2*d^2*e^3 + 8*a^2*b*c^2*d^3*e^2))/(c^2*d^2) + (a*e*x*(2*a^3*b^2*d^5 - 3*
a^4*c*d^5 + 2*b^3*c^2*e^5 + 2*b^5*d^2*e^3 - 2*a*b^4*d^3*e^2 - 2*a^2*b^3*d^4
*e + 8*a^2*c^3*d*e^4 - 8*a^3*c^2*d^3*e^2 - 8*a*b*c^3*e^5 + b^4*c*d*e^4 + 4*
a^3*b*c*d^4*e - 6*a*b^2*c^2*d*e^4 - 12*a*b^3*c*d^2*e^3 + 16*a^2*b*c^2*d^2*e
^3 + 10*a^2*b^2*c*d^3*e^2))/(c^2*d^2) - (a*e*(b^4*e*(b^2 - 4*a*c)^(1/2) - b
^5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) -
5*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c
*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e
+ b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4
*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x -
2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6
*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2
*e^2*x))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4*e*(b^2 - 4*a*c
)^(1/2) - b^5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*
c)^(1/2) - 5*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2)
+ 3*a^2*b*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^
3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(a^3*b^3*d^6 + b^3*c^3*e^6
+ b^6*d^3*e^3 + 4*a^2*c^4*d*e^5 + a^4*c^2*d^5*e + 2*b^4*c^2*d*e^5 + 2*b^5*c
*d^2*e^4 - 4*a^3*c^3*d^3*e^3 - a^4*b*c*d^6 - 3*a*b*c^4*e^6 + 9*a^2*b^2*c^2*
d^3*e^3 - 8*a*b^2*c^3*d*e^5 - 6*a*b^4*c*d^3*e^3 - 9*a*b^3*c^2*d^2*e^4 + 7*a
^2*b*c^3*d^2*e^4))/(c^4*d^4) + (a*e*x*(a^4*b^2*d^6 + 2*a^2*c^4*e^6 + b^4*c^
2*e^6 + b^6*d^2*e^4 - 4*a*b^2*c^3*e^6 - 6*a^3*c^3*d^2*e^4 + 2*a^4*c^2*d^4*e
^2 + 2*b^5*c*d*e^5 + 11*a^2*b^2*c^2*d^2*e^4 - 10*a*b^3*c^2*d*e^5 - 6*a*b^4*
c*d^2*e^4 + 10*a^2*b*c^3*d*e^5))/(c^4*d^4))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^
5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) - 5
*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*
d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b
^2)*(a*d^2 + c*e^2 - b*d*e)) - (a^5*e^5*x)/(c^3*d^3))*(b^4*e*(b^2 - 4*a*c)^
(1/2) - b^5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)
^(1/2) - 5*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) +
3*a^2*b*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a
*c^5*e^2 + 4*a^2*c^4*d^2 - b^2*c^4*e^2 - a*b^2*c^3*d^2 + b^3*c^3*d*e - 4*a*
```

```
b*c^4*d*e)) - (e^4*log(d + e*x))/(a*d^5 + c*d^3*e^2 - b*d^4*e) - (log((((a*
e*(a^3*b^3*d^6 + b^3*c^3*e^6 + b^6*d^3*e^3 + 4*a^2*c^4*d*e^5 + a^4*c^2*d^5*
e + 2*b^4*c^2*d*e^5 + 2*b^5*c*d^2*e^4 - 4*a^3*c^3*d^3*e^3 - a^4*b*c*d^6 - 3
*a*b*c^4*e^6 + 9*a^2*b^2*c^2*d^3*e^3 - 8*a*b^2*c^3*d*e^5 - 6*a*b^4*c*d^3*e^
3 - 9*a*b^3*c^2*d^2*e^4 + 7*a^2*b*c^3*d^2*e^4))/(c^4*d^4) - (((a*e*(a^2*b^3
*d^5 - 4*a*c^4*e^5 + b^2*c^3*e^5 + b^5*d^3*e^2 - 3*a^3*c^2*d^4*e + b^3*c^2*
d*e^4 + b^4*c*d^2*e^3 + 4*a^2*c^3*d^2*e^3 - 2*a^3*b*c*d^5 - 2*a*b^4*d^4*e -
 4*a*b*c^3*d*e^4 - 6*a*b^3*c*d^3*e^2 + 7*a^2*b^2*c*d^4*e - 5*a*b^2*c^2*d^2*
e^3 + 8*a^2*b*c^2*d^3*e^2))/(c^2*d^2) + (a*e*x*(2*a^3*b^2*d^5 - 3*a^4*c*d^5
 + 2*b^3*c^2*e^5 + 2*b^5*d^2*e^3 - 2*a*b^4*d^3*e^2 - 2*a^2*b^3*d^4*e + 8*a^
2*c^3*d*e^4 - 8*a^3*c^2*d^3*e^2 - 8*a*b*c^3*e^5 + b^4*c*d*e^4 + 4*a^3*b*c*d
^4*e - 6*a*b^2*c^2*d*e^4 - 12*a*b^3*c*d^2*e^3 + 16*a^2*b*c^2*d^2*e^3 + 10*a
^2*b^2*c*d^3*e^2))/(c^2*d^2) + (a*e*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*
a^3*c^2*d - a*b^4*d - 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) + 5*a^2*b^2
*c*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*d*(b^2 -
 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2
*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*
x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c
*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*
d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/
(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^5*e + b^4*e*(b^2 - 4*a*c)
^(1/2) - 4*a^3*c^2*d - a*b^4*d - 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2)
+ 5*a^2*b^2*c*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b
*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c
- b^2)*(a*d^2 + c*e^2 - b*d*e)) + (a*e*x*(a^4*b^2*d^6 + 2*a^2*c^4*e^6 + b^4
*c^2*e^6 + b^6*d^2*e^4 - 4*a*b^2*c^3*e^6 - 6*a^3*c^3*d^2*e^4 + 2*a^4*c^2*d^
4*e^2 + 2*b^5*c*d*e^5 + 11*a^2*b^2*c^2*d^2*e^4 - 10*a*b^3*c^2*d*e^5 - 6*a*b
^4*c*d^2*e^4 + 10*a^2*b*c^3*d*e^5))/(c^4*d^4))*(b^5*e + b^4*e*(b^2 - 4*a*c)
^(1/2) - 4*a^3*c^2*d - a*b^4*d - 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2)
+ 5*a^2*b^2*c*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b
*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c
- b^2)*(a*d^2 + c*e^2 - b*d*e)) + (a^4*e^4*(b^2*d^2 + c^2*e^2 - a*c*d^2 + b
*c*d*e))/(c^4*d^4) - (a^5*e^5*x)/(c^3*d^3))*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1
/2) - 4*a^3*c^2*d - a*b^4*d - 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) + 5
*a^2*b^2*c*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*
d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^5*e^2 +
 4*a^2*c^4*d^2 - b^2*c^4*e^2 - a*b^2*c^3*d^2 + b^3*c^3*d*e - 4*a*b*c^4*d*e)
) - (1/(2*c*d) - (x*(b*d + c*e))/(c^2*d^2))/x^2 + (log(x)*(c^2*e^2 - d^2*(a
*c - b^2) + b*c*d*e))/(c^3*d^3)
```

3.70 $\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 343

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = -\frac{(2ad+be)x}{a^2e^3}+\frac{x^2}{2ae^2}+\frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)}$$
$$+\frac{\left(b^5d^2-2b^4cde+8ab^2c^2de-4a^2c^3de+abc^2(5ad^2-3ce^2)-b^3c(5ad^2-ce^2)\right)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}$$
$$+\frac{d^4(3ad^2-e(4bd-5ce))\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)^2}$$
$$+\frac{\left(b^4d^2-2b^3cde+4abc^2de+ac^2(ad^2-ce^2)-b^2c(3ad^2-ce^2)\right)\log\left(c+bx+ax^2\right)}{2a^3\left(ad^2-e(bd-ce)\right)^2}$$

```
[Out] -(2*a*d+b*e)*x/a^2/e^3+1/2*x^2/a/e^2+d^5/e^4/(a*d^2-e*(b*d-c*e))/(e*x+d)+d^
4*(3*a*d^2-e*(4*b*d-5*c*e))*ln(e*x+d)/e^4/(a*d^2-e*(b*d-c*e))^2+1/2*(b^4*d^
2-2*b^3*c*d*e+4*a*b*c^2*d*e+a*c^2*(a*d^2-c*e^2)-b^2*c*(3*a*d^2-c*e^2))*ln(a
*x^2+b*x+c)/a^3/(a*d^2-e*(b*d-c*e))^2+(b^5*d^2-2*b^4*c*d*e+8*a*b^2*c^2*d*e-
4*a^2*c^3*d*e+a*b*c^2*(5*a*d^2-3*c*e^2)-b^3*c*(5*a*d^2-c*e^2))*arctanh((2*a
*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1583, 1642, 648, 632, 212, 642}

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$$
$$= \frac{(-b^2c(3ad^2-ce^2)+4abc^2de+ac^2(ad^2-ce^2)+b^4d^2-2b^3cde)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)^2}$$
$$-\frac{x(2ad+be)}{a^2e^3}$$
$$+\frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)(-4a^2c^3de-b^3c(5ad^2-ce^2)+8ab^2c^2de+abc^2(5ad^2-3ce^2)+b^5d^2-2b^4cde)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}$$
$$+\frac{d^5}{e^4(d+ex)\left(ad^2-e(bd-ce)\right)}+\frac{d^4\log(d+ex)\left(3ad^2-e(4bd-5ce)\right)}{e^4\left(ad^2-e(bd-ce)\right)^2}+\frac{x^2}{2ae^2}$$

```
[In] Int[x^3/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] -(((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2 - e*(b*d -
 c*e))*(d + e*x)) + ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^2*c^2*d*e - 4*a^2*c^3*d
*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) - b^3*c*(5*a*d^2 - c*e^2))*ArcTanh[(b + 2*
a*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2)
+ (d^4*(3*a*d^2 - e*(4*b*d - 5*c*e))*Log[d + e*x])/(e^4*(a*d^2 - e*(b*d - c
*e))^2) + ((b^4*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2*(a*d^2 - c*e^2) -
 b^2*c*(3*a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a^3*(a*d^2 - e*(b*d - c*
e))^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \frac{x^5}{(d+ex)^2\,(c+bx+ax^2)}\,dx$$

$$= \int \left(\frac{-2ad-be}{a^2e^3}+\frac{x}{ae^2}+\frac{d^5}{e^3\,(-ad^2+e(bd-ce))\,(d+ex)^2}+\frac{d^4(3ad^2-e(4bd-5ce))}{e^3\,(ad^2-e(bd-ce))^2\,(d+ex)} + \frac{c(bd-ce)\,(b^2d-2acd-bce)+(b^4d^2-2b^3cde+4abc^2de+ac^2(ad^2-ce^2)-b^2c(3ad^2-ce^2))\,x}{a^2\,(ad^2-e(bd-ce))^2\,(c+bx+ax^2)}\right.$$

$$= -\frac{(2ad+be)x}{a^2e^3}+\frac{x^2}{2ae^2}+\frac{d^5}{e^4\,(ad^2-e(bd-ce))\,(d+ex)} + \frac{d^4(3ad^2-e(4bd-5ce))\log(d+ex)}{e^4\,(ad^2-e(bd-ce))^2} + \frac{\int \frac{c(bd-ce)(b^2d-2acd-bce)+(b^4d^2-2b^3cde+4abc^2de+ac^2(ad^2-ce^2)-b^2c(3ad^2-ce^2))x}{c+bx+ax^2}\,dx}{a^2\,(ad^2-e(bd-ce))^2}$$

$$= -\frac{(2ad+be)x}{a^2e^3}+\frac{x^2}{2ae^2}+\frac{d^5}{e^4\,(ad^2-e(bd-ce))\,(d+ex)} + \frac{d^4(3ad^2-e(4bd-5ce))\log(d+ex)}{e^4\,(ad^2-e(bd-ce))^2} + \frac{(b^4d^2-2b^3cde+4abc^2de+ac^2(ad^2-ce^2)-b^2c(3ad^2-ce^2))\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2a^3\,(ad^2-e(bd-ce))^2} - \frac{(b^5d^2-2b^4cde+8ab^2c^2de-4a^2c^3de+abc^2(5ad^2-3ce^2)-b^3c(5ad^2-ce^2))\int \frac{1}{c+bx+ax^2}\,dx}{2a^3\,(ad^2-e(bd-ce))^2}$$

$$
\begin{aligned}
&= -\frac{(2ad+be)x}{a^2e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)} \\
&+ \frac{d^4(3ad^2-e(4bd-5ce))\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)^2} \\
&+ \frac{(b^4d^2-2b^3cde+4abc^2de+ac^2(ad^2-ce^2)-b^2c(3ad^2-ce^2))\log\left(c+bx+ax^2\right)}{2a^3\left(ad^2-e(bd-ce)\right)^2} \\
&+ \frac{(b^5d^2-2b^4cde+8ab^2c^2de-4a^2c^3de+abc^2(5ad^2-3ce^2)-b^3c(5ad^2-ce^2))\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\right.}{a^3\left(ad^2-e(bd-ce)\right)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(2ad+be)x}{a^2e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)} \\
&+ \frac{(b^5d^2-2b^4cde+8ab^2c^2de-4a^2c^3de+abc^2(5ad^2-3ce^2)-b^3c(5ad^2-ce^2))\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} \\
&+ \frac{d^4(3ad^2-e(4bd-5ce))\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)^2} \\
&+ \frac{(b^4d^2-2b^3cde+4abc^2de+ac^2(ad^2-ce^2)-b^2c(3ad^2-ce^2))\log\left(c+bx+ax^2\right)}{2a^3\left(ad^2-e(bd-ce)\right)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.99

$$
\begin{aligned}
\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx &= -\frac{(2ad+be)x}{a^2e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4\left(ad^2+e(-bd+ce)\right)(d+ex)} \\
&- \frac{(b^5d^2-2b^4cde+8ab^2c^2de-4a^2c^3de+abc^2(5ad^2-3ce^2)+b^3c(-5ad^2+ce^2))\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a^3\sqrt{-b^2+4ac}\left(ad^2+e(-bd+ce)\right)^2} \\
&+ \frac{(3ad^6+d^4e(-4bd+5ce))\log(d+ex)}{e^4\left(ad^2+e(-bd+ce)\right)^2} \\
&+ \frac{(b^4d^2-2b^3cde+4abc^2de+ac^2(ad^2-ce^2)+b^2c(-3ad^2+ce^2))\log(c+x(b+ax))}{2a^3\left(ad^2+e(-bd+ce)\right)^2}
\end{aligned}
$$

```
[In] Integrate[x^3/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] -(((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2 + e*(-(b*d
) + c*e))*(d + e*x)) - ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^2*c^2*d*e - 4*a^2*c^
3*d*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) + b^3*c*(-5*a*d^2 + c*e^2))*ArcTan[(b +
 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^3*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c
*e))^2) + ((3*a*d^6 + d^4*e*(-4*b*d + 5*c*e))*Log[d + e*x])/(e^4*(a*d^2 + e
*(-(b*d) + c*e))^2) + ((b^4*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2*(a*d^
2 - c*e^2) + b^2*c*(-3*a*d^2 + c*e^2))*Log[c + x*(b + a*x)])/(2*a^3*(a*d^2
+ e*(-(b*d) + c*e))^2)
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.05

method	result
default	$-\frac{-\frac{1}{2}ae\,x^2+2adx+bex}{e^3a^2}+\frac{\frac{\left(a^2c^2d^2-3a\,b^2c\,d^2+4ab\,c^2de-a\,c^3e^2+b^4d^2-2b^3cde+b^2c^2e^2\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(-2b\,c^2d^2a+2a\,c^3de+b^3c\,d^2-2b^2c\right.}{}}{\left(a\,d^2-bde+c\,e^2\right)^2}$
risch	Expression too large to display

```
[In] int(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^3/a^2*(-1/2*a*e*x^2+2*a*d*x+b*e*x)+1/(a*d^2-b*d*e+c*e^2)^2/a^2*(1/2*(a
^2*c^2*d^2-3*a*b^2*c*d^2+4*a*b*c^2*d*e-a*c^3*e^2+b^4*d^2-2*b^3*c*d*e+b^2*c^
2*e^2)/a*ln(a*x^2+b*x+c)+2*(-2*b*c^2*d^2*a+2*a*c^3*d*e+b^3*c*d^2-2*b^2*c^2*
d*e+b*c^3*e^2-1/2*(a^2*c^2*d^2-3*a*b^2*c*d^2+4*a*b*c^2*d*e-a*c^3*e^2+b^4*d^
2-2*b^3*c*d*e+b^2*c^2*e^2)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b
^2)^(1/2)))+1/e^4*d^4*(3*a*d^2-4*b*d*e+5*c*e^2)/(a*d^2-b*d*e+c*e^2)^2*ln(e*
x+d)+1/e^4*d^5/(a*d^2-b*d*e+c*e^2)/(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1341 vs. 2(335) = 670.

Time = 64.45 (sec) , antiderivative size = 2703, normalized size of antiderivative = 7.88

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx=\text{Too large to display}$$

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^4*b^2 - 4*a^5*c)*d^7 - 2*(a^3*b^3 - 4*a^4*b*c)*d^6*e + 2*(a^3*b^
2*c - 4*a^4*c^2)*d^5*e^2 + ((a^4*b^2 - 4*a^5*c)*d^4*e^3 - 2*(a^3*b^3 - 4*a^
4*b*c)*d^3*e^4 + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^5 - 2*(a^2*b^3*c
 - 4*a^3*b*c^2)*d*e^6 + (a^2*b^2*c^2 - 4*a^3*c^3)*e^7)*x^3 - (3*(a^4*b^2 -
4*a^5*c)*d^5*e^2 - 4*(a^3*b^3 - 4*a^4*b*c)*d^4*e^3 - (a^2*b^4 - 10*a^3*b^2*
c + 24*a^4*c^2)*d^3*e^4 + 2*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^5 - (
4*a*b^4*c - 19*a^2*b^2*c^2 + 12*a^3*c^3)*d*e^6 + 2*(a*b^3*c^2 - 4*a^2*b*c^3
)*e^7)*x^2 - ((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3*e^4 - 2*(b^4*c - 4*a*b^2*
c^2 + 2*a^2*c^3)*d^2*e^5 + (b^3*c^2 - 3*a*b*c^3)*d*e^6 + ((b^5 - 5*a*b^3*c
+ 5*a^2*b*c^2)*d^2*e^5 - 2*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^6 + (b^3*c
^2 - 3*a*b*c^3)*e^7)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 -
2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) - 2*(2*(a^4*b^2 -
 4*a^5*c)*d^6*e - 3*(a^3*b^3 - 4*a^4*b*c)*d^5*e^2 + 4*(a^3*b^2*c - 4*a^4*c^
2)*d^4*e^3 + (a*b^5 - 6*a^2*b^3*c + 8*a^3*b*c^2)*d^3*e^4 - 2*(a*b^4*c - 5*a
^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^5 + (a*b^3*c^2 - 4*a^2*b*c^3)*d*e^6)*x + ((b^
```

```
6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^4 - 2*(b^5*c - 6*a*b^3*c^
2 + 8*a^2*b*c^3)*d^2*e^5 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*e^6 + ((b^
6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^5 - 2*(b^5*c - 6*a*b^3*c^
2 + 8*a^2*b*c^3)*d*e^6 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^7)*x)*log(a*
x^2 + b*x + c) + 2*(3*(a^4*b^2 - 4*a^5*c)*d^7 - 4*(a^3*b^3 - 4*a^4*b*c)*d^6
*e + 5*(a^3*b^2*c - 4*a^4*c^2)*d^5*e^2 + (3*(a^4*b^2 - 4*a^5*c)*d^6*e - 4*(
a^3*b^3 - 4*a^4*b*c)*d^5*e^2 + 5*(a^3*b^2*c - 4*a^4*c^2)*d^4*e^3)*x)*log(e*
x + d))/((a^5*b^2 - 4*a^6*c)*d^5*e^4 - 2*(a^4*b^3 - 4*a^5*b*c)*d^4*e^5 + (a
^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^3*e^6 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2
*e^7 + (a^3*b^2*c^2 - 4*a^4*c^3)*d*e^8 + ((a^5*b^2 - 4*a^6*c)*d^4*e^5 - 2*(
a^4*b^3 - 4*a^5*b*c)*d^3*e^6 + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^2*e^7
 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d*e^8 + (a^3*b^2*c^2 - 4*a^4*c^3)*e^9)*x), 1/
2*(2*(a^4*b^2 - 4*a^5*c)*d^7 - 2*(a^3*b^3 - 4*a^4*b*c)*d^6*e + 2*(a^3*b^2*c
 - 4*a^4*c^2)*d^5*e^2 + ((a^4*b^2 - 4*a^5*c)*d^4*e^3 - 2*(a^3*b^3 - 4*a^4*b
*c)*d^3*e^4 + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^5 - 2*(a^2*b^3*c -
4*a^3*b*c^2)*d*e^6 + (a^2*b^2*c^2 - 4*a^3*c^3)*e^7)*x^3 - (3*(a^4*b^2 - 4*a
^5*c)*d^5*e^2 - 4*(a^3*b^3 - 4*a^4*b*c)*d^4*e^3 - (a^2*b^4 - 10*a^3*b^2*c +
 24*a^4*c^2)*d^3*e^4 + 2*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^5 - (4*a
*b^4*c - 19*a^2*b^2*c^2 + 12*a^3*c^3)*d*e^6 + 2*(a*b^3*c^2 - 4*a^2*b*c^3)*e
^7)*x^2 + 2*((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3*e^4 - 2*(b^4*c - 4*a*b^2*c
^2 + 2*a^2*c^3)*d^2*e^5 + (b^3*c^2 - 3*a*b*c^3)*d*e^6 + ((b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*d^2*e^5 - 2*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^6 + (b^3*c^
2 - 3*a*b*c^3)*e^7)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x
 + b)/(b^2 - 4*a*c)) - 2*(2*(a^4*b^2 - 4*a^5*c)*d^6*e - 3*(a^3*b^3 - 4*a^4*
b*c)*d^5*e^2 + 4*(a^3*b^2*c - 4*a^4*c^2)*d^4*e^3 + (a*b^5 - 6*a^2*b^3*c + 8
*a^3*b*c^2)*d^3*e^4 - 2*(a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^5 + (a*
b^3*c^2 - 4*a^2*b*c^3)*d*e^6)*x + ((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^
3*c^3)*d^3*e^4 - 2*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d^2*e^5 + (b^4*c^2 -
 5*a*b^2*c^3 + 4*a^2*c^4)*d*e^6 + ((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^
3*c^3)*d^2*e^5 - 2*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*e^6 + (b^4*c^2 - 5
*a*b^2*c^3 + 4*a^2*c^4)*e^7)*x)*log(a*x^2 + b*x + c) + 2*(3*(a^4*b^2 - 4*a^
5*c)*d^7 - 4*(a^3*b^3 - 4*a^4*b*c)*d^6*e + 5*(a^3*b^2*c - 4*a^4*c^2)*d^5*e^
2 + (3*(a^4*b^2 - 4*a^5*c)*d^6*e - 4*(a^3*b^3 - 4*a^4*b*c)*d^5*e^2 + 5*(a^3
*b^2*c - 4*a^4*c^2)*d^4*e^3)*x)*log(e*x + d))/((a^5*b^2 - 4*a^6*c)*d^5*e^4
 - 2*(a^4*b^3 - 4*a^5*b*c)*d^4*e^5 + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^3
*e^6 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2*e^7 + (a^3*b^2*c^2 - 4*a^4*c^3)*d*e^
8 + ((a^5*b^2 - 4*a^6*c)*d^4*e^5 - 2*(a^4*b^3 - 4*a^5*b*c)*d^3*e^6 + (a^3*b
^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^2*e^7 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d*e^8 +
 (a^3*b^2*c^2 - 4*a^4*c^3)*e^9)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.68

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \frac{d^5e^4}{(ad^2e^8-bde^9+ce^{10})(ex+d)} + \frac{(b^4d^2-3\,ab^2cd^2+a^2c^2d^2-2\,b^3cde+4\,abc^2de+b^2c^2e^2-ac^3e^2)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{bde}{(ex+d}\right.}{2\,(a^5d^4-2\,a^4bd^3e+a^3b^2d^2e^2+2\,a^4cd^2e^2-2\,a^3bcde^3+a^3c^2e^4)} + \frac{(b^5d^2e^2-5\,ab^3cd^2e^2+5\,a^2bc^2d^2e^2-2\,b^4cde^3+8\,ab^2c^2de^3-4\,a^2c^3de^3+b^3c^2e^4-3\,abc^3e^4)\arctan\left(-\frac{2\,ad}{}\right.}{(a^5d^4-2\,a^4bd^3e+a^3b^2d^2e^2+2\,a^4cd^2e^2-2\,a^3bcde^3+a^3c^2e^4)\sqrt{-b^2+4\,ac}e^2} + \frac{\left(a^2-\frac{2\,(3\,a^2de+abe^2)}{(ex+d)e}\right)(ex+d)^2}{2\,a^3e^4} - \frac{(3\,a^2d^2+2\,abde+b^2e^2-ace^2)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{a^3e^4}$$

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] d^5*e^4/((a*d^2*e^8 - b*d*e^9 + c*e^10)*(e*x + d)) + 1/2*(b^4*d^2 - 3*a*b^2
*c*d^2 + a^2*c^2*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + b^2*c^2*e^2 - a*c^3*e^
```

```
2)*log(-a + 2*a*d/(e*x + d) - a*d^2/(e*x + d)^2 - b*e/(e*x + d) + b*d*e/(e*
x + d)^2 - c*e^2/(e*x + d)^2)/(a^5*d^4 - 2*a^4*b*d^3*e + a^3*b^2*d^2*e^2 +
2*a^4*c*d^2*e^2 - 2*a^3*b*c*d*e^3 + a^3*c^2*e^4) + (b^5*d^2*e^2 - 5*a*b^3*c
*d^2*e^2 + 5*a^2*b*c^2*d^2*e^2 - 2*b^4*c*d*e^3 + 8*a*b^2*c^2*d*e^3 - 4*a^2*
c^3*d*e^3 + b^3*c^2*e^4 - 3*a*b*c^3*e^4)*arctan(-(2*a*d - 2*a*d^2/(e*x + d)
 - b*e + 2*b*d*e/(e*x + d) - 2*c*e^2/(e*x + d))/(sqrt(-b^2 + 4*a*c)*e))/((a
^5*d^4 - 2*a^4*b*d^3*e + a^3*b^2*d^2*e^2 + 2*a^4*c*d^2*e^2 - 2*a^3*b*c*d*e^
3 + a^3*c^2*e^4)*sqrt(-b^2 + 4*a*c)*e^2) + 1/2*(a^2 - 2*(3*a^2*d*e + a*b*e^
2)/((e*x + d)*e))*(e*x + d)^2/(a^3*e^4) - (3*a^2*d^2 + 2*a*b*d*e + b^2*e^2
- a*c*e^2)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/(a^3*e^4)
```

Mupad [B] (verification not implemented)

Time = 12.48 (sec) , antiderivative size = 3503, normalized size of antiderivative = 10.21

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^3/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(d + e*x)*(3*a*d^6 + 5*c*d^4*e^2 - 4*b*d^5*e))/(c^2*e^8 + a^2*d^4*e^4 +
 b^2*d^2*e^6 - 2*b*c*d*e^7 - 2*a*b*d^3*e^5 + 2*a*c*d^2*e^6) - (log(12*a^5*c
*d^8 - 2*a*c^5*e^8 - 3*a^4*b^2*d^8 + b^2*c^4*e^8 + b^6*d^4*e^4 + 4*a^3*b^3*
d^7*e - 4*b^3*c^3*d*e^7 - 4*b^5*c*d^3*e^5 + b^5*d^4*e^4*(b^2 - 4*a*c)^(1/2)
 + 12*a^2*c^4*d^2*e^6 - 22*a^3*c^3*d^4*e^4 + 8*a^4*c^2*d^6*e^2 + 6*b^4*c^2*
d^2*e^6 - 3*a^4*b*d^8*(b^2 - 4*a*c)^(1/2) + b*c^4*e^8*(b^2 - 4*a*c)^(1/2) -
 6*a^5*d^8*x*(b^2 - 4*a*c)^(1/2) + 12*a*b*c^4*d*e^7 - 16*a^4*b*c*d^7*e - 4*
a^2*c^3*d^3*e^5*(b^2 - 4*a*c)^(1/2) + 20*a^3*c^2*d^5*e^3*(b^2 - 4*a*c)^(1/2
) + 6*b^3*c^2*d^2*e^6*(b^2 - 4*a*c)^(1/2) + a*b*c^4*e^8*x + 24*a^5*c*d^7*e*
x + 14*a^2*b^2*c^2*d^4*e^4 + 4*a*c^4*d*e^7*(b^2 - 4*a*c)^(1/2) + 12*a^4*c*d
^7*e*(b^2 - 4*a*c)^(1/2) + a*c^4*e^8*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^4*c*d^4*
e^4 + a*b^5*d^4*e^4*x - 6*a^4*b^2*d^7*e*x + 8*a^2*c^4*d*e^7*x + 4*a^3*b^2*d
^7*e*(b^2 - 4*a*c)^(1/2) - 4*b^2*c^3*d*e^7*(b^2 - 4*a*c)^(1/2) - 4*b^4*c*d^
3*e^5*(b^2 - 4*a*c)^(1/2) - 24*a*b^2*c^3*d^2*e^6 + 20*a*b^3*c^2*d^3*e^5 - 2
0*a^2*b*c^3*d^3*e^5 - 4*a^2*b^3*c*d^5*e^3 + 16*a^3*b*c^2*d^5*e^3 - 2*a^3*b^
2*c*d^6*e^2 - 4*a^2*b^4*d^5*e^3*x + 11*a^3*b^3*d^6*e^2*x - 8*a^3*c^3*d^3*e^
5*x + 40*a^4*c^2*d^5*e^3*x - 12*a*b*c^3*d^2*e^6*(b^2 - 4*a*c)^(1/2) - 4*a*b
^3*c*d^4*e^4*(b^2 - 4*a*c)^(1/2) - 24*a^3*b*c*d^6*e^2*(b^2 - 4*a*c)^(1/2) +
 a*b^4*d^4*e^4*x*(b^2 - 4*a*c)^(1/2) - 4*a^4*c*d^6*e^2*x*(b^2 - 4*a*c)^(1/2
) + 6*a*b^3*c^2*d^2*e^6*x - 18*a^2*b*c^3*d^2*e^6*x - 15*a^3*b*c^2*d^4*e^4*x
 + 6*a^3*b^2*c*d^5*e^3*x + 12*a*b^2*c^2*d^3*e^5*(b^2 - 4*a*c)^(1/2) - 2*a^2
*b*c^2*d^4*e^4*(b^2 - 4*a*c)^(1/2) + 4*a^2*b^2*c*d^5*e^3*(b^2 - 4*a*c)^(1/2
) + 4*a^2*b^3*d^5*e^3*x*(b^2 - 4*a*c)^(1/2) - 11*a^3*b^2*d^6*e^2*x*(b^2 - 4
*a*c)^(1/2) - 6*a^2*c^3*d^2*e^6*x*(b^2 - 4*a*c)^(1/2) + 11*a^3*c^2*d^4*e^4*
x*(b^2 - 4*a*c)^(1/2) + 16*a^2*b^2*c^2*d^3*e^5*x + 14*a^4*b*d^7*e*x*(b^2 -
```

4*a*c)^(1/2) - 4*a*b^2*c^3*d*e^7*x - 4*a*b^4*c*d^3*e^5*x - 44*a^4*b*c*d^6*e
^2*x - 4*a*b*c^3*d*e^7*x*(b^2 - 4*a*c)^(1/2) - 4*a*b^3*c*d^3*e^5*x*(b^2 - 4
*a*c)^(1/2) + 2*a^3*b*c*d^5*e^3*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c^2*d^2*e^6
x(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d^3*e^5*x*(b^2 - 4*a*c)^(1/2) - 8*a^2*
b^2*c*d^4*e^4*x*(b^2 - 4*a*c)^(1/2))*(b^6*d^2 + b^5*d^2*(b^2 - 4*a*c)^(1/2)
 - 4*a^3*c^3*d^2 + 4*a^2*c^4*e^2 + b^4*c^2*e^2 - 5*a*b^2*c^3*e^2 + b^3*c^2*
e^2*(b^2 - 4*a*c)^(1/2) - 2*b^5*c*d*e + 13*a^2*b^2*c^2*d^2 - 7*a*b^4*c*d^2
+ 12*a*b^3*c^2*d*e - 16*a^2*b*c^3*d*e - 5*a*b^3*c*d^2*(b^2 - 4*a*c)^(1/2) -
 3*a*b*c^3*e^2*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d*e*(b^2 - 4*a*c)^(1/2) + 5*
a^2*b*c^2*d^2*(b^2 - 4*a*c)^(1/2) - 2*b^4*c*d*e*(b^2 - 4*a*c)^(1/2) + 8*a*b
^2*c^2*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^6*c*d^4 - a^5*b^2*d^4 + 4*a^4*c^3*
e^4 + 2*a^4*b^3*d^3*e - a^3*b^2*c^2*e^4 - a^3*b^4*d^2*e^2 + 8*a^5*c^2*d^2*e
^2 - 8*a^5*b*c*d^3*e + 2*a^3*b^3*c*d*e^3 - 8*a^4*b*c^2*d*e^3 + 2*a^4*b^2*c*
d^2*e^2)) - (log(2*a*c^5*e^8 - 12*a^5*c*d^8 + 3*a^4*b^2*d^8 - b^2*c^4*e^8 -
 b^6*d^4*e^4 - 4*a^3*b^3*d^7*e + 4*b^3*c^3*d*e^7 + 4*b^5*c*d^3*e^5 + b^5*d^
4*e^4*(b^2 - 4*a*c)^(1/2) - 12*a^2*c^4*d^2*e^6 + 22*a^3*c^3*d^4*e^4 - 8*a^4
*c^2*d^6*e^2 - 6*b^4*c^2*d^2*e^6 - 3*a^4*b*d^8*(b^2 - 4*a*c)^(1/2) + b*c^4*
e^8*(b^2 - 4*a*c)^(1/2) - 6*a^5*d^8*x*(b^2 - 4*a*c)^(1/2) - 12*a*b*c^4*d*e^
7 + 16*a^4*b*c*d^7*e - 4*a^2*c^3*d^3*e^5*(b^2 - 4*a*c)^(1/2) + 20*a^3*c^2*d
^5*e^3*(b^2 - 4*a*c)^(1/2) + 6*b^3*c^2*d^2*e^6*(b^2 - 4*a*c)^(1/2) - a*b*c^
4*e^8*x - 24*a^5*c*d^7*e*x - 14*a^2*b^2*c^2*d^4*e^4 + 4*a*c^4*d*e^7*(b^2 -
4*a*c)^(1/2) + 12*a^4*c*d^7*e*(b^2 - 4*a*c)^(1/2) + a*c^4*e^8*x*(b^2 - 4*a*
c)^(1/2) + 6*a*b^4*c*d^4*e^4 - a*b^5*d^4*e^4*x + 6*a^4*b^2*d^7*e*x - 8*a^2*
c^4*d*e^7*x + 4*a^3*b^2*d^7*e*(b^2 - 4*a*c)^(1/2) - 4*b^2*c^3*d*e^7*(b^2 -
4*a*c)^(1/2) - 4*b^4*c*d^3*e^5*(b^2 - 4*a*c)^(1/2) + 24*a*b^2*c^3*d^2*e^6 -
 20*a*b^3*c^2*d^3*e^5 + 20*a^2*b*c^3*d^3*e^5 + 4*a^2*b^3*c*d^5*e^3 - 16*a^3
*b*c^2*d^5*e^3 + 2*a^3*b^2*c*d^6*e^2 + 4*a^2*b^4*d^5*e^3*x - 11*a^3*b^3*d^6
*e^2*x + 8*a^3*c^3*d^3*e^5*x - 40*a^4*c^2*d^5*e^3*x - 12*a*b*c^3*d^2*e^6*(b
^2 - 4*a*c)^(1/2) - 4*a*b^3*c*d^4*e^4*(b^2 - 4*a*c)^(1/2) - 24*a^3*b*c*d^6*
e^2*(b^2 - 4*a*c)^(1/2) + a*b^4*d^4*e^4*x*(b^2 - 4*a*c)^(1/2) - 4*a^4*c*d^6
*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c^2*d^2*e^6*x + 18*a^2*b*c^3*d^2*e^6*x
 + 15*a^3*b*c^2*d^4*e^4*x - 6*a^3*b^2*c*d^5*e^3*x + 12*a*b^2*c^2*d^3*e^5*(b
^2 - 4*a*c)^(1/2) - 2*a^2*b*c^2*d^4*e^4*(b^2 - 4*a*c)^(1/2) + 4*a^2*b^2*c*d
^5*e^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*b^3*d^5*e^3*x*(b^2 - 4*a*c)^(1/2) - 11*a
^3*b^2*d^6*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a^2*c^3*d^2*e^6*x*(b^2 - 4*a*c)^(1
/2) + 11*a^3*c^2*d^4*e^4*x*(b^2 - 4*a*c)^(1/2) - 16*a^2*b^2*c^2*d^3*e^5*x +
 14*a^4*b*d^7*e*x*(b^2 - 4*a*c)^(1/2) + 4*a*b^2*c^3*d*e^7*x + 4*a*b^4*c*d^3
*e^5*x + 44*a^4*b*c*d^6*e^2*x - 4*a*b*c^3*d*e^7*x*(b^2 - 4*a*c)^(1/2) - 4*a
*b^3*c*d^3*e^5*x*(b^2 - 4*a*c)^(1/2) + 2*a^3*b*c*d^5*e^3*x*(b^2 - 4*a*c)^(1
/2) + 6*a*b^2*c^2*d^2*e^6*x*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d^3*e^5*x*(b^
2 - 4*a*c)^(1/2) - 8*a^2*b^2*c*d^4*e^4*x*(b^2 - 4*a*c)^(1/2))*(b^6*d^2 - b^
5*d^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^3*d^2 + 4*a^2*c^4*e^2 + b^4*c^2*e^2 - 5
*a*b^2*c^3*e^2 - b^3*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*b^5*c*d*e + 13*a^2*b^2
*c^2*d^2 - 7*a*b^4*c*d^2 + 12*a*b^3*c^2*d*e - 16*a^2*b*c^3*d*e + 5*a*b^3*c*
d^2*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^3*e^2*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^3*d*e

```
*(b^2 - 4*a*c)^(1/2) - 5*a^2*b*c^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*b^4*c*d*e*(b
^2 - 4*a*c)^(1/2) - 8*a*b^2*c^2*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^6*c*d^4 -
 a^5*b^2*d^4 + 4*a^4*c^3*e^4 + 2*a^4*b^3*d^3*e - a^3*b^2*c^2*e^4 - a^3*b^4*
d^2*e^2 + 8*a^5*c^2*d^2*e^2 - 8*a^5*b*c*d^3*e + 2*a^3*b^3*c*d*e^3 - 8*a^4*b
*c^2*d*e^3 + 2*a^4*b^2*c*d^2*e^2)) + x^2/(2*a*e^2) - (x*(b*e^2 + 2*a*d*e))/
(a^2*e^4) + (a^2*d^5)/(e*(a^2*d*e^3 + a^2*e^4*x)*(a*d^2 + c*e^2 - b*d*e))
```

3.71 $\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 274

$$\begin{aligned}\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx &= \frac{x}{ae^2}-\frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)} \\ &\quad -\frac{\left(b^4d^2-2b^3cde+6abc^2de+2ac^2(ad^2-ce^2)-b^2c(4ad^2-ce^2)\right)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} \\ &\quad -\frac{d^3(2ad^2-e(3bd-4ce))\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)^2}-\frac{(bd-ce)\left(b^2d-2acd-bce\right)\log\left(c+bx+ax^2\right)}{2a^2\left(ad^2-e(bd-ce)\right)^2}\end{aligned}$$

```
[Out] x/a/e^2-d^4/e^3/(a*d^2-e*(b*d-c*e))/(e*x+d)-d^3*(2*a*d^2-e*(3*b*d-4*c*e))*l
n(e*x+d)/e^3/(a*d^2-e*(b*d-c*e))^2-1/2*(b*d-c*e)*(-2*a*c*d+b^2*d-b*c*e)*ln(
a*x^2+b*x+c)/a^2/(a*d^2-e*(b*d-c*e))^2-(b^4*d^2-2*b^3*c*d*e+6*a*b*c^2*d*e+2
*a*c^2*(a*d^2-c*e^2)-b^2*c*(4*a*d^2-c*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^
(1/2))/a^2/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used

$= \{1583, 1642, 648, 632, 212, 642\}$

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$$
$$= -\frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(-b^2c(4ad^2-ce^2)+6abc^2de+2ac^2(ad^2-ce^2)+b^4d^2-2b^3cde\right)}{a^2\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2}$$
$$-\frac{(bd-ce)(-2acd+b^2d-bce)\log(ax^2+bx+c)}{2a^2(ad^2-e(bd-ce))^2}$$
$$-\frac{d^4}{e^3(d+ex)(ad^2-e(bd-ce))}-\frac{d^3\log(d+ex)(2ad^2-e(3bd-4ce))}{e^3(ad^2-e(bd-ce))^2}+\frac{x}{ae^2}$$

```
[In] Int[x^2/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] x/(a*e^2) - d^4/(e^3*(a*d^2 - e*(b*d - c*e))*(d + e*x)) - ((b^4*d^2 - 2*b^3
*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) - b^2*c*(4*a*d^2 - c*e^2))
*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*
(b*d - c*e))^2) - (d^3*(2*a*d^2 - e*(3*b*d - 4*c*e))*Log[d + e*x])/(e^3*(a*
d^2 - e*(b*d - c*e))^2) - ((b*d - c*e)*(b^2*d - 2*a*c*d - b*c*e)*Log[c + b*
x + a*x^2])/(2*a^2*(a*d^2 - e*(b*d - c*e))^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^4}{(d+ex)^2\,(c+bx+ax^2)}\,dx \\
&= \int \left(\frac{1}{ae^2}+\frac{d^4}{e^2\,(ad^2-e(bd-ce))\,(d+ex)^2}+\frac{d^3(-2ad^2+e(3bd-4ce))}{e^2\,(ad^2-e(bd-ce))^2\,(d+ex)} \right. \\
&\qquad \left. +\frac{-c(b^2d^2-2bcde-c(ad^2-ce^2))-(bd-ce)\,(b^2d-2acd-bce)\,x}{a\,(ad^2-e(bd-ce))^2\,(c+bx+ax^2)}\right)dx \\
&= \frac{x}{ae^2}-\frac{d^4}{e^3\,(ad^2-e(bd-ce))\,(d+ex)}-\frac{d^3(2ad^2-e(3bd-4ce))\log(d+ex)}{e^3\,(ad^2-e(bd-ce))^2} \\
&\quad +\frac{\int \frac{-c(b^2d^2-2bcde-c(ad^2-ce^2))-(bd-ce)(b^2d-2acd-bce)x}{c+bx+ax^2}\,dx}{a\,(ad^2-e(bd-ce))^2} \\
&= \frac{x}{ae^2}-\frac{d^4}{e^3\,(ad^2-e(bd-ce))\,(d+ex)}-\frac{d^3(2ad^2-e(3bd-4ce))\log(d+ex)}{e^3\,(ad^2-e(bd-ce))^2} \\
&\quad -\frac{((bd-ce)\,(b^2d-2acd-bce))\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2a^2\,(ad^2-e(bd-ce))^2} \\
&\quad +\frac{(b^4d^2-2b^3cde+6abc^2de+2ac^2(ad^2-ce^2)-b^2c(4ad^2-ce^2))\int \frac{1}{c+bx+ax^2}\,dx}{2a^2\,(ad^2-e(bd-ce))^2} \\
&= \frac{x}{ae^2}-\frac{d^4}{e^3\,(ad^2-e(bd-ce))\,(d+ex)}-\frac{d^3(2ad^2-e(3bd-4ce))\log(d+ex)}{e^3\,(ad^2-e(bd-ce))^2} \\
&\quad -\frac{(bd-ce)\,(b^2d-2acd-bce)\log\left(c+bx+ax^2\right)}{2a^2\,(ad^2-e(bd-ce))^2} \\
&\quad -\frac{(b^4d^2-2b^3cde+6abc^2de+2ac^2(ad^2-ce^2)-b^2c(4ad^2-ce^2))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2ax\right)}{a^2\,(ad^2-e(bd-ce))^2}
\end{aligned}$$

$$= \frac{x}{ae^2} - \frac{d^4}{e^3 (ad^2 - e(bd - ce)) (d + ex)} - \frac{(b^4d^2 - 2b^3cde + 6abc^2de + 2ac^2(ad^2 - ce^2) - b^2c(4ad^2 - ce^2)) \tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2 - 4ac}\,(ad^2 - e(bd - ce))^2} - \frac{d^3(2ad^2 - e(3bd - 4ce)) \log(d + ex)}{e^3 (ad^2 - e(bd - ce))^2} - \frac{(bd - ce)(b^2d - 2acd - bce) \log(c + bx + ax^2)}{2a^2 (ad^2 - e(bd - ce))^2}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d + ex)^2}\,dx = \frac{x}{ae^2} - \frac{d^4}{e^3 (ad^2 + e(-bd + ce)) (d + ex)} + \frac{(b^4d^2 - 2b^3cde + 6abc^2de + 2ac^2(ad^2 - ce^2) + b^2c(-4ad^2 + ce^2)) \arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a^2\sqrt{-b^2 + 4ac}\,(ad^2 + e(-bd + ce))^2} - \frac{(2ad^5 + d^3e(-3bd + 4ce)) \log(d + ex)}{e^3 (ad^2 + e(-bd + ce))^2} + \frac{(bd - ce)(-b^2d + 2acd + bce) \log(c + x(b + ax))}{2a^2 (ad^2 + e(-bd + ce))^2}$$

```
[In] Integrate[x^2/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] x/(a*e^2) - d^4/(e^3*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) + ((b^4*d^2 - 2*
b^3*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) + b^2*c*(-4*a*d^2 + c*e
^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^2*Sqrt[-b^2 + 4*a*c]*(a*d^2
 + e*(-(b*d) + c*e))^2) - ((2*a*d^5 + d^3*e*(-3*b*d + 4*c*e))*Log[d + e*x])
/(e^3*(a*d^2 + e*(-(b*d) + c*e))^2) + ((b*d - c*e)*(-(b^2*d) + 2*a*c*d + b*
c*e)*Log[c + x*(b + a*x)])/(2*a^2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.06

method	result
default	$\frac{x}{a\,e^{2}}+\frac{\frac{\left(2 b\,d^{2} c a-2 a\,c^{2} d e-b^{3} d^{2}+2 b^{2} c d e-b\,c^{2} e^{2}\right) \ln \left(a\,x^{2}+b x+c\right)}{2 a}+\frac{2 \left(a\,c^{2} d^{2}-b^{2} c\,d^{2}+2 e d\,c^{2} b-c^{3} e^{2}-\frac{\left(2 b\,d^{2} c a-2 a\,c^{2} d e-b^{3} d^{2}+2 b^{2} c d e-b\,c^{2} e^{2}\right) b}{2 a}\right.}{\sqrt{4 a c-b^{2}}}}{\left(a\,d^{2}-b d e+c\,e^{2}\right)^{2} a}$
risch	Expression too large to display

```
[In] int(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/a/e^2+1/(a*d^2-b*d*e+c*e^2)^2/a*(1/2*(2*a*b*c*d^2-2*a*c^2*d*e-b^3*d^2+2*b
^2*c*d*e-b*c^2*e^2)/a*ln(a*x^2+b*x+c)+2*(a*c^2*d^2-b^2*c*d^2+2*e*d*c^2*b-c^
3*e^2-1/2*(2*a*b*c*d^2-2*a*c^2*d*e-b^3*d^2+2*b^2*c*d*e-b*c^2*e^2)*b/a)/(4*a
*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))-1/e^3*d^4/(a*d^2-b*d*e+c
*e^2)/(e*x+d)-1/e^3*d^3*(2*a*d^2-3*b*d*e+4*c*e^2)/(a*d^2-b*d*e+c*e^2)^2*ln(
e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1060 vs. 2(268) = 536.

Time = 26.47 (sec) , antiderivative size = 2139, normalized size of antiderivative = 7.81

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^3*b^2 - 4*a^4*c)*d^6 - 2*(a^2*b^3 - 4*a^3*b*c)*d^5*e + 2*(a^2*b
^2*c - 4*a^3*c^2)*d^4*e^2 - 2*((a^3*b^2 - 4*a^4*c)*d^4*e^2 - 2*(a^2*b^3 - 4
*a^3*b*c)*d^3*e^3 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^4 - 2*(a*b^3*c
- 4*a^2*b*c^2)*d*e^5 + (a*b^2*c^2 - 4*a^2*c^3)*e^6)*x^2 + ((b^4 - 4*a*b^2*c
 + 2*a^2*c^2)*d^3*e^3 - 2*(b^3*c - 3*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 2*a*c^3)
*d*e^5 + ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^2*e^4 - 2*(b^3*c - 3*a*b*c^2)*d*e
^5 + (b^2*c^2 - 2*a*c^3)*e^6)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x
 + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) - 2*((a^
3*b^2 - 4*a^4*c)*d^5*e - 2*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 + (a*b^4 - 2*a^2*b
^2*c - 8*a^3*c^2)*d^3*e^3 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2*e^4 + (a*b^2*c^2
- 4*a^2*c^3)*d*e^5)*x + ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^3*e^3 - 2*(b^4*c
 - 5*a*b^2*c^2 + 4*a^2*c^3)*d^2*e^4 + (b^3*c^2 - 4*a*b*c^3)*d*e^5 + ((b^5 -
 6*a*b^3*c + 8*a^2*b*c^2)*d^2*e^4 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e
^5 + (b^3*c^2 - 4*a*b*c^3)*e^6)*x)*log(a*x^2 + b*x + c) + 2*(2*(a^3*b^2 - 4
*a^4*c)*d^6 - 3*(a^2*b^3 - 4*a^3*b*c)*d^5*e + 4*(a^2*b^2*c - 4*a^3*c^2)*d^4
*e^2 + (2*(a^3*b^2 - 4*a^4*c)*d^5*e - 3*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 + 4*(
```

```
a^2*b^2*c - 4*a^3*c^2)*d^3*e^3)*x)*log(e*x + d))/((a^4*b^2 - 4*a^5*c)*d^5*e
^3 - 2*(a^3*b^3 - 4*a^4*b*c)*d^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*
d^3*e^5 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e^6 + (a^2*b^2*c^2 - 4*a^3*c^3)*d
*e^7 + ((a^4*b^2 - 4*a^5*c)*d^4*e^4 - 2*(a^3*b^3 - 4*a^4*b*c)*d^3*e^5 + (a^
2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^
7 + (a^2*b^2*c^2 - 4*a^3*c^3)*e^8)*x), -1/2*(2*(a^3*b^2 - 4*a^4*c)*d^6 - 2*
(a^2*b^3 - 4*a^3*b*c)*d^5*e + 2*(a^2*b^2*c - 4*a^3*c^2)*d^4*e^2 - 2*((a^3*b
^2 - 4*a^4*c)*d^4*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d^3*e^3 + (a*b^4 - 2*a^2*b^
2*c - 8*a^3*c^2)*d^2*e^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^5 + (a*b^2*c^2 - 4
*a^2*c^3)*e^6)*x^2 + 2*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3*e^3 - 2*(b^3*c -
3*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 2*a*c^3)*d*e^5 + ((b^4 - 4*a*b^2*c + 2*a^2*
c^2)*d^2*e^4 - 2*(b^3*c - 3*a*b*c^2)*d*e^5 + (b^2*c^2 - 2*a*c^3)*e^6)*x)*sq
rt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - 2*
((a^3*b^2 - 4*a^4*c)*d^5*e - 2*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 + (a*b^4 - 2*a
^2*b^2*c - 8*a^3*c^2)*d^3*e^3 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2*e^4 + (a*b^2*
c^2 - 4*a^2*c^3)*d*e^5)*x + ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^3*e^3 - 2*(b
^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^2*e^4 + (b^3*c^2 - 4*a*b*c^3)*d*e^5 + ((b
^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^2*e^4 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)
*d*e^5 + (b^3*c^2 - 4*a*b*c^3)*e^6)*x)*log(a*x^2 + b*x + c) + 2*(2*(a^3*b^2
 - 4*a^4*c)*d^6 - 3*(a^2*b^3 - 4*a^3*b*c)*d^5*e + 4*(a^2*b^2*c - 4*a^3*c^2)
*d^4*e^2 + (2*(a^3*b^2 - 4*a^4*c)*d^5*e - 3*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 +
 4*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^3)*x)*log(e*x + d))/((a^4*b^2 - 4*a^5*c)*d
^5*e^3 - 2*(a^3*b^3 - 4*a^4*b*c)*d^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c
^2)*d^3*e^5 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e^6 + (a^2*b^2*c^2 - 4*a^3*c^
3)*d*e^7 + ((a^4*b^2 - 4*a^5*c)*d^4*e^4 - 2*(a^3*b^3 - 4*a^4*b*c)*d^3*e^5 +
 (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*
d*e^7 + (a^2*b^2*c^2 - 4*a^3*c^3)*e^8)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(a+c/x**2+b/x)/(e*x+d)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.76

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = -\frac{d^4e^3}{(ad^2e^6-bde^7+ce^8)(ex+d)} - \frac{(b^3d^2-2\,abcd^2-2\,b^2cde+2\,ac^2de+bc^2e^2)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{bde}{(ex+d)^2}-\frac{ce^2}{(ex+d)^2}\right)}{2\,(a^4d^4-2\,a^3bd^3e+a^2b^2d^2e^2+2\,a^3cd^2e^2-2\,a^2bcde^3+a^2c^2e^4)} - \frac{(b^4d^2e^2-4\,ab^2cd^2e^2+2\,a^2c^2d^2e^2-2\,b^3cde^3+6\,abc^2de^3+b^2c^2e^4-2\,ac^3e^4)\arctan\left(-\frac{2\,ad-\frac{2\,ad^2}{ex+d}-be+\frac{2\,bde}{ex+d}-}{\sqrt{-b^2+4\,ace}}\right.}{(a^4d^4-2\,a^3bd^3e+a^2b^2d^2e^2+2\,a^3cd^2e^2-2\,a^2bcde^3+a^2c^2e^4)\sqrt{-b^2+4\,ac}e^2} + \frac{ex+d}{ae^3} + \frac{(2\,ad+be)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{a^2e^3}$$

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -d^4*e^3/((a*d^2*e^6 - b*d*e^7 + c*e^8)*(e*x + d)) - 1/2*(b^3*d^2 - 2*a*b*c
*d^2 - 2*b^2*c*d*e + 2*a*c^2*d*e + b*c^2*e^2)*log(-a + 2*a*d/(e*x + d) - a*
d^2/(e*x + d)^2 - b*e/(e*x + d) + b*d*e/(e*x + d)^2 - c*e^2/(e*x + d)^2)/(a
^4*d^4 - 2*a^3*b*d^3*e + a^2*b^2*d^2*e^2 + 2*a^3*c*d^2*e^2 - 2*a^2*b*c*d*e^
3 + a^2*c^2*e^4) - (b^4*d^2*e^2 - 4*a*b^2*c*d^2*e^2 + 2*a^2*c^2*d^2*e^2 - 2
*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3 + b^2*c^2*e^4 - 2*a*c^3*e^4)*arctan(-(2*a*d
- 2*a*d^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*c*e^2/(e*x + d))/(sqrt(-b
^2 + 4*a*c)*e))/((a^4*d^4 - 2*a^3*b*d^3*e + a^2*b^2*d^2*e^2 + 2*a^3*c*d^2*e
^2 - 2*a^2*b*c*d*e^3 + a^2*c^2*e^4)*sqrt(-b^2 + 4*a*c)*e^2) + (e*x + d)/(a*
e^3) + (2*a*d + b*e)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/(a^2*e^3)
```

Mupad [B] (verification not implemented)

Time = 11.20 (sec) , antiderivative size = 2495, normalized size of antiderivative = 9.11

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Too large to display}$$

```
[In] int(x^2/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] x/(a*e^2) - (log(d + e*x)*(2*a*d^5 + 4*c*d^3*e^2 - 3*b*d^4*e))/(c^2*e^7 + a
^2*d^4*e^3 + b^2*d^2*e^5 - 2*b*c*d*e^6 - 2*a*b*d^3*e^4 + 2*a*c*d^2*e^5) + (
log(8*a^4*c*d^7 + b*c^4*e^7 + c^4*e^7*(b^2 - 4*a*c)^(1/2) - 2*a^3*b^2*d^7 +
 b^5*d^4*e^3 + 3*a^2*b^3*d^6*e - 4*b^2*c^3*d*e^6 - 4*b^4*c*d^3*e^4 + b^4*d^
4*e^3*(b^2 - 4*a*c)^(1/2) - 24*a^2*c^3*d^3*e^4 + 8*a^3*c^2*d^5*e^2 + 6*b^3*
c^2*d^2*e^5 + 8*a*c^4*d*e^6 + 2*a*c^4*e^7*x - 2*a^3*b*d^7*(b^2 - 4*a*c)^(1/
2) - 4*a^4*d^7*x*(b^2 - 4*a*c)^(1/2) - 12*a^3*b*c*d^6*e + 17*a^2*c^2*d^4*e^
3*(b^2 - 4*a*c)^(1/2) + 6*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^(1/2) + 16*a^4*c*d^
6*e*x + 8*a^3*c*d^6*e*(b^2 - 4*a*c)^(1/2) - 4*b*c^3*d*e^6*(b^2 - 4*a*c)^(1/
2) - 18*a*b*c^3*d^2*e^5 - 8*a*b^3*c*d^4*e^3 - 2*a*b^4*d^4*e^3*x - 4*a^3*b^2
*d^6*e*x + 3*a^2*b^2*d^6*e*(b^2 - 4*a*c)^(1/2) - 6*a*c^3*d^2*e^5*(b^2 - 4*a
*c)^(1/2) - 4*b^3*c*d^3*e^4*(b^2 - 4*a*c)^(1/2) + 20*a*b^2*c^2*d^3*e^4 + 17
*a^2*b*c^2*d^4*e^3 - 2*a^2*b^2*c*d^5*e^2 + 8*a^2*b^3*d^5*e^2*x - 12*a^2*c^3
*d^2*e^5*x + 34*a^3*c^2*d^4*e^3*x + 4*a*b*c^2*d^3*e^4*(b^2 - 4*a*c)^(1/2) -
 18*a^2*b*c*d^5*e^2*(b^2 - 4*a*c)^(1/2) + 4*a*b^3*d^4*e^3*x*(b^2 - 4*a*c)^(
1/2) - 4*a^3*c*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c^2*d^2*e^5*x - 4*a^
2*b*c^2*d^3*e^4*x - 8*a^2*b^2*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^3*d*e
^6*x + 12*a^2*c^2*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) + 10*a^3*b*d^6*e*x*(b^2 - 4
*a*c)^(1/2) - 4*a*c^3*d*e^6*x*(b^2 - 4*a*c)^(1/2) - 32*a^3*b*c*d^5*e^2*x +
6*a*b*c^2*d^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 8*a*b^2*c*d^3*e^4*x*(b^2 - 4*a*c)
^(1/2))*(b^5*d^2 + b^4*d^2*(b^2 - 4*a*c)^(1/2) + b^3*c^2*e^2 + 8*a^2*b*c^2*
d^2 + 2*a^2*c^2*d^2*(b^2 - 4*a*c)^(1/2) + b^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) -
 2*b^4*c*d*e - 6*a*b^3*c*d^2 - 4*a*b*c^3*e^2 - 8*a^2*c^3*d*e - 2*a*c^3*e^2*
(b^2 - 4*a*c)^(1/2) + 10*a*b^2*c^2*d*e - 4*a*b^2*c*d^2*(b^2 - 4*a*c)^(1/2)
- 2*b^3*c*d*e*(b^2 - 4*a*c)^(1/2) + 6*a*b*c^2*d*e*(b^2 - 4*a*c)^(1/2)))/(2*
(4*a^5*c*d^4 - a^4*b^2*d^4 + 4*a^3*c^3*e^4 + 2*a^3*b^3*d^3*e - a^2*b^2*c^2*
e^4 - a^2*b^4*d^2*e^2 + 8*a^4*c^2*d^2*e^2 - 8*a^4*b*c*d^3*e + 2*a^2*b^3*c*d
*e^3 - 8*a^3*b*c^2*d*e^3 + 2*a^3*b^2*c*d^2*e^2)) - (log(c^4*e^7*(b^2 - 4*a*
c)^(1/2) - b*c^4*e^7 - 8*a^4*c*d^7 + 2*a^3*b^2*d^7 - b^5*d^4*e^3 - 3*a^2*b^
3*d^6*e + 4*b^2*c^3*d*e^6 + 4*b^4*c*d^3*e^4 + b^4*d^4*e^3*(b^2 - 4*a*c)^(1/
2) + 24*a^2*c^3*d^3*e^4 - 8*a^3*c^2*d^5*e^2 - 6*b^3*c^2*d^2*e^5 - 8*a*c^4*d
*e^6 - 2*a*c^4*e^7*x - 2*a^3*b*d^7*(b^2 - 4*a*c)^(1/2) - 4*a^4*d^7*x*(b^2 -
 4*a*c)^(1/2) + 12*a^3*b*c*d^6*e + 17*a^2*c^2*d^4*e^3*(b^2 - 4*a*c)^(1/2) +
 6*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 16*a^4*c*d^6*e*x + 8*a^3*c*d^6*e*(
b^2 - 4*a*c)^(1/2) - 4*b*c^3*d*e^6*(b^2 - 4*a*c)^(1/2) + 18*a*b*c^3*d^2*e^5
 + 8*a*b^3*c*d^4*e^3 + 2*a*b^4*d^4*e^3*x + 4*a^3*b^2*d^6*e*x + 3*a^2*b^2*d^
```

6*e*(b^2 - 4*a*c)^(1/2) - 6*a*c^3*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 4*b^3*c*d^3*e^4*(b^2 - 4*a*c)^(1/2) - 20*a*b^2*c^2*d^3*e^4 - 17*a^2*b*c^2*d^4*e^3 + 2*a^2*b^2*c*d^5*e^2 - 8*a^2*b^3*d^5*e^2*x + 12*a^2*c^3*d^2*e^5*x - 34*a^3*c^2*d^4*e^3*x + 4*a*b*c^2*d^3*e^4*(b^2 - 4*a*c)^(1/2) - 18*a^2*b*c*d^5*e^2*(b^2 - 4*a*c)^(1/2) + 4*a*b^3*d^4*e^3*x*(b^2 - 4*a*c)^(1/2) - 4*a^3*c*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c^2*d^2*e^5*x + 4*a^2*b*c^2*d^3*e^4*x - 8*a^2*b^2*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + 4*a*b*c^3*d*e^6*x + 12*a^2*c^2*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) + 10*a^3*b*d^6*e*x*(b^2 - 4*a*c)^(1/2) - 4*a*c^3*d*e^6*x*(b^2 - 4*a*c)^(1/2) + 32*a^3*b*c*d^5*e^2*x + 6*a*b*c^2*d^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 8*a*b^2*c*d^3*e^4*x*(b^2 - 4*a*c)^(1/2))*(b^4*d^2*(b^2 - 4*a*c)^(1/2) - b^5*d^2 - b^3*c^2*e^2 - 8*a^2*b*c^2*d^2 + 2*a^2*c^2*d^2*(b^2 - 4*a*c)^(1/2) + b^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 2*b^4*c*d*e + 6*a*b^3*c*d^2 + 4*a*b*c^3*e^2 + 8*a^2*c^3*d*e - 2*a*c^3*e^2*(b^2 - 4*a*c)^(1/2) - 10*a*b^2*c^2*d*e - 4*a*b^2*c*d^2*(b^2 - 4*a*c)^(1/2) - 2*b^3*c*d*e*(b^2 - 4*a*c)^(1/2) + 6*a*b*c^2*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^5*c*d^4 - a^4*b^2*d^4 + 4*a^3*c^3*e^4 + 2*a^3*b^3*d^3*e - a^2*b^2*c^2*e^4 - a^2*b^4*d^2*e^2 + 8*a^4*c^2*d^2*e^2 - 8*a^4*b*c*d^3*e + 2*a^2*b^3*c*d*e^3 - 8*a^3*b*c^2*d*e^3 + 2*a^3*b^2*c*d^2*e^2)) - (a*d^4)/(e*(a*d*e^2 + a*e^3*x)*(a*d^2 + c*e^2 - b*d*e))

3.72 $\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 246

$$\begin{aligned}\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx &= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} \\ &\quad+\frac{\left(b^3d^2-2b^2cde+4ac^2de-bc(3ad^2-ce^2)\right)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} \\ &\quad+\frac{d^2(ad^2-e(2bd-3ce))\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(b^2d^2-2bcde-c(ad^2-ce^2)\right)\log\left(c+bx+ax^2\right)}{2a\left(ad^2-e(bd-ce)\right)^2}\end{aligned}$$

```
[Out] d^3/e^2/(a*d^2-e*(b*d-c*e))/(e*x+d)+d^2*(a*d^2-e*(2*b*d-3*c*e))*ln(e*x+d)/e
^2/(a*d^2-e*(b*d-c*e))^2+1/2*(b^2*d^2-2*b*c*d*e-c*(a*d^2-c*e^2))*ln(a*x^2+b
*x+c)/a/(a*d^2-e*(b*d-c*e))^2+(b^3*d^2-2*b^2*c*d*e+4*a*c^2*d*e-b*c*(3*a*d^2
-c*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/a/(a*d^2-e*(b*d-c*e))^2/(-4*
a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

$= \{1583, 1642, 648, 632, 212, 642\}$

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$$
$$= \frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(-bc(3ad^2-ce^2)+4ac^2de+b^3d^2-2b^2cde\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}$$
$$+\frac{\left(-c(ad^2-ce^2)+b^2d^2-2bcde\right)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)^2}$$
$$+\frac{d^2\log(d+ex)\left(ad^2-e(2bd-3ce)\right)}{e^2\left(ad^2-e(bd-ce)\right)^2}+\frac{d^3}{e^2(d+ex)\left(ad^2-e(bd-ce)\right)}$$

```
[In] Int[x/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] d^3/(e^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*d^2 - 2*b^2*c*d*e + 4*a
*c^2*d*e - b*c*(3*a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(
a*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + (d^2*(a*d^2 - e*(2*b*d - 3
*c*e))*Log[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^2 - 2*b*c*d*
e - c*(a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d - c*e))^2
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^3}{(d+ex)^2\,(c+bx+ax^2)}\,dx\\
&= \int \left(\frac{d^3}{e\,(-ad^2+e(bd-ce))\,(d+ex)^2}+\frac{d^2(ad^2-e(2bd-3ce))}{e\,(ad^2-e(bd-ce))^2\,(d+ex)}\right.\\
&\qquad\left.+\frac{cd(bd-2ce)+(b^2d^2-2bcde-c(ad^2-ce^2))\,x}{(ad^2-e(bd-ce))^2\,(c+bx+ax^2)}\right)dx\\
&= \frac{d^3}{e^2\,(ad^2-e(bd-ce))\,(d+ex)}+\frac{d^2(ad^2-e(2bd-3ce))\log(d+ex)}{e^2\,(ad^2-e(bd-ce))^2}\\
&\quad+\frac{\int \frac{cd(bd-2ce)+(b^2d^2-2bcde-c(ad^2-ce^2))x}{c+bx+ax^2}\,dx}{(ad^2-e(bd-ce))^2}\\
&= \frac{d^3}{e^2\,(ad^2-e(bd-ce))\,(d+ex)}+\frac{d^2(ad^2-e(2bd-3ce))\log(d+ex)}{e^2\,(ad^2-e(bd-ce))^2}\\
&\quad+\frac{(b^2d^2-2bcde-c(ad^2-ce^2))\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2a\,(ad^2-e(bd-ce))^2}\\
&\quad-\frac{(b^3d^2-2b^2cde+4ac^2de-bc(3ad^2-ce^2))\int \frac{1}{c+bx+ax^2}\,dx}{2a\,(ad^2-e(bd-ce))^2}\\
&= \frac{d^3}{e^2\,(ad^2-e(bd-ce))\,(d+ex)}+\frac{d^2(ad^2-e(2bd-3ce))\log(d+ex)}{e^2\,(ad^2-e(bd-ce))^2}\\
&\quad+\frac{(b^2d^2-2bcde-c(ad^2-ce^2))\log(c+bx+ax^2)}{2a\,(ad^2-e(bd-ce))^2}\\
&\quad+\frac{(b^3d^2-2b^2cde+4ac^2de-bc(3ad^2-ce^2))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2ax\right)}{a\,(ad^2-e(bd-ce))^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{d^3}{e^2 (ad^2 - e(bd - ce)) (d + ex)} \\
&+ \frac{(b^3 d^2 - 2b^2 cde + 4ac^2 de - bc(3ad^2 - ce^2)) \tanh^{-1} \left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))^2} \\
&+ \frac{d^2 (ad^2 - e(2bd - 3ce)) \log(d + ex)}{e^2 (ad^2 - e(bd - ce))^2} \\
&+ \frac{(b^2 d^2 - 2bcde - c(ad^2 - ce^2)) \log(c + bx + ax^2)}{2a (ad^2 - e(bd - ce))^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.84

$$
\int \frac{x}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right) (d + ex)^2} \, dx
$$

$$
= \frac{\frac{2d^3(ad^2+e(-bd+ce))}{e^2(d+ex)} - \frac{2(b^3d^2-2b^2cde+4ac^2de+bc(-3ad^2+ce^2)) \arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a\sqrt{-b^2+4ac}} + \frac{2(ad^4+d^2e(-2bd+3ce)) \log(d+ex)}{e^2} + \frac{(b^2d^2-2bc\cdots}{\cdots}}{2 (ad^2 + e(-bd + ce))^2}
$$

```
[In] Integrate[x/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] ((2*d^3*(a*d^2 + e*(-(b*d) + c*e)))/(e^2*(d + e*x)) - (2*(b^3*d^2 - 2*b^2*c
*d*e + 4*a*c^2*d*e + b*c*(-3*a*d^2 + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 +
 4*a*c]])/(a*Sqrt[-b^2 + 4*a*c]) + (2*(a*d^4 + d^2*e*(-2*b*d + 3*c*e))*Log[
d + e*x])/e^2 + ((b^2*d^2 - 2*b*c*d*e + c*(-(a*d^2) + c*e^2))*Log[c + x*(b
+ a*x)])/a)/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.93

method	result
default	$\frac{\frac{\left(-d^2ac+b^2d^2-2bcde+e^2c^2\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(bc\,d^2-2c^2de-\frac{\left(-d^2ac+b^2d^2-2bcde+e^2c^2\right)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{\left(a\,d^2-bde+c\,e^2\right)^2}+\frac{d^2\left(a\,d^2-2bde+3c\,e^2\right)\,\mathrm{l}\cdots}{\left(a\,d^2-bde+c\,e^2\right)^{\cdots}}$
risch	Expression too large to display

```
[In] int(x/(a+c/x^2+b/x)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*d^2-b*d*e+c*e^2)^2*(1/2*(-a*c*d^2+b^2*d^2-2*b*c*d*e+c^2*e^2)/a*ln(a*x^
2+b*x+c)+2*(b*c*d^2-2*c^2*d*e-1/2*(-a*c*d^2+b^2*d^2-2*b*c*d*e+c^2*e^2)*b/a)
/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))+d^2*(a*d^2-2*b*d*e+
```

```
3*c*e^2)/(a*d^2-b*d*e+c*e^2)^2/e^2*ln(e*x+d)+d^3/e^2/(a*d^2-b*d*e+c*e^2)/(e
*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 723 vs. $2(240) = 480$.

Time = 11.05 (sec) , antiderivative size = 1465, normalized size of antiderivative = 5.96

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 2*(a*b^2*c
- 4*a^2*c^2)*d^3*e^2 + (b*c^2*d*e^4 + (b^3 - 3*a*b*c)*d^3*e^2 - 2*(b^2*c -
2*a*c^2)*d^2*e^3 + (b*c^2*e^5 + (b^3 - 3*a*b*c)*d^2*e^3 - 2*(b^2*c - 2*a*c^
2)*d*e^4)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqr
t(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((b^4 - 5*a*b^2*c + 4*a^2*
c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 +
((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2
*c^2 - 4*a*c^3)*e^5)*x)*log(a*x^2 + b*x + c) + 2*((a^2*b^2 - 4*a^3*c)*d^5 -
 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 3*(a*b^2*c - 4*a^2*c^2)*d^3*e^2 + ((a^2*b^2
- 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + 3*(a*b^2*c - 4*a^2*c^2)*
d^2*e^3)*x)*log(e*x + d))/((a^3*b^2 - 4*a^4*c)*d^5*e^2 - 2*(a^2*b^3 - 4*a^3
*b*c)*d^4*e^3 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^3*e^4 - 2*(a*b^3*c - 4*
a^2*b*c^2)*d^2*e^5 + (a*b^2*c^2 - 4*a^2*c^3)*d*e^6 + ((a^3*b^2 - 4*a^4*c)*d
^4*e^3 - 2*(a^2*b^3 - 4*a^3*b*c)*d^3*e^4 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2
)*d^2*e^5 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^6 + (a*b^2*c^2 - 4*a^2*c^3)*e^7)*
x), 1/2*(2*(a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 2*(a*b^2
*c - 4*a^2*c^2)*d^3*e^2 + 2*(b*c^2*d*e^4 + (b^3 - 3*a*b*c)*d^3*e^2 - 2*(b^2
*c - 2*a*c^2)*d^2*e^3 + (b*c^2*e^5 + (b^3 - 3*a*b*c)*d^2*e^3 - 2*(b^2*c - 2
*a*c^2)*d*e^4)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)
/(b^2 - 4*a*c)) + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b
*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*
d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x)*log(a*x
^2 + b*x + c) + 2*((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e +
3*(a*b^2*c - 4*a^2*c^2)*d^3*e^2 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4
*a^2*b*c)*d^3*e^2 + 3*(a*b^2*c - 4*a^2*c^2)*d^2*e^3)*x)*log(e*x + d))/((a^3
*b^2 - 4*a^4*c)*d^5*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d^4*e^3 + (a*b^4 - 2*a^2*
b^2*c - 8*a^3*c^2)*d^3*e^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2*e^5 + (a*b^2*c^2
 - 4*a^2*c^3)*d*e^6 + ((a^3*b^2 - 4*a^4*c)*d^4*e^3 - 2*(a^2*b^3 - 4*a^3*b*c
)*d^3*e^4 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^5 - 2*(a*b^3*c - 4*a^2*
b*c^2)*d*e^6 + (a*b^2*c^2 - 4*a^2*c^3)*e^7)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x/(a+c/x**2+b/x)/(e*x+d)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.71

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$$
$$= \frac{\frac{2\,d^3e^2}{(ad^2e^3-bde^4+ce^5)(ex+d)} + \frac{(b^2d^2e-acd^2e-2\,bcde^2+c^2e^3)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{bde}{(ex+d)^2}-\frac{ce^2}{(ex+d)^2}\right)}{a^3d^4-2\,a^2bd^3e+ab^2d^2e^2+2\,a^2cd^2e^2-2\,abcde^3+ac^2e^4} - \frac{2\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{ae} + \frac{2\,(b\cdots}{\cdots}}{2\,e}$$

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
[Out] 1/2*(2*d^3*e^2/((a*d^2*e^3 - b*d*e^4 + c*e^5)*(e*x + d)) + (b^2*d^2*e - a*c
*d^2*e - 2*b*c*d*e^2 + c^2*e^3)*log(-a + 2*a*d/(e*x + d) - a*d^2/(e*x + d)^
2 - b*e/(e*x + d) + b*d*e/(e*x + d)^2 - c*e^2/(e*x + d)^2)/(a^3*d^4 - 2*a^2
*b*d^3*e + a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2*a*b*c*d*e^3 + a*c^2*e^4) - 2
*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/(a*e) + 2*(b^3*d^2*e^3 - 3*a*b*c*d^
2*e^3 - 2*b^2*c*d*e^4 + 4*a*c^2*d*e^4 + b*c^2*e^5)*arctan(-(2*a*d - 2*a*d^2
/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*c*e^2/(e*x + d))/(sqrt(-b^2 + 4*a*
c)*e))/((a^3*d^4 - 2*a^2*b*d^3*e + a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2*a*b*
c*d*e^3 + a*c^2*e^4)*sqrt(-b^2 + 4*a*c)*e^2))/e
```

Mupad [B] (verification not implemented)

Time = 10.58 (sec) , antiderivative size = 2037, normalized size of antiderivative = 8.28

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Too large to display}$$

```
[In] int(x/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(d + e*x)*(a*d^4 + 3*c*d^2*e^2 - 2*b*d^3*e))/(c^2*e^6 + a^2*d^4*e^2 + b
^2*d^2*e^4 - 2*b*c*d*e^5 - 2*a*b*d^3*e^3 + 2*a*c*d^2*e^4) - (log(a^2*b^2*d^
6 - 4*a^3*c*d^6 - 2*c^4*e^6 - b^4*d^4*e^2 + c^3*e^6*x*(b^2 - 4*a*c)^(1/2) +
 24*a*c^3*d^2*e^4 + 6*b^3*c*d^3*e^3 + 2*b^4*d^3*e^3*x - b^3*d^4*e^2*(b^2 -
4*a*c)^(1/2) - 10*a^2*c^2*d^4*e^2 - 9*b^2*c^2*d^2*e^4 - 2*a*b^3*d^5*e + 4*b
*c^3*d*e^5 - b*c^3*e^6*x + a^2*b*d^6*(b^2 - 4*a*c)^(1/2) + 4*c^3*d*e^5*(b^2
 - 4*a*c)^(1/2) + 2*a^3*d^6*x*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d^5*e + 8*a*c
^3*d*e^5*x - 8*a^3*c*d^5*e*x - 2*a*b^2*d^5*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c*
d^5*e*(b^2 - 4*a*c)^(1/2) - 20*a*b*c^2*d^3*e^3 + 6*a*b^2*c*d^4*e^2 - 6*a*b^
3*d^4*e^2*x + 2*a^2*b^2*d^5*e*x - 3*b^3*c*d^2*e^4*x - 16*a*c^2*d^3*e^3*(b^2
 - 4*a*c)^(1/2) - 3*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d^3*e^3*(b^
2 - 4*a*c)^(1/2) - 2*b^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) - 32*a^2*c^2*d^3*e^3
*x + 4*a*b^2*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) - 12*a*c^2*d^2*e^4*x*(b^2 - 4*a*
c)^(1/2) + 5*a^2*c*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*b^2*c*d^2*e^4*x*(b^2 -
 4*a*c)^(1/2) + 14*a*b*c*d^4*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b*d^5*e*x*(b^2
 - 4*a*c)^(1/2) + 6*a*b*c^2*d^2*e^4*x + 2*a*b^2*c*d^3*e^3*x + 23*a^2*b*c*d^
4*e^2*x + 2*a*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d^2 - 4*a*c^3*e^2 + b
^3*d^2*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 5*
a*b^2*c*d^2 + b*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 8*a*b*c^2*d*e - 3*a*b*c*d^2*(
b^2 - 4*a*c)^(1/2) + 4*a*c^2*d*e*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d*e*(b^2 - 4
*a*c)^(1/2)))/(2*(4*a^4*c*d^4 - a^3*b^2*d^4 + 4*a^2*c^3*e^4 - a*b^2*c^2*e^4
 - a*b^4*d^2*e^2 + 2*a^2*b^3*d^3*e + 8*a^3*c^2*d^2*e^2 + 2*a*b^3*c*d*e^3 -
8*a^3*b*c*d^3*e - 8*a^2*b*c^2*d*e^3 + 2*a^2*b^2*c*d^2*e^2)) - (log(2*c^4*e^
6 + 4*a^3*c*d^6 - a^2*b^2*d^6 + b^4*d^4*e^2 + c^3*e^6*x*(b^2 - 4*a*c)^(1/2)
 - 24*a*c^3*d^2*e^4 - 6*b^3*c*d^3*e^3 - 2*b^4*d^3*e^3*x - b^3*d^4*e^2*(b^2
- 4*a*c)^(1/2) + 10*a^2*c^2*d^4*e^2 + 9*b^2*c^2*d^2*e^4 + 2*a*b^3*d^5*e - 4
*b*c^3*d*e^5 + b*c^3*e^6*x + a^2*b*d^6*(b^2 - 4*a*c)^(1/2) + 4*c^3*d*e^5*(b
^2 - 4*a*c)^(1/2) + 2*a^3*d^6*x*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c*d^5*e - 8*a
*c^3*d*e^5*x + 8*a^3*c*d^5*e*x - 2*a*b^2*d^5*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*
c*d^5*e*(b^2 - 4*a*c)^(1/2) + 20*a*b*c^2*d^3*e^3 - 6*a*b^2*c*d^4*e^2 + 6*a*
b^3*d^4*e^2*x - 2*a^2*b^2*d^5*e*x + 3*b^3*c*d^2*e^4*x - 16*a*c^2*d^3*e^3*(b
^2 - 4*a*c)^(1/2) - 3*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d^3*e^3*(
b^2 - 4*a*c)^(1/2) - 2*b^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) + 32*a^2*c^2*d^3*e
^3*x + 4*a*b^2*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) - 12*a*c^2*d^2*e^4*x*(b^2 - 4*
a*c)^(1/2) + 5*a^2*c*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*b^2*c*d^2*e^4*x*(b^2
 - 4*a*c)^(1/2) + 14*a*b*c*d^4*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b*d^5*e*x*(b
```

```
^2 - 4*a*c)^(1/2) - 6*a*b*c^2*d^2*e^4*x - 2*a*b^2*c*d^3*e^3*x - 23*a^2*b*c*
d^4*e^2*x + 2*a*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d^2 - 4*a*c^3*e^2 -
 b^3*d^2*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e -
5*a*b^2*c*d^2 - b*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 8*a*b*c^2*d*e + 3*a*b*c*d^2
*(b^2 - 4*a*c)^(1/2) - 4*a*c^2*d*e*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d*e*(b^2 -
 4*a*c)^(1/2)))/(2*(4*a^4*c*d^4 - a^3*b^2*d^4 + 4*a^2*c^3*e^4 - a*b^2*c^2*e
^4 - a*b^4*d^2*e^2 + 2*a^2*b^3*d^3*e + 8*a^3*c^2*d^2*e^2 + 2*a*b^3*c*d*e^3
- 8*a^3*b*c*d^3*e - 8*a^2*b*c^2*d*e^3 + 2*a^2*b^2*c*d^2*e^2)) + d^3/(e^2*(d
 + e*x)*(a*d^2 + c*e^2 - b*d*e))
```

3.73 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 194

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = -\frac{d^2}{e\,(ad^2-bde+ce^2)\,(d+ex)} - \frac{(b^2d^2-2bcde-2c(ad^2-ce^2))\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2} + \frac{d(bd-2ce)\log(d+ex)}{(ad^2-e(bd-ce))^2} - \frac{d(bd-2ce)\log\left(c+bx+ax^2\right)}{2\,(ad^2-e(bd-ce))^2}$$

```
[Out] -d^2/e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+d*(b*d-2*c*e)*ln(e*x+d)/(a*d^2-e*(b*d-c*
e))^2-1/2*d*(b*d-2*c*e)*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))^2-(b^2*d^2-2*b*
c*d*e-2*c*(a*d^2-c*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*
d-c*e))^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1459, 1642, 648, 632, 212, 642}

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(-2c(ad^2-ce^2)+b^2d^2-2bcde\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2} - \frac{d^2}{e(d+ex)\,(ad^2-bde+ce^2)} - \frac{d(bd-2ce)\log\left(ax^2+bx+c\right)}{2\,(ad^2-e(bd-ce))^2} + \frac{d(bd-2ce)\log(d+ex)}{(ad^2-e(bd-ce))^2}$$

```
[In] Int[1/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] -(d^2/(e*(a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((b^2*d^2 - 2*b*c*d*e - 2*c*
(a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]
*(a*d^2 - e*(b*d - c*e))^2) + (d*(b*d - 2*c*e)*Log[d + e*x])/(a*d^2 - e*(b*
d - c*e))^2 - (d*(b*d - 2*c*e)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c
*e))^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1459

```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x
_)^(n_.))^(q_.), x_Symbol] :> Int[((d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p)
/x^(2*n*p), x] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[mn, -n] && EqQ[mn2
, 2*mn] && IntegerQ[p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^2}{(d+ex)^2\,(c+bx+ax^2)}\,dx \\
&= \int \left(\frac{d^2}{(ad^2-e(bd-ce))\,(d+ex)^2}+\frac{de(bd-2ce)}{(ad^2-e(bd-ce))^2\,(d+ex)} + \frac{-c(ad^2-ce^2)-ad(bd-2ce)x}{(ad^2-e(bd-ce))^2\,(c+bx+ax^2)}\right)dx \\
&= -\frac{d^2}{e\,(ad^2-bde+ce^2)\,(d+ex)}+\frac{d(bd-2ce)\log(d+ex)}{(ad^2-e(bd-ce))^2}+\frac{\int \frac{-c(ad^2-ce^2)-ad(bd-2ce)x}{c+bx+ax^2}\,dx}{(ad^2-e(bd-ce))^2} \\
&= -\frac{d^2}{e\,(ad^2-bde+ce^2)\,(d+ex)}+\frac{d(bd-2ce)\log(d+ex)}{(ad^2-e(bd-ce))^2} - \frac{(d(bd-2ce))\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2\,(ad^2-e(bd-ce))^2}+\frac{(b^2d^2-2bcde-2c(ad^2-ce^2))\int \frac{1}{c+bx+ax^2}\,dx}{2\,(ad^2-e(bd-ce))^2} \\
&= -\frac{d^2}{e\,(ad^2-bde+ce^2)\,(d+ex)}+\frac{d(bd-2ce)\log(d+ex)}{(ad^2-e(bd-ce))^2} - \frac{d(bd-2ce)\log(c+bx+ax^2)}{2\,(ad^2-e(bd-ce))^2} - \frac{(b^2d^2-2bcde-2c(ad^2-ce^2))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2ax\right)}{(ad^2-e(bd-ce))^2} \\
&= -\frac{d^2}{e\,(ad^2-bde+ce^2)\,(d+ex)}-\frac{(b^2d^2-2bcde-2c(ad^2-ce^2))\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2} + \frac{d(bd-2ce)\log(d+ex)}{(ad^2-e(bd-ce))^2}-\frac{d(bd-2ce)\log(c+bx+ax^2)}{2\,(ad^2-e(bd-ce))^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$
\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \frac{-\frac{2d^2(ad^2+e(-bd+ce))}{e(d+ex)}+\frac{2(b^2d^2-2bcde+2c(-ad^2+ce^2))\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+2d(bd-2ce)\log(d+ex)-d(bd-2ce)\log\ldots}{2\,(ad^2+e(-bd+ce))^2}
$$

```
[In] Integrate[1/((a + c/x^2 + b/x)*(d + e*x)^2),x]
[Out] ((-2*d^2*(a*d^2 + e*(-(b*d) + c*e)))/(e*(d + e*x)) + (2*(b^2*d^2 - 2*b*c*d*
e + 2*c*(-(a*d^2) + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b
^2 + 4*a*c] + 2*d*(b*d - 2*c*e)*Log[d + e*x] - d*(b*d - 2*c*e)*Log[c + x*(b
 + a*x)])/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.97

method	result
default	$\frac{\frac{\left(-ab\,d^{2}+2acde\right)\ln\left(a\,x^{2}+bx+c\right)}{2a}+\frac{2\left(-d^{2}ac+e^{2}c^{2}-\frac{\left(-ab\,d^{2}+2acde\right)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{\left(a\,d^{2}-bde+c\,e^{2}\right)^{2}}-\frac{d^{2}}{e\left(a\,d^{2}-bde+c\,e^{2}\right)\left(ex+d\right)}+\frac{d(bd-2ec)\ln(e}{(a\,d^{2}-bde+c}$
risch	Expression too large to display

```
[In] int(1/(a+c/x^2+b/x)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*d^2-b*d*e+c*e^2)^2*(1/2*(-a*b*d^2+2*a*c*d*e)/a*ln(a*x^2+b*x+c)+2*(-d^2
*a*c+e^2*c^2-1/2*(-a*b*d^2+2*a*c*d*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+
b)/(4*a*c-b^2)^(1/2)))-d^2/e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+d*(b*d-2*c*e)/(a*d
^2-b*d*e+c*e^2)^2*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(188) = 376.

Time = 3.64 (sec) , antiderivative size = 1120, normalized size of antiderivative = 5.77

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b^2 - 4*a^2*c)*d^4 - 2*(b^3 - 4*a*b*c)*d^3*e + 2*(b^2*c - 4*a*c
^2)*d^2*e^2 + (2*b*c*d^2*e^2 - 2*c^2*d*e^3 - (b^2 - 2*a*c)*d^3*e + (2*b*c*d
*e^3 - 2*c^2*e^4 - (b^2 - 2*a*c)*d^2*e^2)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x
^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x +
c)) + ((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*
c)*d^2*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*log(a*x^2 + b*x + c) - 2*((b^3 -
 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*c)*d^2*e^2 -
2*(b^2*c - 4*a*c^2)*d*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5*e - 2*
(a*b^3 - 4*a^2*b*c)*d^4*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^3 - 2*(b^
3*c - 4*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 4*a*c^3)*d*e^5 + ((a^2*b^2 - 4*a^3*c)
*d^4*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^3 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^
2*e^4 - 2*(b^3*c - 4*a*b*c^2)*d*e^5 + (b^2*c^2 - 4*a*c^3)*e^6)*x), -1/2*(2*
(a*b^2 - 4*a^2*c)*d^4 - 2*(b^3 - 4*a*b*c)*d^3*e + 2*(b^2*c - 4*a*c^2)*d^2*e
^2 - 2*(2*b*c*d^2*e^2 - 2*c^2*d*e^3 - (b^2 - 2*a*c)*d^3*e + (2*b*c*d*e^3 -
2*c^2*e^4 - (b^2 - 2*a*c)*d^2*e^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
+ 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + ((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4
*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*c)*d^2*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*
log(a*x^2 + b*x + c) - 2*((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e
```

```
^2 + ((b^3 - 4*a*b*c)*d^2*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*log(e*x + d))
/((a^2*b^2 - 4*a^3*c)*d^5*e - 2*(a*b^3 - 4*a^2*b*c)*d^4*e^2 + (b^4 - 2*a*b^
2*c - 8*a^2*c^2)*d^3*e^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 4*a*c
^3)*d*e^5 + ((a^2*b^2 - 4*a^3*c)*d^4*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^3 +
(b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^4 - 2*(b^3*c - 4*a*b*c^2)*d*e^5 + (b^2*
c^2 - 4*a*c^3)*e^6)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+c/x**2+b/x)/(e*x+d)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.75

$$\begin{aligned}\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx &= -\frac{d^2e}{(ad^2e^2-bde^3+ce^4)(ex+d)} \\ &\quad -\frac{(bd^2-2\,cde)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{bde}{(ex+d)^2}-\frac{ce^2}{(ex+d)^2}\right)}{2\,(a^2d^4-2\,abd^3e+b^2d^2e^2+2\,acd^2e^2-2\,bcde^3+c^2e^4)} \\ &\quad -\frac{(b^2d^2e^2-2\,acd^2e^2-2\,bcde^3+2\,c^2e^4)\arctan\left(-\frac{2\,ad-\frac{2\,ad^2}{ex+d}-be+\frac{2\,bde}{ex+d}-\frac{2\,ce^2}{ex+d}}{\sqrt{-b^2+4\,ace}}\right)}{(a^2d^4-2\,abd^3e+b^2d^2e^2+2\,acd^2e^2-2\,bcde^3+c^2e^4)\sqrt{-b^2+4\,ace^2}}\end{aligned}$$

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -d^2*e/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(e*x + d)) - 1/2*(b*d^2 - 2*c*d*e)*lo
g(-a + 2*a*d/(e*x + d) - a*d^2/(e*x + d)^2 - b*e/(e*x + d) + b*d*e/(e*x + d
)^2 - c*e^2/(e*x + d)^2)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e
^2 - 2*b*c*d*e^3 + c^2*e^4) - (b^2*d^2*e^2 - 2*a*c*d^2*e^2 - 2*b*c*d*e^3 +
2*c^2*e^4)*arctan(-(2*a*d - 2*a*d^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2
*c*e^2/(e*x + d))/(sqrt(-b^2 + 4*a*c)*e))/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2
*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)*e^2)
```

Mupad [B] (verification not implemented)

Time = 11.25 (sec) , antiderivative size = 1585, normalized size of antiderivative = 8.17

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(2*a*b^3*d^4 + b*c^3*e^4 - c^3*e^4*(b^2 - 4*a*c)^(1/2) + 16*a^2*c^2*d^3
*e + 2*b^2*c^2*d*e^3 - b^3*c*d^2*e^2 + a^2*b^2*d^4*x + b^2*c^2*e^4*x - b^4*
d^2*e^2*x - 7*a^2*b*c*d^4 - 16*a*c^3*d*e^3 - 2*a^3*c*d^4*x - 2*a*c^3*e^4*x
+ 2*a*b^2*d^4*(b^2 - 4*a*c)^(1/2) - a^2*c*d^4*(b^2 - 4*a*c)^(1/2) - 6*a*b^2
*c*d^3*e + 2*a*b^3*d^3*e*x + 2*b^3*c*d*e^3*x - 2*b*c^2*d*e^3*(b^2 - 4*a*c)^
(1/2) + 3*a^2*b*d^4*x*(b^2 - 4*a*c)^(1/2) - b*c^2*e^4*x*(b^2 - 4*a*c)^(1/2)
 + 10*a*b*c^2*d^2*e^2 + 14*a*c^2*d^2*e^2*(b^2 - 4*a*c)^(1/2) + b^2*c*d^2*e^
2*(b^2 - 4*a*c)^(1/2) + b^3*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) + 28*a^2*c^2*d^2*
e^2*x - 10*a*b*c*d^3*e*(b^2 - 4*a*c)^(1/2) - 12*a*b*c^2*d*e^3*x - 12*a^2*b*
c*d^3*e*x - 2*a*b^2*d^3*e*x*(b^2 - 4*a*c)^(1/2) + 8*a*c^2*d*e^3*x*(b^2 - 4*
a*c)^(1/2) - 8*a^2*c*d^3*e*x*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d*e^3*x*(b^2 - 4
*a*c)^(1/2) + 2*a*b*c*d^2*e^2*x*(b^2 - 4*a*c)^(1/2))*(d^2*(b^3/2 + (b^2*(b^
2 - 4*a*c)^(1/2))/2) - c*(d^2*(2*a*b + a*(b^2 - 4*a*c)^(1/2)) + d*(b^2*e +
b*e*(b^2 - 4*a*c)^(1/2))) + c^2*(e^2*(b^2 - 4*a*c)^(1/2) + 4*a*d*e)))/(4*a^
3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2
*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*
e + 2*a*b^2*c*d^2*e^2) - (log(2*a*b^3*d^4 + b*c^3*e^4 + c^3*e^4*(b^2 - 4*a*
c)^(1/2) + 16*a^2*c^2*d^3*e + 2*b^2*c^2*d*e^3 - b^3*c*d^2*e^2 + a^2*b^2*d^4
*x + b^2*c^2*e^4*x - b^4*d^2*e^2*x - 7*a^2*b*c*d^4 - 16*a*c^3*d*e^3 - 2*a^3
*c*d^4*x - 2*a*c^3*e^4*x - 2*a*b^2*d^4*(b^2 - 4*a*c)^(1/2) + a^2*c*d^4*(b^2
 - 4*a*c)^(1/2) - 6*a*b^2*c*d^3*e + 2*a*b^3*d^3*e*x + 2*b^3*c*d*e^3*x + 2*b
*c^2*d*e^3*(b^2 - 4*a*c)^(1/2) - 3*a^2*b*d^4*x*(b^2 - 4*a*c)^(1/2) + b*c^2*
e^4*x*(b^2 - 4*a*c)^(1/2) + 10*a*b*c^2*d^2*e^2 - 14*a*c^2*d^2*e^2*(b^2 - 4*
a*c)^(1/2) - b^2*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) - b^3*d^2*e^2*x*(b^2 - 4*a*c
)^(1/2) + 28*a^2*c^2*d^2*e^2*x + 10*a*b*c*d^3*e*(b^2 - 4*a*c)^(1/2) - 12*a*
b*c^2*d*e^3*x - 12*a^2*b*c*d^3*e*x + 2*a*b^2*d^3*e*x*(b^2 - 4*a*c)^(1/2) -
```

```
8*a*c^2*d*e^3*x*(b^2 - 4*a*c)^(1/2) + 8*a^2*c*d^3*e*x*(b^2 - 4*a*c)^(1/2) +
 2*b^2*c*d*e^3*x*(b^2 - 4*a*c)^(1/2) - 2*a*b*c*d^2*e^2*x*(b^2 - 4*a*c)^(1/2
))*(c*(d^2*(2*a*b - a*(b^2 - 4*a*c)^(1/2)) + d*(b^2*e - b*e*(b^2 - 4*a*c)^(
1/2))) - d^2*(b^3/2 - (b^2*(b^2 - 4*a*c)^(1/2))/2) + c^2*(e^2*(b^2 - 4*a*c)
^(1/2) - 4*a*d*e)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4
- b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c
^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) + (log(d + e*x)*(b*d^2 - 2*
c*d*e))/(a^2*d^4 + c^2*e^4 + b^2*d^2*e^2 - 2*a*b*d^3*e - 2*b*c*d*e^3 + 2*a*
c*d^2*e^2) - d^2/(e*(d + e*x)*(a*d^2 + c*e^2 - b*d*e))
```

3.74 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 183

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx = \frac{d}{(ad^2-bde+ce^2)\,(d+ex)} + \frac{(bce^2+ad(bd-4ce))\,\text{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2} - \frac{(ad^2-ce^2)\log(d+ex)}{(ad^2-e(bd-ce))^2} + \frac{(ad^2-ce^2)\log\left(c+bx+ax^2\right)}{2\,(ad^2-e(bd-ce))^2}$$

```
[Out] d/(a*d^2-b*d*e+c*e^2)/(e*x+d)-(a*d^2-c*e^2)*ln(e*x+d)/(a*d^2-e*(b*d-c*e))^2
+1/2*(a*d^2-c*e^2)*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))^2+(b*c*e^2+a*d*(b*d-
4*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c*e))^2/(-4*a*c
+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1583, 814, 648, 632, 212, 642}

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx = \frac{\text{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(ad(bd-4ce)+bce^2\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2} + \frac{(ad^2-ce^2)\log\left(ax^2+bx+c\right)}{2\,(ad^2-e(bd-ce))^2} + \frac{d}{(d+ex)\,(ad^2-bde+ce^2)} - \frac{(ad^2-ce^2)\log(d+ex)}{(ad^2-e(bd-ce))^2}$$

```
[In] Int[1/((a + c/x^2 + b/x)*x*(d + e*x)^2),x]
```

```
[Out] d/((a*d^2 - b*d*e + c*e^2)*(d + e*x)) + ((b*c*e^2 + a*d*(b*d - 4*c*e))*ArcT
anh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*
e))^2) - ((a*d^2 - c*e^2)*Log[d + e*x])/(a*d^2 - e*(b*d - c*e))^2 + ((a*d^2
 - c*e^2)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \frac{x}{(d+ex)^2\,(c+bx+ax^2)}\,dx$$

$$= \int \left(\frac{de}{(-ad^2+e(bd-ce))\,(d+ex)^2} + \frac{e(-ad^2+ce^2)}{(ad^2-e(bd-ce))^2\,(d+ex)} + \frac{ce(2ad-be)+a(ad^2-ce^2)\,x}{(ad^2-e(bd-ce))^2\,(c+bx+ax^2)}\right) dx$$

$$= \frac{d}{(ad^2-bde+ce^2)\,(d+ex)} - \frac{(ad^2-ce^2)\log(d+ex)}{(ad^2-e(bd-ce))^2} + \frac{\int \frac{ce(2ad-be)+a(ad^2-ce^2)x}{c+bx+ax^2}\,dx}{(ad^2-e(bd-ce))^2}$$

$$= \frac{d}{(ad^2-bde+ce^2)\,(d+ex)} - \frac{(ad^2-ce^2)\log(d+ex)}{(ad^2-e(bd-ce))^2} + \frac{(ad^2-ce^2)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2\,(ad^2-e(bd-ce))^2} - \frac{(bce^2+ad(bd-4ce))\int \frac{1}{c+bx+ax^2}\,dx}{2\,(ad^2-e(bd-ce))^2}$$

$$= \frac{d}{(ad^2-bde+ce^2)\,(d+ex)} - \frac{(ad^2-ce^2)\log(d+ex)}{(ad^2-e(bd-ce))^2} + \frac{(ad^2-ce^2)\log\left(c+bx+ax^2\right)}{2\,(ad^2-e(bd-ce))^2} + \frac{(bce^2+ad(bd-4ce))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2ax\right)}{(ad^2-e(bd-ce))^2}$$

$$= \frac{d}{(ad^2-bde+ce^2)\,(d+ex)} + \frac{(bce^2+ad(bd-4ce))\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2} - \frac{(ad^2-ce^2)\log(d+ex)}{(ad^2-e(bd-ce))^2} + \frac{(ad^2-ce^2)\log\left(c+bx+ax^2\right)}{2\,(ad^2-e(bd-ce))^2}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx$$

$$= \frac{\frac{2d(ad^2+e(-bd+ce))}{d+ex} - \frac{2(bce^2+ad(bd-4ce))\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + (-2ad^2+2ce^2)\log(d+ex) + (ad^2-ce^2)\log(c+x(b}{2\,(ad^2+e(-bd+ce))^2}$$

```
[In] Integrate[1/((a + c/x^2 + b/x)*x*(d + e*x)^2),x]
```

```
[Out] ((2*d*(a*d^2 + e*(-(b*d) + c*e)))/(d + e*x) - (2*(b*c*e^2 + a*d*(b*d - 4*c*
e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-2*a*d^2
+ 2*c*e^2)*Log[d + e*x] + (a*d^2 - c*e^2)*Log[c + x*(b + a*x)])/(2*(a*d^2 +
 e*(-(b*d) + c*e))^2)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.02

method	result
default	$\frac{\frac{(a^2d^2-e^2ac)\ln(a\,x^2+bx+c)}{2a}+\frac{2\left(2acde-bc\,e^2-\frac{(a^2d^2-e^2ac)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{(a\,d^2-bde+c\,e^2)^2}+\frac{d}{(a\,d^2-bde+c\,e^2)(ex+d)}-\frac{(a\,d^2-c\,e^2)\ln(ex+}{(a\,d^2-bde+c\,e^2)^2}$
risch	$\frac{d}{(a\,d^2-bde+c\,e^2)(ex+d)}-\frac{\ln(ex+d)a\,d^2}{a^2d^4-2ab\,d^3e+2ac\,d^2e^2+b^2d^2e^2-2bcd\,e^3+c^2e^4}+\frac{\ln(ex+d)c\,e^2}{a^2d^4-2ab\,d^3e+2ac\,d^2e^2+b^2d^2e^2-2bcd\,e^3+c^2e^4}+$

[In] int(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] 1/(a*d^2-b*d*e+c*e^2)^2*(1/2*(a^2*d^2-a*c*e^2)/a*ln(a*x^2+b*x+c)+2*(2*a*c*d*e-b*c*e^2-1/2*(a^2*d^2-a*c*e^2)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))+d/(a*d^2-b*d*e+c*e^2)/(e*x+d)-(a*d^2-c*e^2)/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(177) = 354.

Time = 3.32 (sec) , antiderivative size = 1059, normalized size of antiderivative = 5.79

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx=\text{Too large to display}$$

[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="fricas")

[Out] [1/2*(2*(a*b^2 - 4*a^2*c)*d^3 - 2*(b^3 - 4*a*b*c)*d^2*e + 2*(b^2*c - 4*a*c^2)*d*e^2 + (a*b*d^3 - 4*a*c*d^2*e + b*c*d*e^2 + (a*b*d^2*e - 4*a*c*d*e^2 + b*c*e^3)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2 + ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(a*x^2 + b*x + c) - 2*((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2 + ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x), 1/2*(2*(a*b^2 - 4*a^2*c)*d^3 - 2*(b^3 - 4*a*b*c)*d^2*e + 2*(b^2*c - 4*a*c^2)*d*e^2 + 2*(a*b*d^3 - 4*a*c*d^2*e + b*c*d*e^2 + (a*b*d^2*e - 4*a*c*d*e^2 + b*c*e^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + ((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2 + ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(a*x^2 + b*x + c)

```
- 2*((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2 + ((a*b^2 - 4*a^2*c)*d
^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 - 2
*(a*b^3 - 4*a^2*b*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3
*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*
d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e
^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+c/x**2+b/x)/x/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.79

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx$$
$$=\frac{1}{2}e\left(\frac{(ad^2-ce^2)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{bde}{(ex+d)^2}-\frac{ce^2}{(ex+d)^2}\right)}{a^2d^4e-2\,abd^3e^2+b^2d^2e^3+2\,acd^2e^3-2\,bcde^4+c^2e^5}+\frac{2\,de}{(ad^2e^2-bde^3+ce^4)(ex+d)}+\right.$$

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*e*((a*d^2 - c*e^2)*log(-a + 2*a*d/(e*x + d) - a*d^2/(e*x + d)^2 - b*e/(
e*x + d) + b*d*e/(e*x + d)^2 - c*e^2/(e*x + d)^2)/(a^2*d^4*e - 2*a*b*d^3*e^
2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*b*c*d*e^4 + c^2*e^5) + 2*d*e/((a*d^2*e^
2 - b*d*e^3 + c*e^4)*(e*x + d)) + 2*(a*b*d^2*e - 4*a*c*d*e^2 + b*c*e^3)*arc
tan(-(2*a*d - 2*a*d^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*c*e^2/(e*x +
d))/(sqrt(-b^2 + 4*a*c)*e))/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d
^2*e^2 - 2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)*e^2))
```

Mupad [B] (verification not implemented)

Time = 12.95 (sec) , antiderivative size = 1768, normalized size of antiderivative = 9.66

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(x*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] d/((d + e*x)*(a*d^2 + c*e^2 - b*d*e)) - (log(56*a^3*b^2*c*d^4 - 96*a^4*c^2*
d^4 - 96*a^2*c^4*e^4 - 8*b^4*c^2*e^4 - 8*a^2*b^4*d^4 + 56*a*b^2*c^3*e^4 - 4
*a^3*b^3*d^4*x + 320*a^3*c^3*d^2*e^2 + 8*a*d^3*e*(b^2 - 4*a*c)^(5/2) - 8*c*
d*e^3*(b^2 - 4*a*c)^(5/2) - 3*c*e^4*x*(b^2 - 4*a*c)^(5/2) - 8*b^5*c*e^4*x +
 8*a^2*b*d^4*(b^2 - 4*a*c)^(3/2) - 8*b*c^2*e^4*(b^2 - 4*a*c)^(3/2) + 12*a^3
*d^4*x*(b^2 - 4*a*c)^(3/2) - 6*b*d*e^3*x*(b^2 - 4*a*c)^(5/2) + 16*a^4*b*c*d
^4*x - 112*a^2*b^2*c^2*d^2*e^2 - 8*a*b^2*d^3*e*(b^2 - 4*a*c)^(3/2) + 8*b^2*
c*d*e^3*(b^2 - 4*a*c)^(3/2) + 10*a*d^2*e^2*x*(b^2 - 4*a*c)^(5/2) - 5*b^2*c*
e^4*x*(b^2 - 4*a*c)^(3/2) + 6*b^3*d*e^3*x*(b^2 - 4*a*c)^(3/2) + 16*a*b^3*c^
2*d*e^3 + 8*a*b^4*c*d^2*e^2 - 64*a^2*b*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e - 64*
a^3*b*c^2*d^3*e + 60*a*b^3*c^2*e^4*x - 112*a^2*b*c^3*e^4*x + 4*a*b^5*d^2*e^
2*x - 8*a^2*b^4*d^3*e*x + 256*a^3*c^3*d*e^3*x - 256*a^4*c^2*d^3*e*x - 6*a*b
^2*d^2*e^2*x*(b^2 - 4*a*c)^(3/2) - 160*a^2*b^2*c^2*d*e^3*x - 56*a^2*b^3*c*d
^2*e^2*x + 160*a^3*b*c^2*d^2*e^2*x + 24*a*b^4*c*d*e^3*x - 8*a^2*b*d^3*e*x*(
b^2 - 4*a*c)^(3/2) + 96*a^3*b^2*c*d^3*e*x)*(b^2*((a*d^2)/2 - (c*e^2)/2) - b
*((a*d^2*(b^2 - 4*a*c)^(1/2))/2 + (c*e^2*(b^2 - 4*a*c)^(1/2))/2) - 2*a^2*c*
d^2 + 2*a*c^2*e^2 + 2*a*c*d*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*c*d^4 + 4*a*c^3*
e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3
*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*
e^2) - (log(8*a^2*b^4*d^4 + 96*a^4*c^2*d^4 + 96*a^2*c^4*e^4 + 8*b^4*c^2*e^4
 - 56*a^3*b^2*c*d^4 - 56*a*b^2*c^3*e^4 + 4*a^3*b^3*d^4*x - 320*a^3*c^3*d^2*
e^2 + 8*a*d^3*e*(b^2 - 4*a*c)^(5/2) - 8*c*d*e^3*(b^2 - 4*a*c)^(5/2) - 3*c*e
^4*x*(b^2 - 4*a*c)^(5/2) + 8*b^5*c*e^4*x + 8*a^2*b*d^4*(b^2 - 4*a*c)^(3/2)
- 8*b*c^2*e^4*(b^2 - 4*a*c)^(3/2) + 12*a^3*d^4*x*(b^2 - 4*a*c)^(3/2) - 6*b*
d*e^3*x*(b^2 - 4*a*c)^(5/2) - 16*a^4*b*c*d^4*x + 112*a^2*b^2*c^2*d^2*e^2 -
8*a*b^2*d^3*e*(b^2 - 4*a*c)^(3/2) + 8*b^2*c*d*e^3*(b^2 - 4*a*c)^(3/2) + 10*
a*d^2*e^2*x*(b^2 - 4*a*c)^(5/2) - 5*b^2*c*e^4*x*(b^2 - 4*a*c)^(3/2) + 6*b^3
*d*e^3*x*(b^2 - 4*a*c)^(3/2) - 16*a*b^3*c^2*d*e^3 - 8*a*b^4*c*d^2*e^2 + 64*
```

```
a^2*b*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e + 64*a^3*b*c^2*d^3*e - 60*a*b^3*c^2*e^
4*x + 112*a^2*b*c^3*e^4*x - 4*a*b^5*d^2*e^2*x + 8*a^2*b^4*d^3*e*x - 256*a^3
*c^3*d*e^3*x + 256*a^4*c^2*d^3*e*x - 6*a*b^2*d^2*e^2*x*(b^2 - 4*a*c)^(3/2)
+ 160*a^2*b^2*c^2*d*e^3*x + 56*a^2*b^3*c*d^2*e^2*x - 160*a^3*b*c^2*d^2*e^2*
x - 24*a*b^4*c*d*e^3*x - 8*a^2*b*d^3*e*x*(b^2 - 4*a*c)^(3/2) - 96*a^3*b^2*c
*d^3*e*x)*(b*((a*d^2*(b^2 - 4*a*c)^(1/2))/2 + (c*e^2*(b^2 - 4*a*c)^(1/2))/2
) + b^2*((a*d^2)/2 - (c*e^2)/2) - 2*a^2*c*d^2 + 2*a*c^2*e^2 - 2*a*c*d*e*(b^
2 - 4*a*c)^(1/2)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 -
 b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^
2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) - (log(d + e*x)*(a*d^2 - c*e
^2))/(a^2*d^4 + c^2*e^4 + b^2*d^2*e^2 - 2*a*b*d^3*e - 2*b*c*d*e^3 + 2*a*c*d
^2*e^2)
```

3.75 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 189

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx = -\frac{e}{(ad^2-bde+ce^2)(d+ex)} - \frac{(2a^2d^2+b^2e^2-2ae(bd+ce))\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2} + \frac{e(2ad-be)\log(d+ex)}{(ad^2-e(bd-ce))^2} - \frac{e(2ad-be)\log(c+bx+ax^2)}{2(ad^2-e(bd-ce))^2}$$

```
[Out] -e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+e*(2*a*d-b*e)*ln(e*x+d)/(a*d^2-e*(b*d-c*e))^
2-1/2*e*(2*a*d-b*e)*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))^2-(2*a^2*d^2+b^2*e^
2-2*a*e*(b*d+c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c*e)
)^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

$= \{1583, 723, 814, 648, 632, 212, 642\}$

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)(2a^2d^2-2ae(bd+ce)+b^2e^2)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2} - \frac{e}{(d+ex)(ad^2-bde+ce^2)} - \frac{e(2ad-be)\log(ax^2+bx+c)}{2(ad^2-e(bd-ce))^2} + \frac{e(2ad-be)\log(d+ex)}{(ad^2-e(bd-ce))^2}$$

```
[In] Int[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2),x]
```

```
[Out] -(e/((a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((2*a^2*d^2 + b^2*e^2 - 2*a*e*(b
*d + c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(a*d^
2 - e*(b*d - c*e))^2) + (e*(2*a*d - b*e)*Log[d + e*x])/(a*d^2 - e*(b*d - c*
e))^2 - (e*(2*a*d - b*e)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{(d+ex)^2\,(c+bx+ax^2)}\,dx \\
&= -\frac{e}{(ad^2-bde+ce^2)\,(d+ex)} + \frac{\int \frac{ad-be-aex}{(d+ex)(c+bx+ax^2)}\,dx}{ad^2-bde+ce^2} \\
&= -\frac{e}{(ad^2-bde+ce^2)\,(d+ex)} + \frac{\int \left(\frac{e^2(2ad-be)}{(ad^2-e(bd-ce))(d+ex)} + \frac{a^2d^2+b^2e^2-ae(2bd+ce)-ae(2ad-be)x}{(ad^2-e(bd-ce))(c+bx+ax^2)}\right)dx}{ad^2-bde+ce^2} \\
&= -\frac{e}{(ad^2-bde+ce^2)\,(d+ex)} + \frac{e(2ad-be)\log(d+ex)}{(ad^2-e(bd-ce))^2} + \frac{\int \frac{a^2d^2+b^2e^2-ae(2bd+ce)-ae(2ad-be)x}{c+bx+ax^2}\,dx}{(ad^2-e(bd-ce))^2} \\
&= -\frac{e}{(ad^2-bde+ce^2)\,(d+ex)} + \frac{e(2ad-be)\log(d+ex)}{(ad^2-e(bd-ce))^2} \\
&\quad - \frac{(e(2ad-be))\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2\,(ad^2-e(bd-ce))^2} + \frac{(2a^2d^2+b^2e^2-2ae(bd+ce))\int \frac{1}{c+bx+ax^2}\,dx}{2\,(ad^2-e(bd-ce))^2} \\
&= -\frac{e}{(ad^2-bde+ce^2)\,(d+ex)} + \frac{e(2ad-be)\log(d+ex)}{(ad^2-e(bd-ce))^2} \\
&\quad - \frac{e(2ad-be)\log(c+bx+ax^2)}{2\,(ad^2-e(bd-ce))^2} \\
&\quad - \frac{(2a^2d^2+b^2e^2-2ae(bd+ce))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2ax\right)}{(ad^2-e(bd-ce))^2}
\end{aligned}$$

$$= -\frac{e}{(ad^2 - bde + ce^2)(d + ex)} - \frac{(2a^2d^2 + b^2e^2 - 2ae(bd + ce))\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2 - 4ac}\,(ad^2 - e(bd - ce))^2}$$
$$+ \frac{e(2ad - be)\log(d + ex)}{(ad^2 - e(bd - ce))^2} - \frac{e(2ad - be)\log(c + bx + ax^2)}{2(ad^2 - e(bd - ce))^2}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right) x^2 (d + ex)^2}\,dx$$
$$= \frac{-\frac{2e(ad^2+e(-bd+ce))}{d+ex} + \frac{2(2a^2d^2+b^2e^2-2ae(bd+ce))\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 2e(-2ad + be)\log(d + ex) + e(-2ad + be)\log}{2(ad^2 + e(-bd + ce))^2}$$

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2),x]
```

```
[Out] ((-2*e*(a*d^2 + e*(-(b*d) + c*e)))/(d + e*x) + (2*(2*a^2*d^2 + b^2*e^2 - 2*
a*e*(b*d + c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
 - 2*e*(-2*a*d + b*e)*Log[d + e*x] + e*(-2*a*d + b*e)*Log[c + x*(b + a*x)])
/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.04

method	result
default	$\frac{\frac{(-2a^2de+ab\,e^2)\ln(a\,x^2+bx+c)}{2a}+\frac{2\left(a^2d^2-2abde-e^2ac+b^2e^2-\frac{(-2a^2de+ab\,e^2)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{(a\,d^2-bde+c\,e^2)^2} - \frac{e}{(a\,d^2-bde+c\,e^2)(ex+d)} + \frac{e(2d}{(a\,}$
risch	$-\frac{e}{(a\,d^2-bde+c\,e^2)(ex+d)} + \frac{2e\ln(ex+d)da}{a^2d^4-2ab\,d^3e+2ac\,d^2e^2+b^2d^2e^2-2bcd\,e^3+c^2e^4} - \frac{e^2\ln(ex+d)b}{a^2d^4-2ab\,d^3e+2ac\,d^2e^2+b^2d^2e^2-2bcd\,e^3+c^2e^4}$

```
[In] int(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*d^2-b*d*e+c*e^2)^2*(1/2*(-2*a^2*d*e+a*b*e^2)/a*ln(a*x^2+b*x+c)+2*(a^2*
d^2-2*a*b*d*e-e^2*a*c+b^2*e^2-1/2*(-2*a^2*d*e+a*b*e^2)*b/a)/(4*a*c-b^2)^(1/
2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))-e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+e*(2*
a*d-b*e)/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(183) = 366$.

Time = 1.86 (sec) , antiderivative size = 1079, normalized size of antiderivative = 5.71

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b^2 - 4*a^2*c)*d^2*e - 2*(b^3 - 4*a*b*c)*d*e^2 + 2*(b^2*c - 4*a
*c^2)*e^3 + (2*a^2*d^3 - 2*a*b*d^2*e + (b^2 - 2*a*c)*d*e^2 + (2*a^2*d^2*e -
 2*a*b*d*e^2 + (b^2 - 2*a*c)*e^3)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a
*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + (2
*(a*b^2 - 4*a^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c)*d*e
^2 - (b^3 - 4*a*b*c)*e^3)*x)*log(a*x^2 + b*x + c) - 2*(2*(a*b^2 - 4*a^2*c)*
d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c)*d*e^2 - (b^3 - 4*a*b*c
)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^
4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3
 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^
2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c
^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x), -1/2*(2*(a*b^2 - 4*a^2*c)*d^2*e -
2*(b^3 - 4*a*b*c)*d*e^2 + 2*(b^2*c - 4*a*c^2)*e^3 + 2*(2*a^2*d^3 - 2*a*b*d^
2*e + (b^2 - 2*a*c)*d*e^2 + (2*a^2*d^2*e - 2*a*b*d*e^2 + (b^2 - 2*a*c)*e^3)
*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)
) + (2*(a*b^2 - 4*a^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*
c)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*x)*log(a*x^2 + b*x + c) - 2*(2*(a*b^2 - 4*a
^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c)*d*e^2 - (b^3 - 4
*a*b*c)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b
*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d
^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3
- 4*a^2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4
*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.78

$$\begin{aligned}&\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx\\&=-\frac{e^3}{(ad^2e^2-bde^3+ce^4)(ex+d)}\\&-\frac{(2\,ade-be^2)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{bde}{(ex+d)^2}-\frac{ce^2}{(ex+d)^2}\right)}{2\,(a^2d^4-2\,abd^3e+b^2d^2e^2+2\,acd^2e^2-2\,bcde^3+c^2e^4)}\\&-\frac{(2\,a^2d^2e^2-2\,abde^3+b^2e^4-2\,ace^4)\arctan\left(-\frac{2\,ad-\frac{2\,ad^2}{ex+d}-be+\frac{2\,bde}{ex+d}-\frac{2\,ce^2}{ex+d}}{\sqrt{-b^2+4\,ac}e}\right)}{(a^2d^4-2\,abd^3e+b^2d^2e^2+2\,acd^2e^2-2\,bcde^3+c^2e^4)\sqrt{-b^2+4\,ac}e^2}\end{aligned}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -e^3/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(e*x + d)) - 1/2*(2*a*d*e - b*e^2)*log(
-a + 2*a*d/(e*x + d) - a*d^2/(e*x + d)^2 - b*e/(e*x + d) + b*d*e/(e*x + d)^
2 - c*e^2/(e*x + d)^2)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2
 - 2*b*c*d*e^3 + c^2*e^4) - (2*a^2*d^2*e^2 - 2*a*b*d*e^3 + b^2*e^4 - 2*a*c*
e^4)*arctan(-(2*a*d - 2*a*d^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*c*e^2
/(e*x + d))/(sqrt(-b^2 + 4*a*c)*e))/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 +
 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)*e^2)
```

Mupad [B] (verification not implemented)

Time = 12.75 (sec) , antiderivative size = 1782, normalized size of antiderivative = 9.43

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^2*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(c*e^4*(b^2 - 4*a*c)^(5/2) - 8*b^5*c*e^4 - 8*b^6*e^4*x - 4*a^3*d^4*(b^2
 - 4*a*c)^(3/2) - 4*a^3*b^3*d^4 + 4*b^3*e^4*x*(b^2 - 4*a*c)^(3/2) + 60*a*b^
3*c^2*e^4 - 112*a^2*b*c^3*e^4 + 4*a*b^5*d^2*e^2 - 8*a^2*b^4*d^3*e + 256*a^3
*c^3*d*e^3 - 256*a^4*c^2*d^3*e - 8*a^4*b^2*d^4*x + 32*a^3*c^3*e^4*x + 10*b*
d*e^3*(b^2 - 4*a*c)^(5/2) + 4*b*e^4*x*(b^2 - 4*a*c)^(5/2) + 16*a^4*b*c*d^4
+ 32*a^5*c*d^4*x - 14*a*d^2*e^2*(b^2 - 4*a*c)^(5/2) + 7*b^2*c*e^4*(b^2 - 4*
a*c)^(3/2) - 10*b^3*d*e^3*(b^2 - 4*a*c)^(3/2) - 8*a*d*e^3*x*(b^2 - 4*a*c)^(
5/2) + 24*a*b^4*c*d*e^3 + 64*a*b^4*c*e^4*x + 32*a*b^5*d*e^3*x - 8*a^2*b*d^3
*e*(b^2 - 4*a*c)^(3/2) - 32*a^3*d^3*e*x*(b^2 - 4*a*c)^(3/2) + 96*a^3*b^2*c*
d^3*e + 16*a^3*b^3*d^3*e*x + 18*a*b^2*d^2*e^2*(b^2 - 4*a*c)^(3/2) - 160*a^2
*b^2*c^2*d*e^3 - 56*a^2*b^3*c*d^2*e^2 + 160*a^3*b*c^2*d^2*e^2 - 136*a^2*b^2
*c^2*e^4*x - 40*a^2*b^4*d^2*e^2*x - 448*a^4*c^2*d^2*e^2*x + 48*a^2*b*d^2*e^
2*x*(b^2 - 4*a*c)^(3/2) + 272*a^3*b^2*c*d^2*e^2*x - 64*a^4*b*c*d^3*e*x - 24
*a*b^2*d*e^3*x*(b^2 - 4*a*c)^(3/2) - 240*a^2*b^3*c*d*e^3*x + 448*a^3*b*c^2*
d*e^3*x)*(a*(e^2*(2*b*c - c*(b^2 - 4*a*c)^(1/2)) + e*(b^2*d - b*d*(b^2 - 4*
a*c)^(1/2))) - e^2*(b^3/2 - (b^2*(b^2 - 4*a*c)^(1/2))/2) + a^2*(d^2*(b^2 -
4*a*c)^(1/2) - 4*c*d*e)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^
2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8
*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) - (log(d + e*x)*(b*e^
2 - 2*a*d*e))/(a^2*d^4 + c^2*e^4 + b^2*d^2*e^2 - 2*a*b*d^3*e - 2*b*c*d*e^3
+ 2*a*c*d^2*e^2) - (log(c*e^4*(b^2 - 4*a*c)^(5/2) + 8*b^5*c*e^4 + 8*b^6*e^4
*x - 4*a^3*d^4*(b^2 - 4*a*c)^(3/2) + 4*a^3*b^3*d^4 + 4*b^3*e^4*x*(b^2 - 4*a
*c)^(3/2) - 60*a*b^3*c^2*e^4 + 112*a^2*b*c^3*e^4 - 4*a*b^5*d^2*e^2 + 8*a^2*
b^4*d^3*e - 256*a^3*c^3*d*e^3 + 256*a^4*c^2*d^3*e + 8*a^4*b^2*d^4*x - 32*a^
3*c^3*e^4*x + 10*b*d*e^3*(b^2 - 4*a*c)^(5/2) + 4*b*e^4*x*(b^2 - 4*a*c)^(5/2
) - 16*a^4*b*c*d^4 - 32*a^5*c*d^4*x - 14*a*d^2*e^2*(b^2 - 4*a*c)^(5/2) + 7*
b^2*c*e^4*(b^2 - 4*a*c)^(3/2) - 10*b^3*d*e^3*(b^2 - 4*a*c)^(3/2) - 8*a*d*e^
3*x*(b^2 - 4*a*c)^(5/2) - 24*a*b^4*c*d*e^3 - 64*a*b^4*c*e^4*x - 32*a*b^5*d*
e^3*x - 8*a^2*b*d^3*e*(b^2 - 4*a*c)^(3/2) - 32*a^3*d^3*e*x*(b^2 - 4*a*c)^(3
/2) - 96*a^3*b^2*c*d^3*e - 16*a^3*b^3*d^3*e*x + 18*a*b^2*d^2*e^2*(b^2 - 4*a
*c)^(3/2) + 160*a^2*b^2*c^2*d*e^3 + 56*a^2*b^3*c*d^2*e^2 - 160*a^3*b*c^2*d^
2*e^2 + 136*a^2*b^2*c^2*e^4*x + 40*a^2*b^4*d^2*e^2*x + 448*a^4*c^2*d^2*e^2*
x + 48*a^2*b*d^2*e^2*x*(b^2 - 4*a*c)^(3/2) - 272*a^3*b^2*c*d^2*e^2*x + 64*a
^4*b*c*d^3*e*x - 24*a*b^2*d*e^3*x*(b^2 - 4*a*c)^(3/2) + 240*a^2*b^3*c*d*e^3
*x - 448*a^3*b*c^2*d*e^3*x)*(e^2*(b^3/2 + (b^2*(b^2 - 4*a*c)^(1/2))/2) - a*
(e^2*(2*b*c + c*(b^2 - 4*a*c)^(1/2)) + e*(b^2*d + b*d*(b^2 - 4*a*c)^(1/2)))
```

```
 + a^2*(d^2*(b^2 - 4*a*c)^(1/2) + 4*c*d*e)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a
^2*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e
+ 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) -
e/((d + e*x)*(a*d^2 + c*e^2 - b*d*e))
```

3.76 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 248

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx = \frac{e^2}{d\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{\left(b^3e^2-abe(2bd+3ce)+a^2d(bd+4ce)\right)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} + \frac{\log(x)}{cd^2} - \frac{e^2(3ad^2-e(2bd-ce))\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)^2} - \frac{\left(a^2d^2+b^2e^2-ae(2bd+ce)\right)\log\left(c+bx+ax^2\right)}{2c\left(ad^2-e(bd-ce)\right)^2}$$

```
[Out] e^2/d/(a*d^2-b*d*e+c*e^2)/(e*x+d)+ln(x)/c/d^2-e^2*(3*a*d^2-e*(2*b*d-c*e))*l
n(e*x+d)/d^2/(a*d^2-e*(b*d-c*e))^2-1/2*(a^2*d^2+b^2*e^2-a*e*(2*b*d+c*e))*ln
(a*x^2+b*x+c)/c/(a*d^2-e*(b*d-c*e))^2+(b^3*e^2-a*b*e*(2*b*d+3*c*e)+a^2*d*(b
*d+4*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c/(a*d^2-e*(b*d-c*e))^2/(-
4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used

$= \{1583, 907, 648, 632, 212, 642\}$

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx$$

$$= \frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(a^2d(bd+4ce)-abe(2bd+3ce)+b^3e^2\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} - \frac{\left(a^2d^2-ae(2bd+ce)+b^2e^2\right)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)^2} + \frac{e^2}{d(d+ex)\left(ad^2-e(bd-ce)\right)} - \frac{e^2\log(d+ex)\left(3ad^2-e(2bd-ce)\right)}{d^2\left(ad^2-e(bd-ce)\right)^2} + \frac{\log(x)}{cd^2}$$

```
[In] Int[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2),x]
```

```
[Out] e^2/(d*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*e^2 - a*b*e*(2*b*d + 3*c*
e) + a^2*d*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b
^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + Log[x]/(c*d^2) - (e^2*(3*a*d^2 - e
*(2*b*d - c*e))*Log[d + e*x])/(d^2*(a*d^2 - e*(b*d - c*e))^2) - ((a^2*d^2 +
 b^2*e^2 - a*e*(2*b*d + c*e))*Log[c + b*x + a*x^2])/(2*c*(a*d^2 - e*(b*d -
c*e))^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x(d+ex)^2\left(c+bx+ax^2\right)}\,dx \\
&= \int \left(\frac{1}{cd^2x}+\frac{e^3}{d\left(-ad^2+e(bd-ce)\right)(d+ex)^2}+\frac{e^3(-3ad^2+e(2bd-ce))}{d^2\left(ad^2-e(bd-ce)\right)^2(d+ex)} \right. \\
&\qquad \left. +\frac{-((ad-be)\left(abd-b^2e+2ace\right))-a(a^2d^2+b^2e^2-ae(2bd+ce))\,x}{c\left(ad^2-e(bd-ce)\right)^2\left(c+bx+ax^2\right)}\right)dx \\
&= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{\log(x)}{cd^2}-\frac{e^2(3ad^2-e(2bd-ce))\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)^2} \\
&\quad +\frac{\int \frac{-((ad-be)\left(abd-b^2e+2ace\right))-a\left(a^2d^2+b^2e^2-ae(2bd+ce)\right)x}{c+bx+ax^2}\,dx}{c\left(ad^2-e(bd-ce)\right)^2} \\
&= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{\log(x)}{cd^2}-\frac{e^2(3ad^2-e(2bd-ce))\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)^2} \\
&\quad -\frac{\left(a^2d^2+b^2e^2-ae(2bd+ce)\right)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2c\left(ad^2-e(bd-ce)\right)^2} \\
&\quad -\frac{\left(b^3e^2-abe(2bd+3ce)+a^2d(bd+4ce)\right)\int \frac{1}{c+bx+ax^2}\,dx}{2c\left(ad^2-e(bd-ce)\right)^2}
\end{aligned}$$

$$= \frac{e^2}{d(ad^2 - e(bd - ce))(d + ex)} + \frac{\log(x)}{cd^2} - \frac{e^2(3ad^2 - e(2bd - ce))\log(d + ex)}{d^2(ad^2 - e(bd - ce))^2}$$
$$- \frac{(a^2d^2 + b^2e^2 - ae(2bd + ce))\log(c + bx + ax^2)}{2c(ad^2 - e(bd - ce))^2}$$
$$+ \frac{(b^3e^2 - abe(2bd + 3ce) + a^2d(bd + 4ce))\,\text{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2}\,dx, x, b + 2ax\right)}{c(ad^2 - e(bd - ce))^2}$$

$$= \frac{e^2}{d(ad^2 - e(bd - ce))(d + ex)}$$
$$+ \frac{(b^3e^2 - abe(2bd + 3ce) + a^2d(bd + 4ce))\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2 - 4ac}\,(ad^2 - e(bd - ce))^2}$$
$$+ \frac{\log(x)}{cd^2} - \frac{e^2(3ad^2 - e(2bd - ce))\log(d + ex)}{d^2(ad^2 - e(bd - ce))^2}$$
$$- \frac{(a^2d^2 + b^2e^2 - ae(2bd + ce))\log(c + bx + ax^2)}{2c(ad^2 - e(bd - ce))^2}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x^3(d + ex)^2}\,dx$$
$$= \frac{e^2}{d(ad^2 + e(-bd + ce))(d + ex)}$$
$$- \frac{(b^3e^2 - abe(2bd + 3ce) + a^2d(bd + 4ce))\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{c\sqrt{-b^2 + 4ac}\,(ad^2 + e(-bd + ce))^2}$$
$$+ \frac{\log(x)}{cd^2} - \frac{e^2(3ad^2 + e(-2bd + ce))\log(d + ex)}{(ad^3 + de(-bd + ce))^2}$$
$$+ \frac{(-a^2d^2 - b^2e^2 + ae(2bd + ce))\log(c + x(b + ax))}{2c(ad^2 + e(-bd + ce))^2}$$

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2),x]
```

```
[Out] e^2/(d*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) - ((b^3*e^2 - a*b*e*(2*b*d + 3
*c*e) + a^2*d*(b*d + 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c*Sqr
t[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2) + Log[x]/(c*d^2) - (e^2*(3*a*
d^2 + e*(-2*b*d + c*e))*Log[d + e*x])/(a*d^3 + d*e*(-(b*d) + c*e))^2 + ((-(
a^2*d^2) - b^2*e^2 + a*e*(2*b*d + c*e))*Log[c + x*(b + a*x)])/(2*c*(a*d^2 +
 e*(-(b*d) + c*e))^2)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.13

method	result
default	$\frac{\ln(x)}{c\,d^2}+\frac{\frac{\left(-a^3d^2+2a^2bde+a^2c\,e^2-a\,b^2e^2\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(-a^2b\,d^2-2ecd\,a^2+2a\,b^2de+2ab\,e^2c-b^3e^2-\frac{\left(-a^3d^2+2a^2bde+a^2c\,e^2-a\,b^2e^2\right)b}{2a}\right.}{\sqrt{4ac-b^2}}}{\left(a\,d^2-bde+c\,e^2\right)^2c}$
risch	Expression too large to display

```
[In] int(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/c/d^2+1/(a*d^2-b*d*e+c*e^2)^2/c*(1/2*(-a^3*d^2+2*a^2*b*d*e+a^2*c*e^2-
a*b^2*e^2)/a*ln(a*x^2+b*x+c)+2*(-a^2*b*d^2-2*e*c*d*a^2+2*a*b^2*d*e+2*a*b*e^
2*c-b^3*e^2-1/2*(-a^3*d^2+2*a^2*b*d*e+a^2*c*e^2-a*b^2*e^2)*b/a)/(4*a*c-b^2)
^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))+e^2/d/(a*d^2-b*d*e+c*e^2)/(e*x+
d)-e^2*(3*a*d^2-2*b*d*e+c*e^2)/d^2/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.62

$$\begin{aligned}\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx &= \frac{e^5}{(ad^3e^3-bd^2e^4+cde^5)(ex+d)} \\ &\quad - \frac{(a^2d^2-2\,abde+b^2e^2-ace^2)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{bde}{(ex+d)^2}-\frac{ce^2}{(ex+d)^2}\right)}{2\,(a^2cd^4-2\,abcd^3e+b^2cd^2e^2+2\,ac^2d^2e^2-2\,bc^2de^3+c^3e^4)} \\ &\quad + \frac{(a^2bd^2e^2-2\,ab^2de^3+4\,a^2cde^3+b^3e^4-3\,abce^4)\arctan\left(-\frac{2\,ad-\frac{2\,ad^2}{ex+d}-be+\frac{2\,bde}{ex+d}-\frac{2\,ce^2}{ex+d}}{\sqrt{-b^2+4\,ace}}\right)}{(a^2cd^4-2\,abcd^3e+b^2cd^2e^2+2\,ac^2d^2e^2-2\,bc^2de^3+c^3e^4)\sqrt{-b^2+4\,ace}^2} \\ &\quad + \frac{\log\left(\left|-\frac{d}{ex+d}+1\right|\right)}{cd^2}\end{aligned}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] e^5/((a*d^3*e^3 - b*d^2*e^4 + c*d*e^5)*(e*x + d)) - 1/2*(a^2*d^2 - 2*a*b*d*
e + b^2*e^2 - a*c*e^2)*log(-a + 2*a*d/(e*x + d) - a*d^2/(e*x + d)^2 - b*e/(
e*x + d) + b*d*e/(e*x + d)^2 - c*e^2/(e*x + d)^2)/(a^2*c*d^4 - 2*a*b*c*d^3*
e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*b*c^2*d*e^3 + c^3*e^4) + (a^2*b*d^2
*e^2 - 2*a*b^2*d*e^3 + 4*a^2*c*d*e^3 + b^3*e^4 - 3*a*b*c*e^4)*arctan(-(2*a*
d - 2*a*d^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*c*e^2/(e*x + d))/(sqrt(
-b^2 + 4*a*c)*e))/((a^2*c*d^4 - 2*a*b*c*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2
*e^2 - 2*b*c^2*d*e^3 + c^3*e^4)*sqrt(-b^2 + 4*a*c)*e^2) + log(abs(-d/(e*x +
 d) + 1))/(c*d^2)
```

Mupad [B] (verification not implemented)

Time = 27.77 (sec) , antiderivative size = 3510, normalized size of antiderivative = 14.15

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^3*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log((a^4*e^4)/(d*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e^5*x)/(d^2*(a*d^2 + c*
e^2 - b*d*e)^2) - (((a*e^3*(3*a^3*b*d^4 + b^3*c*e^4 - b^4*d*e^3 + 5*a*b^3*d
^2*e^2 - 7*a^2*b^2*d^3*e + 8*a^2*c^2*d*e^3 - 3*a*b*c^2*e^4 + 9*a^3*c*d^3*e
- a*b^2*c*d*e^3 - 8*a^2*b*c*d^2*e^2))/(d^2*(a*d^2 + c*e^2 - b*d*e)^2) + (((
a*e*(a^3*b*d^5 - 4*a*c^3*e^5 + b^2*c^2*e^5 - b^4*d^2*e^3 + 3*a*b^3*d^3*e^2
- 3*a^2*b^2*d^4*e - 8*a^2*c^2*d^2*e^3 + 4*a^3*c*d^4*e - b^3*c*d*e^4 + 4*a*b
*c^2*d*e^4 + 6*a*b^2*c*d^2*e^3 - 9*a^2*b*c*d^3*e^2))/(a*d^3 - b*d^2*e + c*d
*e^2) + (a*e*x*(3*a^4*d^5 + 2*b^3*c*e^5 - 4*b^4*d*e^4 + 9*a*b^3*d^2*e^3 + 4
*a^2*c^2*d*e^4 + 19*a^3*c*d^3*e^2 - 3*a^2*b^2*d^3*e^2 - 8*a*b*c^2*e^5 - 5*a
^3*b*d^4*e + 15*a*b^2*c*d*e^4 - 36*a^2*b*c*d^2*e^3))/(a*d^3 - b*d^2*e + c*d
*e^2) - (a*e*(b^4*e^2 - 4*a^3*c*d^2 + b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2
*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 + a^2*b*d^2*(b^2 - 4*a*c
)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^2*d*e*(b^
2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*
c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e
^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^
2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c
^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x
))/(2*c*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 +
b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5
*a*b^2*c*e^2 + a^2*b*d^2*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*
(b^2 - 4*a*c)^(1/2) - 2*a*b^2*d*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 -
4*a*c)^(1/2)))/(2*c*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a*e^3*x*(9*
a^4*d^4 + b^4*e^4 + 2*a^2*c^2*e^4 - 6*a^3*c*d^2*e^2 + 8*a^2*b^2*d^2*e^2 - 4
*a*b^2*c*e^4 - 4*a*b^3*d*e^3 - 12*a^3*b*d^3*e + 10*a^2*b*c*d*e^3))/(d^2*(a*
d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 + b^3*e^2*(b^2 - 4*a*c)^(1/
2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 + a^2*b*d^2*
(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a
*b^2*d*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c*(4*a*
c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 + b^3*e^2*(b^2
- 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2
+ a^2*b*d^2*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*(b^2 - 4*a*c)
^(1/2) - 2*a*b^2*d*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2))
)/(2*(4*a*c^4*e^4 + 4*a^3*c^2*d^4 - b^2*c^3*e^4 - a^2*b^2*c*d^4 + 2*b^3*c^2
*d*e^3 - b^4*c*d^2*e^2 + 8*a^2*c^3*d^2*e^2 - 8*a*b*c^3*d*e^3 + 2*a*b^3*c*d^
3*e - 8*a^2*b*c^2*d^3*e + 2*a*b^2*c^2*d^2*e^2)) + (log((a^4*e^4)/(d*(a*d^2
```

```
+ c*e^2 - b*d*e)^2) + (a^4*e^5*x)/(d^2*(a*d^2 + c*e^2 - b*d*e)^2) - (((a*e^
3*(3*a^3*b*d^4 + b^3*c*e^4 - b^4*d*e^3 + 5*a*b^3*d^2*e^2 - 7*a^2*b^2*d^3*e
+ 8*a^2*c^2*d*e^3 - 3*a*b*c^2*e^4 + 9*a^3*c*d^3*e - a*b^2*c*d*e^3 - 8*a^2*b
*c*d^2*e^2))/(d^2*(a*d^2 + c*e^2 - b*d*e)^2) + (((a*e*(a^3*b*d^5 - 4*a*c^3*
e^5 + b^2*c^2*e^5 - b^4*d^2*e^3 + 3*a*b^3*d^3*e^2 - 3*a^2*b^2*d^4*e - 8*a^2
*c^2*d^2*e^3 + 4*a^3*c*d^4*e - b^3*c*d*e^4 + 4*a*b*c^2*d*e^4 + 6*a*b^2*c*d^
2*e^3 - 9*a^2*b*c*d^3*e^2))/(a*d^3 - b*d^2*e + c*d*e^2) + (a*e*x*(3*a^4*d^5
 + 2*b^3*c*e^5 - 4*b^4*d*e^4 + 9*a*b^3*d^2*e^3 + 4*a^2*c^2*d*e^4 + 19*a^3*c
*d^3*e^2 - 3*a^2*b^2*d^3*e^2 - 8*a*b*c^2*e^5 - 5*a^3*b*d^4*e + 15*a*b^2*c*d
*e^4 - 36*a^2*b*c*d^2*e^3))/(a*d^3 - b*d^2*e + c*d*e^2) - (a*e*(b^4*e^2 - 4
*a^3*c*d^2 - b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*
a*b^3*d*e - 5*a*b^2*c*e^2 - a^2*b*d^2*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e +
 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^2*d*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*
c*d*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2
 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*
c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e
*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*
d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/(2*c*(4*a*c - b^2)*(a*
d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 - b^3*e^2*(b^2 - 4*a*c)^(1/
2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 - a^2*b*d^2*
(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e + 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) + 2*a
*b^2*d*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c*(4*a*
c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a*e^3*x*(9*a^4*d^4 + b^4*e^4 + 2*a^2
*c^2*e^4 - 6*a^3*c*d^2*e^2 + 8*a^2*b^2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*a*b^3*d*
e^3 - 12*a^3*b*d^3*e + 10*a^2*b*c*d*e^3))/(d^2*(a*d^2 + c*e^2 - b*d*e)^2))*
(b^4*e^2 - 4*a^3*c*d^2 - b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*
c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 - a^2*b*d^2*(b^2 - 4*a*c)^(1/2) + 8*a
^2*b*c*d*e + 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^2*d*e*(b^2 - 4*a*c)^(1
/2) - 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c*(4*a*c - b^2)*(a*d^2 + c*e^2 -
 b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 - b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*
d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 - a^2*b*d^2*(b^2 - 4*a*c)
^(1/2) + 8*a^2*b*c*d*e + 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^2*d*e*(b^2
 - 4*a*c)^(1/2) - 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^4*e^4 + 4*a^3
*c^2*d^4 - b^2*c^3*e^4 - a^2*b^2*c*d^4 + 2*b^3*c^2*d*e^3 - b^4*c*d^2*e^2 +
8*a^2*c^3*d^2*e^2 - 8*a*b*c^3*d*e^3 + 2*a*b^3*c*d^3*e - 8*a^2*b*c^2*d^3*e +
 2*a*b^2*c^2*d^2*e^2)) - (log(d + e*x)*(c*e^4 + 3*a*d^2*e^2 - 2*b*d*e^3))/(
a^2*d^6 + b^2*d^4*e^2 + c^2*d^2*e^4 - 2*a*b*d^5*e + 2*a*c*d^4*e^2 - 2*b*c*d
^3*e^3) + log(x)/(c*d^2) + e^2/(d*(d + e*x)*(a*d^2 + c*e^2 - b*d*e))
```

3.77 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 291

$$\begin{aligned}\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx &= -\frac{1}{cd^2x}-\frac{e^3}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)} \\ &\quad+\frac{\left(2a^3cd^2-b^4e^2+2ab^2e(bd+2ce)-a^2(b^2d^2+6bcde+2c^2e^2)\right)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} \\ &\quad-\frac{(bd+2ce)\log(x)}{c^2d^3}+\frac{e^3\left(4ad^2-e(3bd-2ce)\right)\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)^2} \\ &\quad+\frac{(ad-be)\left(abd-b^2e+2ace\right)\log\left(c+bx+ax^2\right)}{2c^2\left(ad^2-e(bd-ce)\right)^2}\end{aligned}$$

```
[Out] -1/c/d^2/x-e^3/d^2/(a*d^2-e*(b*d-c*e))/(e*x+d)-(b*d+2*c*e)*ln(x)/c^2/d^3+e^
3*(4*a*d^2-e*(3*b*d-2*c*e))*ln(e*x+d)/d^3/(a*d^2-e*(b*d-c*e))^2+1/2*(a*d-b*
e)*(a*b*d+2*a*c*e-b^2*e)*ln(a*x^2+b*x+c)/c^2/(a*d^2-e*(b*d-c*e))^2+(2*a^3*c
*d^2-b^4*e^2+2*a*b^2*e*(b*d+2*c*e)-a^2*(b^2*d^2+6*b*c*d*e+2*c^2*e^2))*arcta
nh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1583, 907, 648, 632, 212, 642}

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx$$
$$= \frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(2a^3cd^2-a^2(b^2d^2+6bcde+2c^2e^2)+2ab^2e(bd+2ce)+b^4(-e^2)\right)}{c^2\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2}$$
$$+\frac{(ad-be)\left(abd+2ace+b^2(-e)\right)\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2-e(bd-ce)\right)^2}-\frac{e^3}{d^2(d+ex)\left(ad^2-e(bd-ce)\right)}$$
$$+\frac{e^3\log(d+ex)\left(4ad^2-e(3bd-2ce)\right)}{d^3\left(ad^2-e(bd-ce)\right)^2}-\frac{\log(x)(bd+2ce)}{c^2d^3}-\frac{1}{cd^2x}$$

```
[In] Int[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2),x]
```

```
[Out] -(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((2*a^3*c*d^
2 - b^4*e^2 + 2*a*b^2*e*(b*d + 2*c*e) - a^2*(b^2*d^2 + 6*b*c*d*e + 2*c^2*e^
2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 -
 e*(b*d - c*e))^2) - ((b*d + 2*c*e)*Log[x])/(c^2*d^3) + (e^3*(4*a*d^2 - e*(
3*b*d - 2*c*e))*Log[d + e*x])/(d^3*(a*d^2 - e*(b*d - c*e))^2) + ((a*d - b*e
)*(a*b*d - b^2*e + 2*a*c*e)*Log[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d -
c*e))^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^2(d+ex)^2\left(c+bx+ax^2\right)}\,dx \\
&= \int \left(\frac{1}{cd^2x^2}+\frac{-bd-2ce}{c^2d^3x}+\frac{e^4}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)^2}+\frac{e^4(4ad^2-e(3bd-2ce))}{d^3\left(ad^2-e(bd-ce)\right)^2(d+ex)}\right. \\
&\quad \left.+\frac{-a^3cd^2+b^4e^2-ab^2e(2bd+3ce)+a^2(b^2d^2+4bcde+c^2e^2)+a(ad-be)\left(abd-b^2e+2ace\right)x}{c^2\left(ad^2-e(bd-ce)\right)^2\left(c+bx+ax^2\right)}\right) \\
&= -\frac{1}{cd^2x}-\frac{e^3}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)} \\
&\quad -\frac{(bd+2ce)\log(x)}{c^2d^3}+\frac{e^3(4ad^2-e(3bd-2ce))\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)^2} \\
&\quad +\frac{\int \frac{-a^3cd^2+b^4e^2-ab^2e(2bd+3ce)+a^2\left(b^2d^2+4bcde+c^2e^2\right)+a(ad-be)\left(abd-b^2e+2ace\right)x}{c+bx+ax^2}\,dx}{c^2\left(ad^2-e(bd-ce)\right)^2}
\end{aligned}$$

$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2(ad^2 - e(bd - ce))(d + ex)} - \frac{(bd + 2ce)\log(x)}{c^2d^3} + \frac{e^3(4ad^2 - e(3bd - 2ce))\log(d + ex)}{d^3(ad^2 - e(bd - ce))^2} + \frac{((ad - be)(abd - b^2e + 2ace))\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2c^2(ad^2 - e(bd - ce))^2} - \frac{(2a^3cd^2 - b^4e^2 + 2ab^2e(bd + 2ce) - a^2(b^2d^2 + 6bcde + 2c^2e^2))\int \frac{1}{c+bx+ax^2}\,dx}{2c^2(ad^2 - e(bd - ce))^2}$$

$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2(ad^2 - e(bd - ce))(d + ex)} - \frac{(bd + 2ce)\log(x)}{c^2d^3} + \frac{e^3(4ad^2 - e(3bd - 2ce))\log(d + ex)}{d^3(ad^2 - e(bd - ce))^2} + \frac{(ad - be)(abd - b^2e + 2ace)\log(c + bx + ax^2)}{2c^2(ad^2 - e(bd - ce))^2} + \frac{(2a^3cd^2 - b^4e^2 + 2ab^2e(bd + 2ce) - a^2(b^2d^2 + 6bcde + 2c^2e^2))\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b + 2ax\right)}{c^2(ad^2 - e(bd - ce))^2}$$

$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2(ad^2 - e(bd - ce))(d + ex)} + \frac{(2a^3cd^2 - b^4e^2 + 2ab^2e(bd + 2ce) - a^2(b^2d^2 + 6bcde + 2c^2e^2))\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2 - 4ac}\,(ad^2 - e(bd - ce))^2} - \frac{(bd + 2ce)\log(x)}{c^2d^3} + \frac{e^3(4ad^2 - e(3bd - 2ce))\log(d + ex)}{d^3(ad^2 - e(bd - ce))^2} + \frac{(ad - be)(abd - b^2e + 2ace)\log(c + bx + ax^2)}{2c^2(ad^2 - e(bd - ce))^2}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x^4(d + ex)^2}\,dx$$

$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2(ad^2 + e(-bd + ce))(d + ex)} + \frac{(-2a^3cd^2 + b^4e^2 - 2ab^2e(bd + 2ce) + a^2(b^2d^2 + 6bcde + 2c^2e^2))\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2 + 4ac}\,(ad^2 + e(-bd + ce))^2} - \frac{(bd + 2ce)\log(x)}{c^2d^3} + \frac{e^3(4ad^2 + e(-3bd + 2ce))\log(d + ex)}{d^3(ad^2 + e(-bd + ce))^2} + \frac{(ad - be)(abd - b^2e + 2ace)\log(c + x(b + ax))}{2c^2(ad^2 + e(-bd + ce))^2}$$

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2),x]
```

```
[Out] -(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) + ((-2*a^3*
c*d^2 + b^4*e^2 - 2*a*b^2*e*(b*d + 2*c*e) + a^2*(b^2*d^2 + 6*b*c*d*e + 2*c^
2*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*(a*
d^2 + e*(-(b*d) + c*e))^2) - ((b*d + 2*c*e)*Log[x])/(c^2*d^3) + (e^3*(4*a*d
^2 + e*(-3*b*d + 2*c*e))*Log[d + e*x])/(d^3*(a*d^2 + e*(-(b*d) + c*e))^2) +
 ((a*d - b*e)*(a*b*d - b^2*e + 2*a*c*e)*Log[c + x*(b + a*x)])/(2*c^2*(a*d^2
 + e*(-(b*d) + c*e))^2)
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.19

method	result
default	$-\frac{1}{c\,d^{2}x}+\frac{\left(-bd-2ec\right)\ln\left(x\right)}{c^{2}d^{3}}+\frac{\frac{\left(a^{3}b\,d^{2}+2a^{3}cde-2a^{2}b^{2}de-2a^{2}bc\,e^{2}+b^{3}e^{2}a\right)\ln\left(a\,x^{2}+bx+c\right)}{2a}+\frac{2\left(-a^{3}c\,d^{2}+b^{2}d^{2}a^{2}+4a^{2}bcde+a^{2}c^{2}e^{2}-2a\,b\right.}{}}{\left(a\,d^{2}-bde+c\,e^{2}\right)^{2}}$
risch	Expression too large to display

```
[In] int(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c/d^2/x+(-b*d-2*c*e)/c^2/d^3*ln(x)+1/(a*d^2-b*d*e+c*e^2)^2/c^2*(1/2*(a^3
*b*d^2+2*a^3*c*d*e-2*a^2*b^2*d*e-2*a^2*b*c*e^2+a*b^3*e^2)/a*ln(a*x^2+b*x+c)
+2*(-a^3*c*d^2+b^2*d^2*a^2+4*a^2*b*c*d*e+a^2*c^2*e^2-2*a*b^3*d*e-3*a*b^2*c*
e^2+b^4*e^2-1/2*(a^3*b*d^2+2*a^3*c*d*e-2*a^2*b^2*d*e-2*a^2*b*c*e^2+a*b^3*e^
2)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))-e^3/d^2/(a*d
^2-b*d*e+c*e^2)/(e*x+d)+e^3*(4*a*d^2-3*b*d*e+2*c*e^2)/d^3/(a*d^2-b*d*e+c*e^
2)^2*ln(e*x+d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.69

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx=-\frac{e^7}{(ad^4e^4-bd^3e^5+cd^2e^6)(ex+d)}+\frac{(a^2bd^2-2\,ab^2de+2\,a^2cde+b^3e^2-2\,abce^2)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{bde}{(ex+d)^2}-\frac{ce^2}{(ex+d)^2}\right)}{2\,(a^2c^2d^4-2\,abc^2d^3e+b^2c^2d^2e^2+2\,ac^3d^2e^2-2\,bc^3de^3+c^4e^4)}-\frac{(a^2b^2d^2e^2-2\,a^3cd^2e^2-2\,ab^3de^3+6\,a^2bcde^3+b^4e^4-4\,ab^2ce^4+2\,a^2c^2e^4)\arctan\left(-\frac{2\,ad-\frac{2\,ad^2}{ex+d}-be+\frac{2\,bde}{ex+d}-\frac{2\,c}{ex}}{\sqrt{-b^2+4\,ace}}\right.}{(a^2c^2d^4-2\,abc^2d^3e+b^2c^2d^2e^2+2\,ac^3d^2e^2-2\,bc^3de^3+c^4e^4)\sqrt{-b^2+4\,ace}^2}+\frac{e}{cd^3\left(\frac{d}{ex+d}-1\right)}-\frac{(bde+2\,ce^2)\log\left(\left|-\frac{d}{ex+d}+1\right|\right)}{c^2d^3e}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -e^7/((a*d^4*e^4 - b*d^3*e^5 + c*d^2*e^6)*(e*x + d)) + 1/2*(a^2*b*d^2 - 2*a
*b^2*d*e + 2*a^2*c*d*e + b^3*e^2 - 2*a*b*c*e^2)*log(-a + 2*a*d/(e*x + d) -
```

```
a*d^2/(e*x + d)^2 - b*e/(e*x + d) + b*d*e/(e*x + d)^2 - c*e^2/(e*x + d)^2)/
(a^2*c^2*d^4 - 2*a*b*c^2*d^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 - 2*b*c^
3*d*e^3 + c^4*e^4) - (a^2*b^2*d^2*e^2 - 2*a^3*c*d^2*e^2 - 2*a*b^3*d*e^3 + 6
*a^2*b*c*d*e^3 + b^4*e^4 - 4*a*b^2*c*e^4 + 2*a^2*c^2*e^4)*arctan(-(2*a*d -
2*a*d^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*c*e^2/(e*x + d))/(sqrt(-b^2
 + 4*a*c)*e))/((a^2*c^2*d^4 - 2*a*b*c^2*d^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d
^2*e^2 - 2*b*c^3*d*e^3 + c^4*e^4)*sqrt(-b^2 + 4*a*c)*e^2) + e/(c*d^3*(d/(e*
x + d) - 1)) - (b*d*e + 2*c*e^2)*log(abs(-d/(e*x + d) + 1))/(c^2*d^3*e)
```

Mupad [B] (verification not implemented)

Time = 34.25 (sec) , antiderivative size = 4948, normalized size of antiderivative = 17.00

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^4*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(d + e*x)*(2*c*e^5 + 4*a*d^2*e^3 - 3*b*d*e^4))/(a^2*d^7 + b^2*d^5*e^2 +
 c^2*d^3*e^4 - 2*a*b*d^6*e + 2*a*c*d^5*e^2 - 2*b*c*d^4*e^3) - (1/(c*d) + (x
*(2*c*e^3 + a*d^2*e - b*d*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)))/(d*x + e*x
^2) - (log((((a*e*(a^5*b*d^8 + 4*b^3*c^3*e^8 + b^6*d^3*e^5 - 2*a*b^5*d^4*e^
4 - 2*a^4*b^2*d^7*e + 16*a^2*c^4*d*e^7 - 4*b^4*c^2*d*e^7 - b^5*c*d^2*e^6 +
a^2*b^4*d^5*e^3 + a^3*b^3*d^6*e^2 + 16*a^3*c^3*d^3*e^5 + a^4*c^2*d^5*e^3 -
12*a*b*c^4*e^8 + 2*a^5*c*d^7*e - 16*a^2*b^2*c^2*d^3*e^5 + 4*a*b^2*c^3*d*e^7
 - 2*a^4*b*c*d^6*e^2 + 13*a*b^3*c^2*d^2*e^6 - 20*a^2*b*c^3*d^2*e^6 + a^2*b^
3*c*d^4*e^4 + 8*a^3*b*c^2*d^4*e^4))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)^2) - (
((a*e*(a^4*c*d^6 + 8*a*c^4*e^6 - a^3*b^2*d^6 - 2*b^2*c^3*e^6 + b^5*d^3*e^3
- 3*a*b^4*d^4*e^2 + 3*a^2*b^3*d^5*e + b^3*c^2*d*e^5 + b^4*c*d^2*e^4 + 8*a^2
*c^3*d^2*e^4 - 7*a^3*c^2*d^4*e^2 - 4*a*b*c^3*d*e^5 - 7*a^3*b*c*d^5*e - 7*a*
b^3*c*d^3*e^3 - 6*a*b^2*c^2*d^2*e^4 + 12*a^2*b*c^2*d^3*e^3 + 12*a^2*b^2*c*d
^4*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(b^5*e^2 + b^4*e^2*(b^2 - 4
*a*c)^(1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/
2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e - 4*a^3*b*c*d^2 - 6*a*
b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/2) + 10*a^2*b^2*c*
d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) +
 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*
c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d
^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*
b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8
*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c
 - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) - (2*a*e*x*(a*d - b*e)*(a^3*b*d^5 + 8*a*
c^3*e^5 - 2*b^2*c^2*e^5 + b^4*d^2*e^3 - a*b^3*d^3*e^2 - a^2*b^2*d^4*e + 16*
a^2*c^2*d^2*e^3 + 2*a^3*c*d^4*e + 2*b^3*c*d*e^4 - 8*a*b*c^2*d*e^4 - 8*a*b^2
*c*d^2*e^3 + 4*a^2*b*c*d^3*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)))*(b^5*e^2
```

+ b^4*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2
*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e - 4*
a^3*b*c*d^2 - 6*a*b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/
2) + 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^
2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)
*(a*d^2 + c*e^2 - b*d*e)^2) + (a*e*x*(a^6*d^8 + 8*a^2*c^4*e^8 + 4*b^4*c^2*e
^8 + b^6*d^2*e^6 - 16*a*b^2*c^3*e^8 - 2*a*b^5*d^3*e^5 + 2*a^5*c*d^6*e^2 + a
^2*b^4*d^4*e^4 + a^4*b^2*d^6*e^2 + 8*a^3*c^3*d^2*e^6 + 18*a^4*c^2*d^4*e^4 -
 2*a^5*b*d^7*e - 4*b^5*c*d*e^7 - 26*a^2*b^2*c^2*d^2*e^6 + 8*a*b^3*c^2*d*e^7
 + 4*a*b^4*c*d^2*e^6 + 16*a^2*b*c^3*d*e^7 + 6*a^4*b*c*d^5*e^3 + 10*a^2*b^3*
c*d^3*e^5 - 18*a^3*b^2*c*d^4*e^4))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)^2))*(b^
5*e^2 + b^4*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b
^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*
e - 4*a^3*b*c*d^2 - 6*a*b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*
c)^(1/2) + 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d
e(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c
- b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e^4*(b*d + 2*c*e)*(3*a*d^2 + 2*c*e
^2 - 3*b*d*e))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)^2) + (4*a^5*e^4*x*(a*d - b*
e))/(c^2*d^2*(a*d^2 + c*e^2 - b*d*e)^2))*(b^5*e^2 + b^4*e^2*(b^2 - 4*a*c)^(
1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*
a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e - 4*a^3*b*c*d^2 - 6*a*b^3*c*e
^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/2) + 10*a^2*b^2*c*d*e - 4
*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*
b*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^5*e^4 + 4*a^3*c^3*d^4 - b^2*c^4*e^4
 + 2*b^3*c^3*d*e^3 - a^2*b^2*c^2*d^4 + 8*a^2*c^4*d^2*e^2 - b^4*c^2*d^2*e^2
- 8*a*b*c^4*d*e^3 + 2*a*b^3*c^2*d^3*e - 8*a^2*b*c^3*d^3*e + 2*a*b^2*c^3*d^2
*e^2)) + (log((a^4*e^4*(b*d + 2*c*e)*(3*a*d^2 + 2*c*e^2 - 3*b*d*e))/(c^2*d^
4*(a*d^2 + c*e^2 - b*d*e)^2) - (((a*e*(a^5*b*d^8 + 4*b^3*c^3*e^8 + b^6*d^3*
e^5 - 2*a*b^5*d^4*e^4 - 2*a^4*b^2*d^7*e + 16*a^2*c^4*d*e^7 - 4*b^4*c^2*d*e^
7 - b^5*c*d^2*e^6 + a^2*b^4*d^5*e^3 + a^3*b^3*d^6*e^2 + 16*a^3*c^3*d^3*e^5
+ a^4*c^2*d^5*e^3 - 12*a*b*c^4*e^8 + 2*a^5*c*d^7*e - 16*a^2*b^2*c^2*d^3*e^5
 + 4*a*b^2*c^3*d*e^7 - 2*a^4*b*c*d^6*e^2 + 13*a*b^3*c^2*d^2*e^6 - 20*a^2*b*
c^3*d^2*e^6 + a^2*b^3*c*d^4*e^4 + 8*a^3*b*c^2*d^4*e^4))/(c^2*d^4*(a*d^2 + c
*e^2 - b*d*e)^2) - (((a*e*(b^4*e^2*(b^2 - 4*a*c)^(1/2) - b^5*e^2 - a^2*b^3*
d^2 - 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^
2 - 4*a*c)^(1/2) + 2*a*b^4*d*e + 4*a^3*b*c*d^2 + 6*a*b^3*c*e^2 + 8*a^3*c^2*
d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/2) - 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b
^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 -
4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*
d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6
*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*
d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*
a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2
 - b*d*e)^2) - (a*e*(a^4*c*d^6 + 8*a*c^4*e^6 - a^3*b^2*d^6 - 2*b^2*c^3*e^6
+ b^5*d^3*e^3 - 3*a*b^4*d^4*e^2 + 3*a^2*b^3*d^5*e + b^3*c^2*d*e^5 + b^4*c*d

```
^2*e^4 + 8*a^2*c^3*d^2*e^4 - 7*a^3*c^2*d^4*e^2 - 4*a*b*c^3*d*e^5 - 7*a^3*b*
c*d^5*e - 7*a*b^3*c*d^3*e^3 - 6*a*b^2*c^2*d^2*e^4 + 12*a^2*b*c^2*d^3*e^3 +
12*a^2*b^2*c*d^4*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)) + (2*a*e*x*(a*d - b*
e)*(a^3*b*d^5 + 8*a*c^3*e^5 - 2*b^2*c^2*e^5 + b^4*d^2*e^3 - a*b^3*d^3*e^2 -
 a^2*b^2*d^4*e + 16*a^2*c^2*d^2*e^3 + 2*a^3*c*d^4*e + 2*b^3*c*d*e^4 - 8*a*b
*c^2*d*e^4 - 8*a*b^2*c*d^2*e^3 + 4*a^2*b*c*d^3*e^2))/(c*d^2*(a*d^2 + c*e^2
- b*d*e)))*(b^4*e^2*(b^2 - 4*a*c)^(1/2) - b^5*e^2 - a^2*b^3*d^2 - 8*a^2*b*c
^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2
) + 2*a*b^4*d*e + 4*a^3*b*c*d^2 + 6*a*b^3*c*e^2 + 8*a^3*c^2*d*e - 2*a^3*c*d
^2*(b^2 - 4*a*c)^(1/2) - 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/
2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1/2)))/
(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a*e*x*(a^6*d^8 + 8*a^2*c
^4*e^8 + 4*b^4*c^2*e^8 + b^6*d^2*e^6 - 16*a*b^2*c^3*e^8 - 2*a*b^5*d^3*e^5 +
 2*a^5*c*d^6*e^2 + a^2*b^4*d^4*e^4 + a^4*b^2*d^6*e^2 + 8*a^3*c^3*d^2*e^6 +
18*a^4*c^2*d^4*e^4 - 2*a^5*b*d^7*e - 4*b^5*c*d*e^7 - 26*a^2*b^2*c^2*d^2*e^6
 + 8*a*b^3*c^2*d*e^7 + 4*a*b^4*c*d^2*e^6 + 16*a^2*b*c^3*d*e^7 + 6*a^4*b*c*d
^5*e^3 + 10*a^2*b^3*c*d^3*e^5 - 18*a^3*b^2*c*d^4*e^4))/(c^2*d^4*(a*d^2 + c*
e^2 - b*d*e)^2))*(b^4*e^2*(b^2 - 4*a*c)^(1/2) - b^5*e^2 - a^2*b^3*d^2 - 8*a
^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c
)^(1/2) + 2*a*b^4*d*e + 4*a^3*b*c*d^2 + 6*a*b^3*c*e^2 + 8*a^3*c^2*d*e - 2*a
^3*c*d^2*(b^2 - 4*a*c)^(1/2) - 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*
c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1
/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (4*a^5*e^4*x*(a*d -
 b*e))/(c^2*d^2*(a*d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2*(b^2 - 4*a*c)^(1/2) -
b^5*e^2 - a^2*b^3*d^2 - 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) +
 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^4*d*e + 4*a^3*b*c*d^2 + 6*a*b^3*
c*e^2 + 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/2) - 10*a^2*b^2*c*d*e
- 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a
^2*b*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^5*e^4 + 4*a^3*c^3*d^4 - b^2*c^4*
e^4 + 2*b^3*c^3*d*e^3 - a^2*b^2*c^2*d^4 + 8*a^2*c^4*d^2*e^2 - b^4*c^2*d^2*e
^2 - 8*a*b*c^4*d*e^3 + 2*a*b^3*c^2*d^3*e - 8*a^2*b*c^3*d^3*e + 2*a*b^2*c^3*
d^2*e^2)) - (log(x)*(b*d + 2*c*e))/(c^2*d^3)
```

3.78 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 372

$$\begin{aligned}\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx &= -\frac{1}{2cd^2x^2}+\frac{bd+2ce}{c^2d^3x}+\frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)}\\ &+\frac{\left(b^5e^2-a^3cd(3bd+4ce)-ab^3e(2bd+5ce)+a^2b(b^2d^2+8bcde+5c^2e^2)\right)\operatorname{arctanh}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}\\ &+\frac{\left(b^2d^2+2bcde-c(ad^2-3ce^2)\right)\log(x)}{c^3d^4}-\frac{e^4(5ad^2-e(4bd-3ce))\log(d+ex)}{d^4\left(ad^2-e(bd-ce)\right)^2}\\ &+\frac{\left(a^3cd^2-b^4e^2+ab^2e(2bd+3ce)-a^2(b^2d^2+4bcde+c^2e^2)\right)\log\left(c+bx+ax^2\right)}{2c^3\left(ad^2-e(bd-ce)\right)^2}\end{aligned}$$

```
[Out] -1/2/c/d^2/x^2+(b*d+2*c*e)/c^2/d^3/x+e^4/d^3/(a*d^2-e*(b*d-c*e))/(e*x+d)+(b
^2*d^2+2*b*c*d*e-c*(a*d^2-3*c*e^2))*ln(x)/c^3/d^4-e^4*(5*a*d^2-e*(4*b*d-3*c
*e))*ln(e*x+d)/d^4/(a*d^2-e*(b*d-c*e))^2+1/2*(a^3*c*d^2-b^4*e^2+a*b^2*e*(2*
b*d+3*c*e)-a^2*(b^2*d^2+4*b*c*d*e+c^2*e^2))*ln(a*x^2+b*x+c)/c^3/(a*d^2-e*(b
*d-c*e))^2+(b^5*e^2-a^3*c*d*(3*b*d+4*c*e)-a*b^3*e*(2*b*d+5*c*e)+a^2*b*(b^2*
d^2+8*b*c*d*e+5*c^2*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(a*d^2-
e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1583, 907, 648, 632, 212, 642}

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right) x^5 (d+e x)^2} \, dx$$
$$= \frac{\operatorname{arctanh}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)\left(-a^3cd(3bd+4ce)+a^2b(b^2d^2+8bcde+5c^2e^2)-ab^3e(2bd+5ce)+b^5e^2\right)}{c^3\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2}$$
$$+\frac{\left(a^3cd^2-a^2(b^2d^2+4bcde+c^2e^2)+ab^2e(2bd+3ce)+b^4(-e^2)\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)^2}$$
$$+\frac{\log(x)\left(-c(ad^2-3ce^2)+b^2d^2+2bcde\right)}{c^3d^4}-\frac{e^4\log(d+ex)\left(5ad^2-e(4bd-3ce)\right)}{d^4\left(ad^2-e(bd-ce)\right)^2}$$
$$+\frac{e^4}{d^3(d+ex)\left(ad^2-e(bd-ce)\right)}+\frac{bd+2ce}{c^2d^3x}-\frac{1}{2cd^2x^2}$$

```
[In] Int[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2),x]
```

```
[Out] -1/2*1/(c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 - e*(b*d -
 c*e))*(d + e*x)) + ((b^5*e^2 - a^3*c*d*(3*b*d + 4*c*e) - a*b^3*e*(2*b*d +
5*c*e) + a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c^2*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[
b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^
2 + 2*b*c*d*e - c*(a*d^2 - 3*c*e^2))*Log[x])/(c^3*d^4) - (e^4*(5*a*d^2 - e*
(4*b*d - 3*c*e))*Log[d + e*x])/(d^4*(a*d^2 - e*(b*d - c*e))^2) + ((a^3*c*d^
2 - b^4*e^2 + a*b^2*e*(2*b*d + 3*c*e) - a^2*(b^2*d^2 + 4*b*c*d*e + c^2*e^2)
)*Log[c + b*x + a*x^2])/(2*c^3*(a*d^2 - e*(b*d - c*e))^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^3(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cd^2x^3} + \frac{-bd-2ce}{c^2d^3x^2} + \frac{b^2d^2+2bcde-c(ad^2-3ce^2)}{c^3d^4x} + \frac{e^5}{d^3\left(-ad^2+e(bd-ce)\right)(d+ex)^2} + \frac{e^5(-5ad^2+e(4bd-3ce))}{d^4\left(ad^2-e(bd-ce)\right)^2(d+ex)} + \frac{-((abd-b^2e+ace)\left(ab^2d-2a^2cd-b^3e+3abce\right))+a(a^3cd^2-b^4e^2+ab^2e(2bd+3ce)-a^2(b^2d}{c^3\left(ad^2-e(bd-ce)\right)^2\left(c+bx+ax^2\right)}\right.$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{(b^2d^2+2bcde-c(ad^2-3ce^2))\log(x)}{c^3d^4} - \frac{e^4(5ad^2-e(4bd-3ce))\log(d+ex)}{d^4\left(ad^2-e(bd-ce)\right)^2} + \frac{\int \frac{-((abd-b^2e+ace)\left(ab^2d-2a^2cd-b^3e+3abce\right))+a\left(a^3cd^2-b^4e^2+ab^2e(2bd+3ce)-a^2\left(b^2d^2+4bcde+c^2e^2\right)\right)x}{c+bx+ax^2}\,dx}{c^3\left(ad^2-e(bd-ce)\right)^2}$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3(ad^2-e(bd-ce))(d+ex)} + \frac{(b^2d^2+2bcde-c(ad^2-3ce^2))\log(x)}{c^3d^4} - \frac{e^4(5ad^2-e(4bd-3ce))\log(d+ex)}{d^4(ad^2-e(bd-ce))^2} + \frac{(a^3cd^2-b^4e^2+ab^2e(2bd+3ce)-a^2(b^2d^2+4bcde+c^2e^2))\int\frac{b+2ax}{c+bx+ax^2}\,dx}{2c^3(ad^2-e(bd-ce))^2} - \frac{(b^5e^2-a^3cd(3bd+4ce)-ab^3e(2bd+5ce)+a^2b(b^2d^2+8bcde+5c^2e^2))\int\frac{1}{c+bx+ax^2}\,dx}{2c^3(ad^2-e(bd-ce))^2}$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3(ad^2-e(bd-ce))(d+ex)} + \frac{(b^2d^2+2bcde-c(ad^2-3ce^2))\log(x)}{c^3d^4} - \frac{e^4(5ad^2-e(4bd-3ce))\log(d+ex)}{d^4(ad^2-e(bd-ce))^2} + \frac{(a^3cd^2-b^4e^2+ab^2e(2bd+3ce)-a^2(b^2d^2+4bcde+c^2e^2))\log(c+bx+ax^2)}{2c^3(ad^2-e(bd-ce))^2} + \frac{(b^5e^2-a^3cd(3bd+4ce)-ab^3e(2bd+5ce)+a^2b(b^2d^2+8bcde+5c^2e^2))\,\mathrm{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x\right.}{c^3(ad^2-e(bd-ce))^2}$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3(ad^2-e(bd-ce))(d+ex)} + \frac{(b^5e^2-a^3cd(3bd+4ce)-ab^3e(2bd+5ce)+a^2b(b^2d^2+8bcde+5c^2e^2))\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2} + \frac{(b^2d^2+2bcde-c(ad^2-3ce^2))\log(x)}{c^3d^4} - \frac{e^4(5ad^2-e(4bd-3ce))\log(d+ex)}{d^4(ad^2-e(bd-ce))^2} + \frac{(a^3cd^2-b^4e^2+ab^2e(2bd+3ce)-a^2(b^2d^2+4bcde+c^2e^2))\log(c+bx+ax^2)}{2c^3(ad^2-e(bd-ce))^2}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.99

$$\int\frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx = -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3(ad^2+e(-bd+ce))(d+ex)} + \frac{(-b^5e^2+a^3cd(3bd+4ce)+ab^3e(2bd+5ce)-a^2b(b^2d^2+8bcde+5c^2e^2))\arctan\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{c^3\sqrt{-b^2+4ac}\,(ad^2+e(-bd+ce))^2} + \frac{(b^2d^2+2bcde+c(-ad^2+3ce^2))\log(x)}{c^3d^4} - \frac{e^4(5ad^2+e(-4bd+3ce))\log(d+ex)}{d^4(ad^2+e(-bd+ce))^2} - \frac{(-a^3cd^2+b^4e^2-ab^2e(2bd+3ce)+a^2(b^2d^2+4bcde+c^2e^2))\log(c+x(b+ax))}{2c^3(ad^2+e(-bd+ce))^2}$$

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2),x]
```

```
[Out] -1/2*1/(c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 + e*(-(b*d
) + c*e))*(d + e*x)) + ((-(b^5*e^2) + a^3*c*d*(3*b*d + 4*c*e) + a*b^3*e*(2*
b*d + 5*c*e) - a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c^2*e^2))*ArcTan[(b + 2*a*x)/
Sqrt[-b^2 + 4*a*c]])/(c^3*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2)
+ ((b^2*d^2 + 2*b*c*d*e + c*(-(a*d^2) + 3*c*e^2))*Log[x])/(c^3*d^4) - (e^4*
(5*a*d^2 + e*(-4*b*d + 3*c*e))*Log[d + e*x])/(d^4*(a*d^2 + e*(-(b*d) + c*e)
)^2) - ((-(a^3*c*d^2) + b^4*e^2 - a*b^2*e*(2*b*d + 3*c*e) + a^2*(b^2*d^2 +
4*b*c*d*e + c^2*e^2))*Log[c + x*(b + a*x)])/(2*c^3*(a*d^2 + e*(-(b*d) + c*e
))^2)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.22

method	result
default	$-\frac{1}{2c\,d^2x^2}-\frac{-bd-2ec}{x\,c^2d^3}+\frac{\left(-d^2ac+b^2d^2+2bcde+3e^2c^2\right)\ln(x)}{d^4c^3}+\frac{\frac{\left(a^4c\,d^2-a^3b^2d^2-4a^3bcde-a^3c^2e^2+2a^2b^3de+3a^2b^2c\,e^2-a\,b^4e^2\right)\ln\left(a\,x \cdots\right.}{2a}}{\cdots}$
risch	Expression too large to display

```
[In] int(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c/d^2/x^2-(-b*d-2*c*e)/x/c^2/d^3+1/d^4/c^3*(-a*c*d^2+b^2*d^2+2*b*c*d*e
+3*c^2*e^2)*ln(x)+1/(a*d^2-b*d*e+c*e^2)^2/c^3*(1/2*(a^4*c*d^2-a^3*b^2*d^2-4
*a^3*b*c*d*e-a^3*c^2*e^2+2*a^2*b^3*d*e+3*a^2*b^2*c*e^2-a*b^4*e^2)/a*ln(a*x^
2+b*x+c)+2*(2*a^3*b*c*d^2+2*a^3*d*e*c^2-a^2*b^3*d^2-6*a^2*b^2*c*d*e-3*a^2*c
^2*e^2*b+2*a*b^4*d*e+4*e^2*a*c*b^3-b^5*e^2-1/2*(a^4*c*d^2-a^3*b^2*d^2-4*a^3
*b*c*d*e-a^3*c^2*e^2+2*a^2*b^3*d*e+3*a^2*b^2*c*e^2-a*b^4*e^2)*b/a)/(4*a*c-b
^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))+e^4/d^3/(a*d^2-b*d*e+c*e^2)/
(e*x+d)-e^4*(5*a*d^2-4*b*d*e+3*c*e^2)/d^4/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.61

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx = \frac{e^9}{(ad^5e^5-bd^4e^6+cd^3e^7)(ex+d)} - \frac{(a^2b^2d^2-a^3cd^2-2\,ab^3de+4\,a^2bcde+b^4e^2-3\,ab^2ce^2+a^2c^2e^2)\log\left(-a+\frac{2\,ad}{ex+d}-\frac{ad^2}{(ex+d)^2}-\frac{be}{ex+d}+\frac{b}{(ex}\right.}{2\,(a^2c^3d^4-2\,abc^3d^3e+b^2c^3d^2e^2+2\,ac^4d^2e^2-2\,bc^4de^3+c^5e^4)} + \frac{(a^2b^3d^2e^2-3\,a^3bcd^2e^2-2\,ab^4de^3+8\,a^2b^2cde^3-4\,a^3c^2de^3+b^5e^4-5\,ab^3ce^4+5\,a^2bc^2e^4)\arctan\left(-\frac{2\,ad}{}\right.}{(a^2c^3d^4-2\,abc^3d^3e+b^2c^3d^2e^2+2\,ac^4d^2e^2-2\,bc^4de^3+c^5e^4)\sqrt{-b^2+4\,ace^2}} + \frac{(b^2d^2e-acd^2e+2\,bcde^2+3\,c^2e^3)\log\left(\left|-\frac{d}{ex+d}+1\right|\right)}{c^3d^4e} + \frac{2\,bcde+5\,c^2e^2-\frac{2\,(bcd^2e^2+3\,c^2de^3)}{(ex+d)e}}{2\,c^3d^4\left(\frac{d}{ex+d}-1\right)^2}$$

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="giac")
[Out] e^9/((a*d^5*e^5 - b*d^4*e^6 + c*d^3*e^7)*(e*x + d)) - 1/2*(a^2*b^2*d^2 - a^
3*c*d^2 - 2*a*b^3*d*e + 4*a^2*b*c*d*e + b^4*e^2 - 3*a*b^2*c*e^2 + a^2*c^2*e
```

```
^2)*log(-a + 2*a*d/(e*x + d) - a*d^2/(e*x + d)^2 - b*e/(e*x + d) + b*d*e/(e
*x + d)^2 - c*e^2/(e*x + d)^2)/(a^2*c^3*d^4 - 2*a*b*c^3*d^3*e + b^2*c^3*d^2
*e^2 + 2*a*c^4*d^2*e^2 - 2*b*c^4*d*e^3 + c^5*e^4) + (a^2*b^3*d^2*e^2 - 3*a^
3*b*c*d^2*e^2 - 2*a*b^4*d*e^3 + 8*a^2*b^2*c*d*e^3 - 4*a^3*c^2*d*e^3 + b^5*e
^4 - 5*a*b^3*c*e^4 + 5*a^2*b*c^2*e^4)*arctan(-(2*a*d - 2*a*d^2/(e*x + d) -
b*e + 2*b*d*e/(e*x + d) - 2*c*e^2/(e*x + d))/(sqrt(-b^2 + 4*a*c)*e))/((a^2*
c^3*d^4 - 2*a*b*c^3*d^3*e + b^2*c^3*d^2*e^2 + 2*a*c^4*d^2*e^2 - 2*b*c^4*d*e
^3 + c^5*e^4)*sqrt(-b^2 + 4*a*c)*e^2) + (b^2*d^2*e - a*c*d^2*e + 2*b*c*d*e^
2 + 3*c^2*e^3)*log(abs(-d/(e*x + d) + 1))/(c^3*d^4*e) + 1/2*(2*b*c*d*e + 5*
c^2*e^2 - 2*(b*c*d^2*e^2 + 3*c^2*d*e^3)/((e*x + d)*e))/(c^3*d^4*(d/(e*x + d
) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 47.17 (sec) , antiderivative size = 7144, normalized size of antiderivative = 19.20

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right) x^5 (d+e x)^2} \, dx = \text{Too large to display}$$

```
[In] int(1/(x^5*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] ((x*(2*b*d + 3*c*e))/(2*c^2*d^2) - 1/(2*c*d) + (x^2*(3*c^2*e^4 - b^2*d^2*e^
2 + a*b*d^3*e - b*c*d*e^3 + 2*a*c*d^2*e^2))/(c^2*d^3*(a*d^2 + c*e^2 - b*d*e
)))/(d*x^2 + e*x^3) - (log(d + e*x)*(3*c*e^6 + 5*a*d^2*e^4 - 4*b*d*e^5))/(a
^2*d^8 + b^2*d^6*e^2 + c^2*d^4*e^4 - 2*a*b*d^7*e + 2*a*c*d^6*e^2 - 2*b*c*d^
5*e^3) + (log((((27*a^2*b*c^6*e^11 - 9*a*b^3*c^5*e^11 - a*b^8*d^5*e^6 - a^6
*b^3*d^10*e - 36*a^3*c^6*d*e^10 + 2*a^2*b^7*d^6*e^5 - a^3*b^6*d^7*e^4 - a^4
*b^5*d^8*e^3 + 2*a^5*b^4*d^9*e^2 - 36*a^4*c^5*d^3*e^8 + 4*a^5*c^4*d^5*e^6 +
 3*a^6*c^3*d^7*e^4 + a^7*b*c*d^10*e - 39*a^2*b^3*c^4*d^2*e^9 - 15*a^2*b^4*c
^3*d^3*e^8 + 7*a^2*b^5*c^2*d^4*e^7 + 53*a^3*b^2*c^4*d^3*e^8 + 7*a^3*b^3*c^3
*d^4*e^7 - 33*a^3*b^4*c^2*d^5*e^6 + 20*a^4*b^2*c^3*d^5*e^6 + 33*a^4*b^3*c^2
*d^6*e^5 - 9*a^5*b^2*c^2*d^7*e^4 + 6*a*b^4*c^4*d*e^10 - 2*a*b^7*c*d^4*e^7 +
 5*a*b^5*c^3*d^2*e^9 + a*b^6*c^2*d^3*e^8 + 12*a^2*b^6*c*d^5*e^6 + 51*a^3*b*
c^5*d^2*e^9 - 16*a^3*b^5*c*d^6*e^5 - 27*a^4*b*c^4*d^4*e^7 + 6*a^4*b^4*c*d^7
*e^4 - 19*a^5*b*c^3*d^6*e^5 + 3*a^5*b^3*c*d^8*e^3 - a^6*b*c^2*d^8*e^3 - 4*a
^6*b^2*c*d^9*e^2)/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2) + (((a*e*(12*a*c^5*e^
7 - a^3*b^3*d^7 - 3*b^2*c^4*e^7 + b^6*d^4*e^3 - 3*a*b^5*d^5*e^2 + 3*a^2*b^4
*d^6*e + 4*a^4*c^2*d^6*e + b^3*c^3*d*e^6 + b^5*c*d^3*e^4 + 8*a^2*c^4*d^2*e^
5 - 8*a^3*c^3*d^4*e^3 + b^4*c^2*d^2*e^5 + 2*a^4*b*c*d^7 - 4*a*b*c^4*d*e^6 +
 18*a^2*b^2*c^2*d^4*e^3 - 8*a*b^4*c*d^4*e^3 - 10*a^3*b^2*c*d^6*e - 6*a*b^2*
c^3*d^2*e^5 - 7*a*b^3*c^2*d^3*e^4 + 12*a^2*b*c^3*d^3*e^4 + 15*a^2*b^3*c*d^5
*e^2 - 16*a^3*b*c^2*d^5*e^2))/(c^2*d^3*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(4*a
^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*
e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a
*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^
```

2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x -
6*a*b^2*c*d^2*e^2*x)*(b^6*e^2 + b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 +
4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 + a^2*b^3*d^2*(b^2 - 4*a*c
)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d
*e - 16*a^3*b*c^2*d*e - 3*a^3*b*c*d^2*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^2*(
b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*e^2*(b
^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b^2*c*d*e*(b^2
- 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) - (a*e*x*(
2*a^4*b^2*d^7 - 3*a^5*c*d^7 + 6*b^3*c^3*e^7 - 2*b^6*d^3*e^4 + 4*a*b^5*d^4*e
^3 - 4*a^3*b^3*d^6*e + 24*a^2*c^4*d*e^6 - 5*b^4*c^2*d*e^6 - b^5*c*d^2*e^5 +
32*a^3*c^3*d^3*e^4 - 7*a^4*c^2*d^5*e^2 - 24*a*b*c^4*e^7 + 9*a^4*b*c*d^6*e
- 36*a^2*b^2*c^2*d^3*e^4 + 14*a*b^2*c^3*d*e^6 + 15*a*b^4*c*d^3*e^4 + 16*a*b
^3*c^2*d^2*e^5 - 48*a^2*b*c^3*d^2*e^5 - 24*a^2*b^3*c*d^4*e^3 + 32*a^3*b*c^2
*d^4*e^3 + 4*a^3*b^2*c*d^5*e^2))/(c^2*d^3*(a*d^2 + c*e^2 - b*d*e)))*(b^6*e^
2 + b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e
^2 - 5*a^3*b^2*c*d^2 + a^2*b^3*d^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a
^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d*e - 16*a^3*b*c^2*d*e - 3*a^
3*b*c*d^2*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c
^2*d*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*
d*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a
c - b^2)(a*d^2 + c*e^2 - b*d*e)^2) - (x*(18*a^3*c^6*e^11 + 9*a*b^4*c^4*e^
11 + a*b^8*d^4*e^7 + a^7*b^2*d^10*e - 36*a^2*b^2*c^5*e^11 - 2*a^2*b^7*d^5*e
^6 + a^3*b^6*d^6*e^5 + a^5*b^4*d^8*e^3 - 2*a^6*b^3*d^9*e^2 + 6*a^4*c^5*d^2*
e^9 - 10*a^5*c^4*d^4*e^7 - 12*a^6*c^3*d^6*e^5 + 3*a^7*c^2*d^8*e^3 + 44*a^2*
b^4*c^3*d^2*e^9 - 2*a^2*b^5*c^2*d^3*e^8 - 85*a^3*b^2*c^4*d^2*e^9 - 46*a^3*b
^3*c^3*d^3*e^8 + 45*a^3*b^4*c^2*d^4*e^7 - 42*a^4*b^2*c^3*d^4*e^7 - 56*a^4*b
^3*c^2*d^5*e^6 + 19*a^5*b^2*c^2*d^6*e^5 - 6*a*b^5*c^3*d*e^10 + 2*a*b^7*c*d^
3*e^8 + 42*a^3*b*c^5*d*e^10 + 2*a^7*b*c*d^9*e^2 - 5*a*b^6*c^2*d^2*e^9 + 6*a
^2*b^3*c^4*d*e^10 - 12*a^2*b^6*c*d^4*e^7 + 16*a^3*b^5*c*d^5*e^6 + 88*a^4*b*
c^4*d^3*e^8 - 6*a^4*b^4*c*d^6*e^5 + 62*a^5*b*c^3*d^5*e^6 - 2*a^6*b*c^2*d^7*
e^4 - 2*a^6*b^2*c*d^8*e^3))/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2))*(b^6*e^2 +
b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2
- 5*a^3*b^2*c*d^2 + a^2*b^3*d^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*
b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d*e - 16*a^3*b*c^2*d*e - 3*a^3*b
*c*d^2*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*
d*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e
*(b^2 - 4*a*c)^(1/2) + 8*a^2*b^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c
- b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e^4*(a^2*b^2*d^5 - 9*b*c^3*e^5 - a
^3*c*d^5 + 4*b^4*d^3*e^2 + 6*b^2*c^2*d*e^4 + 5*b^3*c*d^2*e^3 + 3*a^2*c^2*d^
3*e^2 - 5*a*b^3*d^4*e + 7*a^2*b*c*d^4*e - 12*a*b*c^2*d^2*e^3 - 14*a*b^2*c*d
^3*e^2))/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2) - (a^5*e^5*x*(9*c^3*e^4 + 4*a*
b^2*d^4 + a^2*c*d^4 - 4*b^3*d^3*e + 12*a*c^2*d^2*e^2 - 5*b^2*c*d^2*e^2 - 6*
b*c^2*d*e^3 + 8*a*b*c*d^3*e))/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2))*(b^6*e^2
+ b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^
2 - 5*a^3*b^2*c*d^2 + a^2*b^3*d^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^

2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d*e - 16*a^3*b*c^2*d*e - 3*a^3
*b*c*d^2*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^
2*d*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d
e(b^2 - 4*a*c)^(1/2) + 8*a^2*b^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^6*
e^4 + 4*a^3*c^4*d^4 - b^2*c^5*e^4 + 2*b^3*c^4*d*e^3 - a^2*b^2*c^3*d^4 + 8*a
^2*c^5*d^2*e^2 - b^4*c^3*d^2*e^2 - 8*a*b*c^5*d*e^3 + 2*a*b^3*c^3*d^3*e - 8*
a^2*b*c^4*d^3*e + 2*a*b^2*c^4*d^2*e^2)) + (log((((27*a^2*b*c^6*e^11 - 9*a*b
^3*c^5*e^11 - a*b^8*d^5*e^6 - a^6*b^3*d^10*e - 36*a^3*c^6*d*e^10 + 2*a^2*b^
7*d^6*e^5 - a^3*b^6*d^7*e^4 - a^4*b^5*d^8*e^3 + 2*a^5*b^4*d^9*e^2 - 36*a^4*
c^5*d^3*e^8 + 4*a^5*c^4*d^5*e^6 + 3*a^6*c^3*d^7*e^4 + a^7*b*c*d^10*e - 39*a
^2*b^3*c^4*d^2*e^9 - 15*a^2*b^4*c^3*d^3*e^8 + 7*a^2*b^5*c^2*d^4*e^7 + 53*a^
3*b^2*c^4*d^3*e^8 + 7*a^3*b^3*c^3*d^4*e^7 - 33*a^3*b^4*c^2*d^5*e^6 + 20*a^4
*b^2*c^3*d^5*e^6 + 33*a^4*b^3*c^2*d^6*e^5 - 9*a^5*b^2*c^2*d^7*e^4 + 6*a*b^4
*c^4*d*e^10 - 2*a*b^7*c*d^4*e^7 + 5*a*b^5*c^3*d^2*e^9 + a*b^6*c^2*d^3*e^8 +
12*a^2*b^6*c*d^5*e^6 + 51*a^3*b*c^5*d^2*e^9 - 16*a^3*b^5*c*d^6*e^5 - 27*a^
4*b*c^4*d^4*e^7 + 6*a^4*b^4*c*d^7*e^4 - 19*a^5*b*c^3*d^6*e^5 + 3*a^5*b^3*c*
d^8*e^3 - a^6*b*c^2*d^8*e^3 - 4*a^6*b^2*c*d^9*e^2)/(c^4*d^6*(a*d^2 + c*e^2
- b*d*e)^2) + (((a*e*(12*a*c^5*e^7 - a^3*b^3*d^7 - 3*b^2*c^4*e^7 + b^6*d^4*
e^3 - 3*a*b^5*d^5*e^2 + 3*a^2*b^4*d^6*e + 4*a^4*c^2*d^6*e + b^3*c^3*d*e^6 +
b^5*c*d^3*e^4 + 8*a^2*c^4*d^2*e^5 - 8*a^3*c^3*d^4*e^3 + b^4*c^2*d^2*e^5 +
2*a^4*b*c*d^7 - 4*a*b*c^4*d*e^6 + 18*a^2*b^2*c^2*d^4*e^3 - 8*a*b^4*c*d^4*e^
3 - 10*a^3*b^2*c*d^6*e - 6*a*b^2*c^3*d^2*e^5 - 7*a*b^3*c^2*d^3*e^4 + 12*a^2
*b*c^3*d^3*e^4 + 15*a^2*b^3*c*d^5*e^2 - 16*a^3*b*c^2*d^5*e^2))/(c^2*d^3*(a*
d^2 + c*e^2 - b*d*e)) + (a*e*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e
^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*
a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3
*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^
2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x)*(b^6*e^2 - b^5*e^2*(b
^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2
*c*d^2 - a^2*b^3*d^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2
- 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d*e - 16*a^3*b*c^2*d*e + 3*a^3*b*c*d^2*(b^2
- 4*a*c)^(1/2) + 5*a*b^3*c*e^2*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*d*e*(b^2 -
4*a*c)^(1/2) - 5*a^2*b*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^4*d*e*(b^2 - 4*a
*c)^(1/2) - 8*a^2*b^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d
^2 + c*e^2 - b*d*e)^2) - (a*e*x*(2*a^4*b^2*d^7 - 3*a^5*c*d^7 + 6*b^3*c^3*e^
7 - 2*b^6*d^3*e^4 + 4*a*b^5*d^4*e^3 - 4*a^3*b^3*d^6*e + 24*a^2*c^4*d*e^6 -
5*b^4*c^2*d*e^6 - b^5*c*d^2*e^5 + 32*a^3*c^3*d^3*e^4 - 7*a^4*c^2*d^5*e^2 -
24*a*b*c^4*e^7 + 9*a^4*b*c*d^6*e - 36*a^2*b^2*c^2*d^3*e^4 + 14*a*b^2*c^3*d*
e^6 + 15*a*b^4*c*d^3*e^4 + 16*a*b^3*c^2*d^2*e^5 - 48*a^2*b*c^3*d^2*e^5 - 24
*a^2*b^3*c*d^4*e^3 + 32*a^3*b*c^2*d^4*e^3 + 4*a^3*b^2*c*d^5*e^2))/(c^2*d^3*
(a*d^2 + c*e^2 - b*d*e)))*(b^6*e^2 - b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*
d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 - a^2*b^3*d^2*(b^2 -
4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^
3*c*d*e - 16*a^3*b*c^2*d*e + 3*a^3*b*c*d^2*(b^2 - 4*a*c)^(1/2) + 5*a*b^3*c*
e^2*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) - 5*a^2*b*c^2*e

```
^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b^2*c*d*e*
(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) - (x*
(18*a^3*c^6*e^11 + 9*a*b^4*c^4*e^11 + a*b^8*d^4*e^7 + a^7*b^2*d^10*e - 36*a
^2*b^2*c^5*e^11 - 2*a^2*b^7*d^5*e^6 + a^3*b^6*d^6*e^5 + a^5*b^4*d^8*e^3 - 2
*a^6*b^3*d^9*e^2 + 6*a^4*c^5*d^2*e^9 - 10*a^5*c^4*d^4*e^7 - 12*a^6*c^3*d^6*
e^5 + 3*a^7*c^2*d^8*e^3 + 44*a^2*b^4*c^3*d^2*e^9 - 2*a^2*b^5*c^2*d^3*e^8 -
85*a^3*b^2*c^4*d^2*e^9 - 46*a^3*b^3*c^3*d^3*e^8 + 45*a^3*b^4*c^2*d^4*e^7 -
42*a^4*b^2*c^3*d^4*e^7 - 56*a^4*b^3*c^2*d^5*e^6 + 19*a^5*b^2*c^2*d^6*e^5 -
6*a*b^5*c^3*d*e^10 + 2*a*b^7*c*d^3*e^8 + 42*a^3*b*c^5*d*e^10 + 2*a^7*b*c*d^
9*e^2 - 5*a*b^6*c^2*d^2*e^9 + 6*a^2*b^3*c^4*d*e^10 - 12*a^2*b^6*c*d^4*e^7 +
 16*a^3*b^5*c*d^5*e^6 + 88*a^4*b*c^4*d^3*e^8 - 6*a^4*b^4*c*d^6*e^5 + 62*a^5
*b*c^3*d^5*e^6 - 2*a^6*b*c^2*d^7*e^4 - 2*a^6*b^2*c*d^8*e^3))/(c^4*d^6*(a*d^
2 + c*e^2 - b*d*e)^2))*(b^6*e^2 - b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2
 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 - a^2*b^3*d^2*(b^2 - 4*a
*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c
*d*e - 16*a^3*b*c^2*d*e + 3*a^3*b*c*d^2*(b^2 - 4*a*c)^(1/2) + 5*a*b^3*c*e^2
*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) - 5*a^2*b*c^2*e^2*
(b^2 - 4*a*c)^(1/2) + 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b^2*c*d*e*(b^
2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e
^4*(a^2*b^2*d^5 - 9*b*c^3*e^5 - a^3*c*d^5 + 4*b^4*d^3*e^2 + 6*b^2*c^2*d*e^4
 + 5*b^3*c*d^2*e^3 + 3*a^2*c^2*d^3*e^2 - 5*a*b^3*d^4*e + 7*a^2*b*c*d^4*e -
12*a*b*c^2*d^2*e^3 - 14*a*b^2*c*d^3*e^2))/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^
2) - (a^5*e^5*x*(9*c^3*e^4 + 4*a*b^2*d^4 + a^2*c*d^4 - 4*b^3*d^3*e + 12*a*c
^2*d^2*e^2 - 5*b^2*c*d^2*e^2 - 6*b*c^2*d*e^3 + 8*a*b*c*d^3*e))/(c^4*d^6*(a*
d^2 + c*e^2 - b*d*e)^2))*(b^6*e^2 - b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d
^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 - a^2*b^3*d^2*(b^2 - 4
*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3
*c*d*e - 16*a^3*b*c^2*d*e + 3*a^3*b*c*d^2*(b^2 - 4*a*c)^(1/2) + 5*a*b^3*c*e
^2*(b^2 - 4*a*c)^(1/2) + 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) - 5*a^2*b*c^2*e^
2*(b^2 - 4*a*c)^(1/2) + 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b^2*c*d*e*(
b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^6*e^4 + 4*a^3*c^4*d^4 - b^2*c^5*e^4 + 2*b^3*
c^4*d*e^3 - a^2*b^2*c^3*d^4 + 8*a^2*c^5*d^2*e^2 - b^4*c^3*d^2*e^2 - 8*a*b*c
^5*d*e^3 + 2*a*b^3*c^3*d^3*e - 8*a^2*b*c^4*d^3*e + 2*a*b^2*c^4*d^2*e^2)) +
(log(x)*(3*c^2*e^2 - d^2*(a*c - b^2) + 2*b*c*d*e))/(c^3*d^4)
```

3.79 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 981

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx =$$

$$-\frac{2(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2(3b^2d^2-29bcde+50c^2e^2))\sqrt{a+}}{3465a^4e^4}$$

$$+\frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex}$$

$$+\frac{2(233a^3d^3+48b^3e^3+abe^2(67bd-157ce)+4a^2de(18bd-37ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{3465a^3e^4}$$

$$-\frac{2(29a^2d^2+8b^2e^2+ae(19bd-18ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{5/2}}{693a^2e^4}$$

$$+\frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{7/2}}{99ae^4}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(128a^5d^5+128b^5e^5-4a^4d^3e(14bd-27ce)-8ab^3e^4(7bd+87ce)-a^2be^3(37b^2d^2-258bcde}{346}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(ad^2-e(bd-ce))(128a^4d^4-64b^4e^4-4ab^2e^3(7bd-69ce)+4a^3d^2e(2bd+3ce)-3a^2e^2(}{34}$$

```
[Out] 2/3465*(233*a^3*d^3+48*b^3*e^3+a*b*e^2*(67*b*d-157*c*e)+4*a^2*d*e*(18*b*d-3
7*c*e))*x*(e*x+d)^(3/2)*(a+c/x^2+b/x)^(1/2)/a^3/e^4-2/693*(29*a^2*d^2+8*b^2
```

```
*e^2+a*e*(19*b*d-18*c*e))*x*(e*x+d)^(5/2)*(a+c/x^2+b/x)^(1/2)/a^2/e^4+2/99*
(a*d+b*e)*x*(e*x+d)^(7/2)*(a+c/x^2+b/x)^(1/2)/a/e^4-2/3465*(187*a^4*d^4+64*
b^4*e^4+4*a*b^2*e^3*(7*b*d-69*c*e)-4*a^3*d^2*e*(2*b*d+3*c*e)+3*a^2*e^2*(3*b
^2*d^2-29*b*c*d*e+50*c^2*e^2))*x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/a^4/e^4+
2/11*x^5*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)+1/3465*(128*a^5*d^5+128*b^5*e^5-
4*a^4*d^3*e*(14*b*d-27*c*e)-8*a*b^3*e^4*(7*b*d+87*c*e)-a^2*b*e^3*(37*b^2*d^
2-258*b*c*d*e-771*c^2*e^2)-a^3*d*e^2*(37*b^2*d^2-135*b*c*d*e+156*c^2*e^2))*
x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/
(-4*a*c+b^2))^(1/2)/a^5/e^5/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^
2)^(1/2))))^(1/2)-2/3465*(a*d^2-e*(b*d-c*e))*(128*a^4*d^4-64*b^4*e^4-4*a*b^
2*e^3*(7*b*d-69*c*e)+4*a^3*d^2*e*(2*b*d+3*c*e)-3*a^2*e^2*(3*b^2*d^2-29*b*c*
d*e+50*c^2*e^2))*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/
2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+
c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a
^5/e^5/(a*x^2+b*x+c)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 3.62 (sec) , antiderivative size = 981, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used

$= \{1587, 932, 1667, 857, 732, 435, 430\}$

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx$$

$$= \frac{2}{11}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}x^5 + \frac{2(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(d+ex)^{7/2}x}{99ae^4}$$
$$- \frac{2(29a^2d^2+8b^2e^2+ae(19bd-18ce))\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(d+ex)^{5/2}x}{693a^2e^4}$$
$$+ \frac{2(233a^3d^3+4a^2e(18bd-37ce)d+48b^3e^3+abe^2(67bd-157ce))\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(d+ex)^{3/2}x}{3465a^3e^4}$$
$$+ \frac{\sqrt{2}\sqrt{b^2-4ac}(128a^5d^5-4a^4e(14bd-27ce)d^3-a^3e^2(37b^2d^2-135bced+156c^2e^2)d+128b^5e^5-8ab^3e^4}{346}$$
$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}(ad^2-e(bd-ce))(128a^4d^4+4a^3e(2bd+3ce)d^2-64b^4e^4-4ab^2e^3(7bd-69ce)-3a^2e^2(}{34}$$
$$- \frac{2(187a^4d^4-4a^3e(2bd+3ce)d^2+64b^4e^4+4ab^2e^3(7bd-69ce)+3a^2e^2(3b^2d^2-29bced+50c^2e^2))\sqrt{a+}}{3465a^4e^4}$$

```
[In] Int[Sqrt[a + c/x^2 + b/x]*x^4*Sqrt[d + e*x],x]
```

```
[Out] (-2*(187*a^4*d^4 + 64*b^4*e^4 + 4*a*b^2*e^3*(7*b*d - 69*c*e) - 4*a^3*d^2*e*
(2*b*d + 3*c*e) + 3*a^2*e^2*(3*b^2*d^2 - 29*b*c*d*e + 50*c^2*e^2))*Sqrt[a +
 c/x^2 + b/x]*x*Sqrt[d + e*x])/(3465*a^4*e^4) + (2*Sqrt[a + c/x^2 + b/x]*x^
5*Sqrt[d + e*x])/11 + (2*(233*a^3*d^3 + 48*b^3*e^3 + a*b*e^2*(67*b*d - 157*
c*e) + 4*a^2*d*e*(18*b*d - 37*c*e))*Sqrt[a + c/x^2 + b/x]*x*(d + e*x)^(3/2)
)/(3465*a^3*e^4) - (2*(29*a^2*d^2 + 8*b^2*e^2 + a*e*(19*b*d - 18*c*e))*Sqrt
[a + c/x^2 + b/x]*x*(d + e*x)^(5/2))/(693*a^2*e^4) + (2*(a*d + b*e)*Sqrt[a
+ c/x^2 + b/x]*x*(d + e*x)^(7/2))/(99*a*e^4) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
128*a^5*d^5 + 128*b^5*e^5 - 4*a^4*d^3*e*(14*b*d - 27*c*e) - 8*a*b^3*e^4*(7*
b*d + 87*c*e) - a^2*b*e^3*(37*b^2*d^2 - 258*b*c*d*e - 771*c^2*e^2) - a^3*d*
e^2*(37*b^2*d^2 - 135*b*c*d*e + 156*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[
d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
- 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3465*a^5*e^5*Sqrt[(a*(d
+ e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) - (2*Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))*(128*a^4*d^4 - 64*b^4*e^4 - 4*a
*b^2*e^3*(7*b*d - 69*c*e) + 4*a^3*d^2*e*(2*b*d + 3*c*e) - 3*a^2*e^2*(3*b^2*
d^2 - 29*b*c*d*e + 50*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/
```

```
(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*
a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
])/(3465*a^5*e^5*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
+ 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
```

```
-1]
```

Rule 1587

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p]), Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int x^3\sqrt{d+ex}\sqrt{c+bx+ax^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{x^3(-3cd-2(bd+ce)x-(ad+be)x^2)}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{11\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex} + \frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{7/2}}{99ae^4} - \frac{\left(2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{\frac{1}{2}d^3e(ad+be)(bd+7ce)+\frac{1}{2}d^2e(ad+be)(2ad^2+e(11bd+21ce))x+\frac{3}{2}de^2(ad+be)(5ad^2+e(9bd+7ce))x^2+\frac{1}{2}e\cdots}{\sqrt{d+ex}\sqrt{c+bx+\cdots}}}{99ae^5\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex}$$
$$-\frac{2(29a^2d^2+8b^2e^2+ae(19bd-18ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{5/2}}{693a^2e^4}$$
$$+\frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{7/2}}{99ae^4}$$
$$-\frac{\left(4\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{-\frac{1}{2}d^2e^5\left(4b^2e^2(bd+5ce)+a^2d^2(11bd+48ce)+ae\left(6b^2d^2+14bcde-45c^2e^2\right)\right)-\frac{1}{4}de^5\left(44a^3d^4+16b^2e^3(4bd\right.}{}}{}$$

$$= \frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex}$$
$$+\frac{2(233a^3d^3+48b^3e^3+abe^2(67bd-157ce)+4a^2de(18bd-37ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{3465a^3e^4}$$
$$-\frac{2(29a^2d^2+8b^2e^2+ae(19bd-18ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{5/2}}{693a^2e^4}$$
$$+\frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{7/2}}{99ae^4}$$
$$-\frac{\left(8\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{\frac{3}{8}de^8\left(16b^3e^3(bd+3ce)+a^3d^3(41bd+73ce)+abe^2\left(9b^2d^2-52bcde-157c^2e^2\right)+2a^2de\left(2b^2d^2-12bcde+c^2e\right.\right.}{}}{}$$

$$=$$
$$-\frac{2(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2(3b^2d^2-29bcde+50c^2e}{3465a^4e^4}$$
$$+\frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex}$$
$$+\frac{2(233a^3d^3+48b^3e^3+abe^2(67bd-157ce)+4a^2de(18bd-37ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{3465a^3e^4}$$
$$-\frac{2(29a^2d^2+8b^2e^2+ae(19bd-18ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{5/2}}{693a^2e^4}$$
$$+\frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{7/2}}{99ae^4}$$
$$-\frac{\left(16\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{-\frac{3}{8}e^{10}\left(16a^4d^4(2bd-ce)+32b^4e^4(bd+ce)-a^3d^2e\left(10b^2d^2-26bcde+9c^2e^2\right)-2ab^2e^3\left(5b^2d^2+98bcde+\right.\right.}{}}{}$$

$$= -\frac{2(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2(3b^2d^2-29bcde+50c^2e^2)}{3465a^4e^4}$$

$$+\frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex}$$

$$+\frac{2(233a^3d^3+48b^3e^3+abe^2(67bd-157ce)+4a^2de(18bd-37ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{3465a^3e^4}$$

$$-\frac{2(29a^2d^2+8b^2e^2+ae(19bd-18ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{5/2}}{693a^2e^4}$$

$$+\frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{7/2}}{99ae^4}$$

$$+\frac{\Big((128a^5d^5+128b^5e^5-4a^4d^3e(14bd-27ce)-8ab^3e^4(7bd+87ce)-a^2be^3(37b^2d^2-258bcde-7}{3465a^4e^5\sqrt{c+bx+a}}$$

$$-\frac{\Big(16\big(\frac{3}{16}de^{10}(128a^5d^5+128b^5e^5-4a^4d^3e(14bd-27ce)-8ab^3e^4(7bd+87ce)-a^2be^3(37b^2d^2-25}{}$$

$$= -\frac{2(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2(3b^2d^2-29bcde+50c^2e^2)}{3465a^4e^4}$$

$$+\frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex}$$

$$+\frac{2(233a^3d^3+48b^3e^3+abe^2(67bd-157ce)+4a^2de(18bd-37ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{3465a^3e^4}$$

$$-\frac{2(29a^2d^2+8b^2e^2+ae(19bd-18ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{5/2}}{693a^2e^4}$$

$$+\frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{7/2}}{99ae^4}$$

$$+\frac{\Big(\sqrt{2}\sqrt{b^2-4ac}(128a^5d^5+128b^5e^5-4a^4d^3e(14bd-27ce)-8ab^3e^4(7bd+87ce)-a^2be^3(37b^2d^2}{}$$

$$-\frac{\Big(32\sqrt{2}\sqrt{b^2-4ac}\big(\frac{3}{16}de^{10}(128a^5d^5+128b^5e^5-4a^4d^3e(14bd-27ce)-8ab^3e^4(7bd+87ce)-a^2b}{}$$

$$= -\frac{2(187a^4d^4 + 64b^4e^4 + 4ab^2e^3(7bd - 69ce) - 4a^3d^2e(2bd + 3ce) + 3a^2e^2(3b^2d^2 - 29bcde + 50c^2e}{3465a^4e^4}$$

$$+ \frac{2}{11}\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x^5\sqrt{d + ex}$$

$$+ \frac{2(233a^3d^3 + 48b^3e^3 + abe^2(67bd - 157ce) + 4a^2de(18bd - 37ce))\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x(d + ex)^{3/2}}{3465a^3e^4}$$

$$- \frac{2(29a^2d^2 + 8b^2e^2 + ae(19bd - 18ce))\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x(d + ex)^{5/2}}{693a^2e^4}$$

$$+ \frac{2(ad + be)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x(d + ex)^{7/2}}{99ae^4}$$

$$+ \frac{\sqrt{2}\sqrt{b^2 - 4ac}(128a^5d^5 + 128b^5e^5 - 4a^4d^3e(14bd - 27ce) - 8ab^3e^4(7bd + 87ce) - a^2be^3(37b^2d^2}{}$$

$$- \frac{2\sqrt{2}\sqrt{b^2 - 4ac}(ad^2 - bde + ce^2)(128a^4d^4 + 8a^3bd^3e - 9a^2b^2d^2e^2 + 12a^3cd^2e^2 - 28ab^3de^3 + 87a}{}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.47 (sec) , antiderivative size = 10904, normalized size of antiderivative = 11.12

$$\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x^4\sqrt{d + ex}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*x^4*Sqrt[d + e*x],x]
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5003 vs. 2(899) = 1798.

Time = 2.70 (sec) , antiderivative size = 5004, normalized size of antiderivative = 5.10

method	result	size
risch	Expression too large to display	5004
default	Expression too large to display	11938

```
[In] int(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 920, normalized size of antiderivative = 0.94

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx =$$

$$-\frac{2\left((128\,a^6d^6-120\,a^5bd^5e-3\,(11\,a^4b^2-68\,a^5c)d^4e^2-(20\,a^3b^3-87\,a^4bc)d^3e^3-3\,(11\,a^2b^4-53\,a^3b^2c+\right.}{}$$

```
[In] integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
[Out] -2/10395*((128*a^6*d^6 - 120*a^5*b*d^5*e - 3*(11*a^4*b^2 - 68*a^5*c)*d^4*e^
2 - (20*a^3*b^3 - 87*a^4*b*c)*d^3*e^3 - 3*(11*a^2*b^4 - 53*a^3*b^2*c + 34*a
^4*c^2)*d^2*e^4 - 3*(40*a*b^5 - 246*a^2*b^3*c + 329*a^3*b*c^2)*d*e^5 + (128
*b^6 - 888*a*b^4*c + 1599*a^2*b^2*c^2 - 450*a^3*c^3)*e^6)*sqrt(a*e)*weierst
rassPInverse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e^2), -4/27*(
2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e
^3)/(a^3*e^3), 1/3*(3*a*e*x + a*d + b*e)/(a*e)) + 3*(128*a^6*d^5*e - 56*a^5
*b*d^4*e^2 - (37*a^4*b^2 - 108*a^5*c)*d^3*e^3 - (37*a^3*b^3 - 135*a^4*b*c)*
d^2*e^4 - 2*(28*a^2*b^4 - 129*a^3*b^2*c + 78*a^4*c^2)*d*e^5 + (128*a*b^5 -
696*a^2*b^3*c + 771*a^3*b*c^2)*e^6)*sqrt(a*e)*weierstrassZeta(4/3*(a^2*d^2
- a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e^2), -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e
- 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(a^3*e^3), weierstrass
PInverse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e^2), -4/27*(2*a^
3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/
(a^3*e^3), 1/3*(3*a*e*x + a*d + b*e)/(a*e))) - 3*(315*a^6*e^6*x^5 + 35*(a^6
*d*e^5 + a^5*b*e^6)*x^4 - 10*(4*a^6*d^2*e^4 - a^5*b*d*e^5 + (4*a^4*b^2 - 9*
a^5*c)*e^6)*x^3 + (48*a^6*d^3*e^3 - 13*a^5*b*d^2*e^4 - (13*a^4*b^2 - 32*a^5
*c)*d*e^5 + (48*a^3*b^3 - 157*a^4*b*c)*e^6)*x^2 - 2*(32*a^6*d^4*e^2 - 10*a^
```

```
5*b*d^3*e^3 - (9*a^4*b^2 - 23*a^5*c)*d^2*e^4 - 5*(2*a^3*b^3 - 7*a^4*b*c)*d*
e^5 + (32*a^2*b^4 - 138*a^3*b^2*c + 75*a^4*c^2)*e^6)*x)*sqrt(e*x + d)*sqrt(
(a*x^2 + b*x + c)/x^2))/(a^6*e^6)
```

Sympy [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx=\int x^4\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

```
[In] integrate(x**4*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
[Out] Integral(x**4*sqrt(d + e*x)*sqrt(a + b/x + c/x**2), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx=\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^4\,dx$$

```
[In] integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^4, x)
```

Giac [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx=\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^4\,dx$$

```
[In] integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx=\int x^4\,\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

```
[In] int(x^4*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
[Out] int(x^4*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

3.80 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 778

$$\begin{aligned}\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx &= \frac{2(19a^3d^3-6a^2cde^2+8b^3e^3+3abe^2(bd-9ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{315a^3e^3} \\ &\quad+\frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex} \\ &\quad-\frac{4(8a^2d^2+3b^2e^2+ae(4bd-7ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{315a^2e^3} \\ &\quad+\frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{5/2}}{63ae^3} \\ &\quad-\frac{2\sqrt{2}\sqrt{b^2-4ac}(8a^4d^4+8b^4e^4-a^3d^2e(4bd-9ce)-4ab^2e^3(bd+9ce)-3a^2e^2(b^2d^2-5bcde-7c^2e^2))\sqrt{a}}{315a^4e^4\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+} \\ &\quad+\frac{2\sqrt{2}\sqrt{b^2-4ac}(16a^3d^3+6a^2cde^2-8b^3e^3-3abe^2(bd-9ce))(ad^2-e(bd-ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d}{2ad-\left(b+\right.}}}{315a^4e^4\sqrt{d+ex}(c+bx+}\end{aligned}$$

```
[Out] -4/315*(8*a^2*d^2+3*b^2*e^2+a*e*(4*b*d-7*c*e))*x*(e*x+d)^(3/2)*(a+c/x^2+b/x
)^(1/2)/a^2/e^3+2/63*(a*d+b*e)*x*(e*x+d)^(5/2)*(a+c/x^2+b/x)^(1/2)/a/e^3+2/
315*(19*a^3*d^3-6*a^2*c*d*e^2+8*b^3*e^3+3*a*b*e^2*(b*d-9*c*e))*x*(a+c/x^2+b
```

```
/x)^(1/2)*(e*x+d)^(1/2)/a^3/e^3+2/9*x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)-2
/315*(8*a^4*d^4+8*b^4*e^4-a^3*d^2*e*(4*b*d-9*c*e)-4*a*b^2*e^3*(b*d+9*c*e)-3
*a^2*e^2*(b^2*d^2-5*b*c*d*e-7*c^2*e^2))*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b
^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*
d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x
)^(1/2)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/a^4/e^4/(a*x^2+
b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/315*(16*a^3*d^3
+6*a^2*c*d*e^2-8*b^3*e^3-3*a*b*e^2*(b*d-9*c*e))*(a*d^2-e*(b*d-c*e))*x*Ellip
ticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-
2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4
*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(
a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a^4/e^4/(a*x^2+b*x+c)/(e*
x+d)^(1/2)
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 778, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1587, 932, 1667, 857, 732, 435, 430}

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx$$

$$= -\frac{4x(d+ex)^{3/2}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(8a^2d^2+ae(4bd-7ce)+3b^2e^2)}{315a^2e^3}$$

$$+\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(19a^3d^3-6a^2cde^2+3abe^2(bd-9ce)+8b^3e^3)}{315a^3e^3}$$

$$-\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}(8a^4d^4-a^3d^2e(4bd-9ce)-3a^2e^2(b^2d^2-5bcde-}{315a^4e^4(ax^2+bx+c)\sqrt{\frac{}{2ad-}}}$$

$$+\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}(16a^3d^3+6a^2cde^2-3abe^2(bd-9ce)-8b^3e^3)(ad^2-e(bd}{315a^4e^4\sqrt{d+ex}(ax^2+b}$$

$$+\frac{2x(d+ex)^{5/2}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(ad+be)}{63ae^3}+\frac{2}{9}x^4\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}$$

```
[In] Int[Sqrt[a + c/x^2 + b/x]*x^3*Sqrt[d + e*x],x]
```

```
[Out] (2*(19*a^3*d^3 - 6*a^2*c*d*e^2 + 8*b^3*e^3 + 3*a*b*e^2*(b*d - 9*c*e))*Sqrt[
a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(315*a^3*e^3) + (2*Sqrt[a + c/x^2 + b/x]*
```

```
x^4*Sqrt[d + e*x])/9 - (4*(8*a^2*d^2 + 3*b^2*e^2 + a*e*(4*b*d - 7*c*e))*Sqr
t[a + c/x^2 + b/x]*x*(d + e*x)^(3/2))/(315*a^2*e^3) + (2*(a*d + b*e)*Sqrt[a
 + c/x^2 + b/x]*x*(d + e*x)^(5/2))/(63*a*e^3) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(8*a^4*d^4 + 8*b^4*e^4 - a^3*d^2*e*(4*b*d - 9*c*e) - 4*a*b^2*e^3*(b*d + 9
*c*e) - 3*a^2*e^2*(b^2*d^2 - 5*b*c*d*e - 7*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*
x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315*a^4*e^4*Sqrt[
(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (2*
Sqrt[2]*Sqrt[b^2 - 4*a*c]*(16*a^3*d^3 + 6*a^2*c*d*e^2 - 8*b^3*e^3 - 3*a*b*e
^2*(b*d - 9*c*e))*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(
d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))
/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt
[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 -
4*a*c])*e)])/(315*a^4*e^4*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
+ 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1587

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p]), Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int x^2\sqrt{d+ex}\sqrt{c+bx+ax^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{x^2(-3cd-2(bd+ce)x-(ad+be)x^2)}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{9\sqrt{c+bx+ax^2}}$$

$$=\frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}+\frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{5/2}}{63ae^3}-\frac{\left(2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{\frac{1}{2}d^2e(ad+be)(bd+5ce)+de(ad+be)\left(ad^2+e(4bd+5ce)\right)x+\frac{1}{2}e^2\left(11a^2d^3+8ade(3bd-2ce)+be^2(13bd+5ce)\right)}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}}{63ae^4\sqrt{c+bx+ax^2}}$$

$$=\frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}-\frac{4(8a^2d^2+3b^2e^2+ae(4bd-7ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{315a^2e^3}+\frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{5/2}}{63ae^3}-\frac{\left(4\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{-\frac{1}{4}de^4\left(6b^2e^2(bd+3ce)+a^2d^2(11bd+23ce)+3ae\left(b^2d^2-5bcde-14c^2e^2\right)\right)-\frac{1}{2}e^4\left(11a^3d^4+a^2d^2e(23bd-15c\right.}{\sqrt{d+ex}\sqrt{c}}}{315a^2e^7\sqrt{c+bx+a}}$$

$$=\frac{2(19a^3d^3-6a^2cde^2+8b^3e^3+3abe^2(bd-9ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{315a^3e^3}+\frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}-\frac{4(8a^2d^2+3b^2e^2+ae(4bd-7ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{315a^2e^3}+\frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{5/2}}{63ae^3}-\frac{\left(8\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{\frac{3}{8}e^6\left(4a^3d^3(2bd-ce)+8b^3e^3(bd+ce)-3a^2de\left(b^2d^2-3bcde-12c^2e^2\right)-3abe^2\left(b^2d^2+14bcde+9c^2e^2\right)\right)+\frac{3}{4}e^6(}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}}{945a^3e^9\sqrt{c+bx+ax^2}}$$

$$= \frac{2(19a^3d^3 - 6a^2cde^2 + 8b^3e^3 + 3abe^2(bd - 9ce))\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x\sqrt{d + ex}}{315a^3e^3}$$
$$+ \frac{2}{9}\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x^4\sqrt{d + ex}$$
$$- \frac{4(8a^2d^2 + 3b^2e^2 + ae(4bd - 7ce))\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x(d + ex)^{3/2}}{315a^2e^3}$$
$$+ \frac{2(ad + be)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x(d + ex)^{5/2}}{63ae^3}$$
$$- \frac{\left(2(8a^4d^4 + 8b^4e^4 - a^3d^2e(4bd - 9ce) - 4ab^2e^3(bd + 9ce) - 3a^2e^2(b^2d^2 - 5bcde - 7c^2e^2))\sqrt{a +}\right.}{315a^3e^4\sqrt{c + bx + ax^2}}$$
$$- \frac{\left(8\left(-\frac{3}{4}de^6(8a^4d^4 + 8b^4e^4 - a^3d^2e(4bd - 9ce) - 4ab^2e^3(bd + 9ce) - 3a^2e^2(b^2d^2 - 5bcde - 7c^2e^2)\right.\right.}{}$$

$$= \frac{2(19a^3d^3 - 6a^2cde^2 + 8b^3e^3 + 3abe^2(bd - 9ce))\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x\sqrt{d + ex}}{315a^3e^3}$$
$$+ \frac{2}{9}\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x^4\sqrt{d + ex}$$
$$- \frac{4(8a^2d^2 + 3b^2e^2 + ae(4bd - 7ce))\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x(d + ex)^{3/2}}{315a^2e^3}$$
$$+ \frac{2(ad + be)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x(d + ex)^{5/2}}{63ae^3}$$
$$- \frac{\left(2\sqrt{2}\sqrt{b^2 - 4ac}(8a^4d^4 + 8b^4e^4 - a^3d^2e(4bd - 9ce) - 4ab^2e^3(bd + 9ce) - 3a^2e^2(b^2d^2 - 5bcde -\right.}{315a^4e^4\sqrt{\frac{a(d+}{2ad-be-\sqrt{}}}}$$
$$- \frac{\left(16\sqrt{2}\sqrt{b^2 - 4ac}\left(-\frac{3}{4}de^6(8a^4d^4 + 8b^4e^4 - a^3d^2e(4bd - 9ce) - 4ab^2e^3(bd + 9ce) - 3a^2e^2(b^2d^2 -\right.\right.}{}$$

$$= \frac{2(19a^3d^3 - 6a^2cde^2 + 8b^3e^3 + 3abe^2(bd - 9ce))\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x\sqrt{d + ex}}{315a^3e^3}$$
$$+ \frac{2}{9}\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x^4\sqrt{d + ex}$$
$$- \frac{4(8a^2d^2 + 3b^2e^2 + ae(4bd - 7ce))\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x(d + ex)^{3/2}}{315a^2e^3}$$
$$+ \frac{2(ad + be)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x(d + ex)^{5/2}}{63ae^3}$$
$$- \frac{2\sqrt{2}\sqrt{b^2 - 4ac}(8a^4d^4 + 8b^4e^4 - a^3d^2e(4bd - 9ce) - 4ab^2e^3(bd + 9ce) - 3a^2e^2(b^2d^2 - 5bcde - 7c^2}{315a^4e^4\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4}\right.}}}$$
$$+ \frac{2\sqrt{2}\sqrt{b^2 - 4ac}(ad^2 - bde + ce^2)(16a^3d^3 - 3ab^2de^2 + 6a^2cde^2 - 8b^3e^3 + 27abce^3)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x}{315a^4e^4\sqrt{d + ex}(c + b}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.88 (sec) , antiderivative size = 7531, normalized size of antiderivative = 9.68

$$\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}x^3\sqrt{d + ex}\, dx = \text{Result too large to show}$$

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*x^3*Sqrt[d + e*x],x]
[Out] Result too large to show
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3660 vs. 2(702) = 1404.

Time = 2.06 (sec) , antiderivative size = 3661, normalized size of antiderivative = 4.71

method	result	size
risch	Expression too large to display	3661
default	Expression too large to display	9182

```
[In] int(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(35*a^3*e^3*x^3+5*a^3*d*e^2*x^2+5*a^2*b*e^3*x^2-6*a^3*d^2*e*x+2*a^2*b
*d*e^2*x+14*a^2*c*e^3*x-6*a*b^2*e^3*x+8*a^3*d^3-3*a^2*b*d^2*e+8*a^2*c*d*e^2
-3*a*b^2*d*e^2-27*a*b*c*e^3+8*b^3*e^3)*(e*x+d)^(1/2)/a^3/e^3*((a*x^2+b*x+c)
/x^2)^(1/2)*x-1/315/a^3/e^3*(16*a^3*b*d^4*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))
/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*
a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b
+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x
^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(
b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/
e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+16*b^4*d*e^3*(1/e*d-1/2*(b+(-4*a
*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((
x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1
/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)
)^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d
)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^
(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+16*b^3*c*e^4*(1/e*
d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2)
)/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)
^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+
b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*Ellip
ticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b
+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))-8*a^
3*c*d^3*e*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4
*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(
-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+
1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c
*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),
((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/
a))^(1/2))-6*a^2*b^2*d^3*e*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/
(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))
/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(
1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*
x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^
(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4
*a*c+b^2)^(1/2))/a))^(1/2))-6*a*b^3*d^2*e^2*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2
))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-
4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*
(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e
*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2
*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-
1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+72*a^2*c^2*d*e^3*(1/e*d-1/2*(b
+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1
/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/
a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/
2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x
```

```
+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c
+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))-54*a*b*c^2*e
^4*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^
2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*
a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+
(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/
2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*
d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/
2))+18*a^2*b*c*d^2*e^2*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e
*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/
(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2)
)/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+
b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2
))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c
+b^2)^(1/2))/a))^(1/2))-84*b^2*c*d*e^3*a*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/
a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a
*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+
(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^
3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b
+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e
*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+2*(16*a^4*d^4-8*a^3*b*d^3*e+18*a^
3*c*d^2*e^2-6*a^2*b^2*d^2*e^2+30*a^2*b*c*d*e^3+42*a^2*c^2*e^4-8*a*b^3*d*e^3
-72*a*b^2*c*e^4+16*b^4*e^4)*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)
/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2)
)/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^
(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e
*x^2+b*d*x+c*e*x+c*d)^(1/2)*((-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)*Ellipti
cE(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(
-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+1/2*(-
b+(-4*a*c+b^2)^(1/2))/a*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/
2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*
c+b^2)^(1/2))/a))^(1/2))))*((a*x^2+b*x+c)/x^2)^(1/2)*x/(a*x^2+b*x+c)*((a*x^
2+b*x+c)*(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 734, normalized size of antiderivative = 0.94

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx$$

$$= \frac{2\left((16\,a^5d^5-16\,a^4bd^4e-5\,(a^3b^2-6\,a^4c)d^3e^2-(5\,a^2b^3-21\,a^3bc)d^2e^3-2\,(8\,ab^4-42\,a^2b^2c+33\,a^3c^2)de^4\right.}{}$$

```
[In] integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*((16*a^5*d^5 - 16*a^4*b*d^4*e - 5*(a^3*b^2 - 6*a^4*c)*d^3*e^2 - (5*a^
2*b^3 - 21*a^3*b*c)*d^2*e^3 - 2*(8*a*b^4 - 42*a^2*b^2*c + 33*a^3*c^2)*d*e^4
 + (16*b^5 - 96*a*b^3*c + 123*a^2*b*c^2)*e^5)*sqrt(a*e)*weierstrassPInverse
(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e^2), -4/27*(2*a^3*d^3 -
3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(a^3*e^3
), 1/3*(3*a*e*x + a*d + b*e)/(a*e)) + 6*(8*a^5*d^4*e - 4*a^4*b*d^3*e^2 - 3*
(a^3*b^2 - 3*a^4*c)*d^2*e^3 - (4*a^2*b^3 - 15*a^3*b*c)*d*e^4 + (8*a*b^4 - 3
6*a^2*b^2*c + 21*a^3*c^2)*e^5)*sqrt(a*e)*weierstrassZeta(4/3*(a^2*d^2 - a*b
*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e^2), -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(
a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(a^3*e^3), weierstrassPInve
rse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e^2), -4/27*(2*a^3*d^3
 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(a^3*
e^3), 1/3*(3*a*e*x + a*d + b*e)/(a*e))) + 3*(35*a^5*e^5*x^4 + 5*(a^5*d*e^4
+ a^4*b*e^5)*x^3 - 2*(3*a^5*d^2*e^3 - a^4*b*d*e^4 + (3*a^3*b^2 - 7*a^4*c)*e
^5)*x^2 + (8*a^5*d^3*e^2 - 3*a^4*b*d^2*e^3 - (3*a^3*b^2 - 8*a^4*c)*d*e^4 +
(8*a^2*b^3 - 27*a^3*b*c)*e^5)*x)*sqrt(e*x + d)*sqrt((a*x^2 + b*x + c)/x^2))
/(a^5*e^5)
```

Sympy [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx=\int x^3\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

```
[In] integrate(x**3*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(d + e*x)*sqrt(a + b/x + c/x**2), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx=\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^3\,dx$$

```
[In] integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^3, x)
```

Giac [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx=\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^3\,dx$$

```
[In] integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx=\int x^3\,\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

```
[In] int(x^3*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
[Out] int(x^3*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

3.81 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 636

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx$$
$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-4be)x)}{105a^2e^2}$$
$$+\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}(c+bx+ax^2)}{7a}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(8a^3d^3+8b^3e^3-a^2de(5bd-16ce)-abe^2(5bd+29ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx\cdots}{b^2-4\cdots}}}{105a^3e^3\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+bx+ax^2)}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(8a^2d^2-4b^2e^2-ae(bd-10ce))(ad^2-e(bd-ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\cdots}}{105a^3e^3\sqrt{d+ex}(c+bx+ax^2)}$$

```
[Out] -2/105*x*(4*a^2*d^2+4*b^2*e^2-a*e*(2*b*d-5*c*e)-3*a*e*(a*d-4*b*e)*x)*(a+c/x
^2+b/x)^(1/2)*(e*x+d)^(1/2)/a^2/e^2+2/7*x*(a*x^2+b*x+c)*(a+c/x^2+b/x)^(1/2)
*(e*x+d)^(1/2)/a+1/105*(8*a^3*d^3+8*b^3*e^3-a^2*d*e*(5*b*d-16*c*e)-a*b*e^2*
(5*b*d+29*c*e))*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2
))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-
a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/a^3/e^3/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d
-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/105*(8*a^2*d^2-4*b^2*e^2-a*e*(b*d-10*c*
```

```
e))*(a*d^2-e*(b*d-c*e))*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b
^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x
^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^
(1/2)/a^3/e^3/(a*x^2+b*x+c)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1587, 846, 828, 857, 732, 435, 430}

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx$$

$$= -\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(4a^2d^2-ae(2bd-5ce)-3aex(ad-4be)+4b^2e^2)}{105a^2e^2}$$

$$-\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}(8a^2d^2-ae(bd-10ce)-4b^2e^2)\left(ad^2-e(bd-ce)\right)\sqrt{\frac{a(\ldots}{2ad-e(\ldots}}\ldots}{105a^3e^3\sqrt{d+ex}\,(ax^2+bx+c)}$$

$$+\frac{\sqrt{2}x\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}(8a^3d^3-a^2de(5bd-16ce)-abe^2(5bd+29ce)+8b^3\ldots}{105a^3e^3\,(ax^2+bx+c)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$+\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(ax^2+bx+c)}{7a}$$

```
[In] Int[Sqrt[a + c/x^2 + b/x]*x^2*Sqrt[d + e*x],x]
```

```
[Out] (-2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*(4*a^2*d^2 + 4*b^2*e^2 - a*e*(2*b
*d - 5*c*e) - 3*a*e*(a*d - 4*b*e)*x))/(105*a^2*e^2) + (2*Sqrt[a + c/x^2 + b
/x]*x*Sqrt[d + e*x]*(c + b*x + a*x^2))/(7*a) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
8*a^3*d^3 + 8*b^3*e^3 - a^2*d*e*(5*b*d - 16*c*e) - a*b*e^2*(5*b*d + 29*c*e)
)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])
*e)]])/(105*a^3*e^3*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
(c + b*x + a*x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*a^2*d^2 - 4*b^2*e^2 -
a*e*(b*d - 10*c*e))*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a
*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2
))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sq
```

```
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(105*a^3*e^3*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
```

```
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1587

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p]), Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int x\sqrt{d+ex}\sqrt{c+bx+ax^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}(c+bx+ax^2)}{7a} + \frac{\left(2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{\left(\frac{1}{2}(-3bd-ce)+\frac{1}{2}(ad-4be)x\right)\sqrt{c+bx+ax^2}}{\sqrt{d+ex}}\,dx}{7a\sqrt{c+bx+ax^2}}$$

$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-4be)x)}{105a^2e^2} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}(c+bx+ax^2)}{7a} - \frac{\left(4\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{\frac{1}{2}\left(-a^2d^2(2bd-ce)-2b^2e^2(bd+ce)+ae\left(b^2d^2+9bcde+5c^2e^2\right)\right)-\frac{1}{4}\left(8a^3d^3+8b^3e^3-a^2de(5bd-16ce)-abe^2(\ldots}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}}{105a^2e^2\sqrt{c+bx+ax^2}}$$

$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-4be)x)}{105a^2e^2}$$
$$+\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}(c+bx+ax^2)}{7a}$$
$$-\frac{\left((-8a^3d^3-8b^3e^3+a^2de(5bd-16ce)+abe^2(5bd+29ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{\sqrt{d+ex}}{\sqrt{c+bx+ax^2}}\,dx}{105a^2e^3\sqrt{c+bx+ax^2}}$$
$$-\frac{\left(4\left(-\frac{1}{4}d(-8a^3d^3-8b^3e^3+a^2de(5bd-16ce)+abe^2(5bd+29ce))+\frac{1}{2}e(-a^2d^2(2bd-ce)-2b^2e\right.\right.}{105a^2e^3\sqrt{c+bx+a}}$$

$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-4be)x)}{105a^2e^2}$$
$$+\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}(c+bx+ax^2)}{7a}$$
$$-\frac{\left(\sqrt{2}\sqrt{b^2-4ac}(-8a^3d^3-8b^3e^3+a^2de(5bd-16ce)+abe^2(5bd+29ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+e\right.}{105a^3e^3\sqrt{\frac{a(d+ex)}{2ad-be-\sqrt{b^2-4ace}}}(c+bx}$$
$$-\frac{\left(8\sqrt{2}\sqrt{b^2-4ac}\left(-\frac{1}{4}d(-8a^3d^3-8b^3e^3+a^2de(5bd-16ce)+abe^2(5bd+29ce))+\frac{1}{2}e(-a^2d^2(2b\right.\right.}{}$$

$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-4be)x)}{105a^2e^2}$$
$$+\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}(c+bx+ax^2)}{7a}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(8a^3d^3+8b^3e^3-a^2de(5bd-16ce)-abe^2(5bd+29ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{}}{105a^3e^3\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+bx+}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(ad^2-bde+ce^2)(8a^2d^2-abde-4b^2e^2+10ace^2)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4\right.}}}}{105a^3e^3\sqrt{d+ex}(c+bx+ax}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.03 (sec) , antiderivative size = 1314, normalized size of antiderivative = 2.07

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx$$
$$= x\sqrt{d+ex}\left(\frac{4(-2a^2d^2+abde-2b^2e^2+5ace^2)}{105a^2e^2}+\frac{2(ad+be)x}{35ae}+\frac{2x^2}{7}\right)\sqrt{a+\frac{c+bx}{x^2}}$$
$$+\frac{x(d+ex)^{3/2}\sqrt{a+\frac{c+bx}{x^2}}\left(4\sqrt{\frac{ad^2+e(-bd+ce)}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}(8a^3d^3+8b^3e^3+a^2de(-5bd+16ce)-abe^2(5bd+29ce)\right.}{}$$

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*x^2*Sqrt[d + e*x],x]
```

```
[Out] x*Sqrt[d + e*x]*((4*(-2*a^2*d^2 + a*b*d*e - 2*b^2*e^2 + 5*a*c*e^2))/(105*a^
2*e^2) + (2*(a*d + b*e)*x)/(35*a*e) + (2*x^2)/7)*Sqrt[a + (c + b*x)/x^2] +
(x*(d + e*x)^(3/2)*Sqrt[a + (c + b*x)/x^2]*(4*Sqrt[(a*d^2 + e*(-(b*d) + c*e
))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(8*a^3*d^3 + 8*b^3*e^3 + a^2*d
*e*(-5*b*d + 16*c*e) - a*b*e^2*(5*b*d + 29*c*e))*(a*(-1 + d/(d + e*x))^2 +
(e*(b - (b*d)/(d + e*x) + (c*e)/(d + e*x)))/(d + e*x)) - (I*Sqrt[2]*(2*a*d
- b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(8*a^3*d^3 + 8*b^3*e^3 + a^2*d*e*(-5*b*d +
 16*c*e) - a*b*e^2*(5*b*d + 29*c*e))*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*c*e
^2)/(d + e*x) - 2*a*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*a
*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*e
^2)/(d + e*x) + 2*a*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*a
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d
^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x
]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4
*a*c)*e^2]))])/Sqrt[d + e*x] + (I*Sqrt[2]*(8*b^3*e^3*(-(b*e) + Sqrt[(b^2 -
4*a*c)*e^2]) + a^3*(-4*c*d^2*e^2 + 8*d^3*Sqrt[(b^2 - 4*a*c)*e^2]) + a*b*e^2
*(13*b^2*d*e + 37*b*c*e^2 - 5*b*d*Sqrt[(b^2 - 4*a*c)*e^2] - 29*c*e*Sqrt[(b^
2 - 4*a*c)*e^2]) + a^2*e*(b^2*d^2*e - 4*c*e*(5*c*e^2 - 4*d*Sqrt[(b^2 - 4*a*
c)*e^2]) - b*d*(52*c*e^2 + 5*d*Sqrt[(b^2 - 4*a*c)*e^2])))*Sqrt[(Sqrt[(b^2 -
 4*a*c)*e^2] - (2*c*e^2)/(d + e*x) - 2*a*d*(-1 + d/(d + e*x)) + b*e*(-1 + (
2*d)/(d + e*x)))/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 -
 4*a*c)*e^2] + (2*c*e^2)/(d + e*x) + 2*a*d*(-1 + d/(d + e*x)) + b*(e - (2*d
```

```
*e)/(d + e*x)))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*EllipticF[I*ArcSi
nh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)
*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d
- b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/(210*a^3*e^4*Sqrt[(a*d^
2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[c + b*
x + a*x^2]*Sqrt[((d + e*x)^2*(a*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e
*x) + (c*e)/(d + e*x)))/(d + e*x)))/e^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2661 vs. 2(572) = 1144.

Time = 1.84 (sec) , antiderivative size = 2662, normalized size of antiderivative = 4.19

method	result	size
risch	Expression too large to display	2662
default	Expression too large to display	6302

```
[In] int(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*(-15*a^2*e^2*x^2-3*a^2*d*e*x-3*a*b*e^2*x+4*a^2*d^2-2*a*b*d*e-10*a*c*
e^2+4*b^2*e^2)*(e*x+d)^(1/2)/a^2/e^2*((a*x^2+b*x+c)/x^2)^(1/2)*x+1/105/e^2/
a^2*(8*a^2*b*d^3*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2
*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*
d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(
-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+
c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))
^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^
(1/2))/a))^(1/2))+8*b^3*d*e^2*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*
d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/
2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2
)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b
*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^
2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+
(-4*a*c+b^2)^(1/2))/a))^(1/2))+8*b^2*c*e^3*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2)
)/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4
*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(
b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*
x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*
(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1
/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))-20*a*c^2*e^3*(1/e*d-1/2*(b+(-4*
a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*(
(x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(
1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a
))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*
```

```
d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)
^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))-4*a^2*c*d^2*e*(1/
e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/
2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^
2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*
c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*Ell
ipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*
(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))-4*
a*b^2*d^2*e*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(
-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2
*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*
d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x
+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2
),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2)
)/a))^(1/2))-36*a*b*c*d*e^2*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)
/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2)
)/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^
(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e
*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)
^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-
4*a*c+b^2)^(1/2))/a))^(1/2))+2*(8*a^3*d^3-5*a^2*b*d^2*e+16*a^2*c*d*e^2-5*a*
b^2*d*e^2-29*a*b*c*e^3+8*b^3*e^3)*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+
1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)
^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c
+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x
^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*((-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)*E
llipticE(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/
2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+
1/2*(-b+(-4*a*c+b^2)^(1/2))/a*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^
2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+
(-4*a*c+b^2)^(1/2))/a))^(1/2))))*((a*x^2+b*x+c)/x^2)^(1/2)*x/(a*x^2+b*x+c)*
((a*x^2+b*x+c)*(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 598, normalized size of antiderivative = 0.94

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx = -\frac{2\left((8\,a^4d^4-9\,a^3bd^3e-2\,(2\,a^2b^2-11\,a^3c)d^2e^2-(9\,ab^3-41\,a^2bc)de^3+(8\,b^4-41\,ab^2c+30\,a^2c^2)e^4)\sqrt{ae}\right.}{}$$

```
[In] integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*((8*a^4*d^4 - 9*a^3*b*d^3*e - 2*(2*a^2*b^2 - 11*a^3*c)*d^2*e^2 - (9*
a*b^3 - 41*a^2*b*c)*d*e^3 + (8*b^4 - 41*a*b^2*c + 30*a^2*c^2)*e^4)*sqrt(a*e
)*weierstrassPInverse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e^2)
, -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9
*a*b*c)*e^3)/(a^3*e^3), 1/3*(3*a*e*x + a*d + b*e)/(a*e)) + 3*(8*a^4*d^3*e -
 5*a^3*b*d^2*e^2 - (5*a^2*b^2 - 16*a^3*c)*d*e^3 + (8*a*b^3 - 29*a^2*b*c)*e^
4)*sqrt(a*e)*weierstrassZeta(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)/(a
^2*e^2), -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*
b^3 - 9*a*b*c)*e^3)/(a^3*e^3), weierstrassPInverse(4/3*(a^2*d^2 - a*b*d*e +
 (b^2 - 3*a*c)*e^2)/(a^2*e^2), -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2
- 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(a^3*e^3), 1/3*(3*a*e*x + a*d + b
*e)/(a*e))) - 3*(15*a^4*e^4*x^3 + 3*(a^4*d*e^3 + a^3*b*e^4)*x^2 - 2*(2*a^4*
d^2*e^2 - a^3*b*d*e^3 + (2*a^2*b^2 - 5*a^3*c)*e^4)*x)*sqrt(e*x + d)*sqrt((a
*x^2 + b*x + c)/x^2))/(a^4*e^4)
```

Sympy [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx=\int x^2\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

```
[In] integrate(x**2*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
[Out] Integral(x**2*sqrt(d + e*x)*sqrt(a + b/x + c/x**2), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx=\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^2\,dx$$

```
[In] integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^2, x)
```

Giac [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx=\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^2\,dx$$

```
[In] integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx=\int x^2\,\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

```
[In] int(x^2*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
[Out] int(x^2*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

3.82 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 550

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{5e}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}(a^2d^2+b^2e^2-ae(bd+3ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4\cdots}}{\sqrt{b^2-\cdots}}}}{\sqrt{2}\cdots}\right.\right.}{15a^2e^2\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+bx+ax^2)\cdots}$$

$$+ \frac{2\sqrt{2}\sqrt{b^2-4ac}(2ad-be)\left(ad^2-e(bd-ce)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\,\text{EllipticF}\cdots}{15a^2e^2\sqrt{d+ex}(c+bx+ax^2)\cdots}$$

```
[Out] 2/5*x*(e*x+d)^(3/2)*(a+c/x^2+b/x)^(1/2)/e-2/15*(2*a*d-b*e)*x*(a+c/x^2+b/x)^
(1/2)*(e*x+d)^(1/2)/a/e-2/15*(a^2*d^2+b^2*e^2-a*e*(b*d+3*c*e))*x*EllipticE(
1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(
-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+
b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2)
)^(1/2)/a^2/e^2/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^
(1/2)+2/15*(2*a*d-b*e)*(a*d^2-e*(b*d-c*e))*x*EllipticF(1/2*((b+2*a*x+(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2
*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+
b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4
*a*c+b^2)^(1/2))))^(1/2)/a^2/e^2/(a*x^2+b*x+c)/(e*x+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1587, 748, 846, 857, 732, 435, 430}

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx$$

$$= \frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}(2ad-be)\left(ad^2-e(bd-ce)\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\Bigg(\text{ar}}{15a^2e^2\sqrt{d+ex}\,(ax^2+bx+c)}$$

$$-\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\left(a^2d^2-ae(bd+3ce)+b^2e^2\right)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2}}{\sqrt{b^2-4c}}}}{\sqrt{2}}\right.\right.}{15a^2e^2\,(ax^2+bx+c)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$+\frac{2x(d+ex)^{3/2}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{5e}-\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(2ad-be)}{15ae}$$

```
[In] Int[Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x],x]
```

```
[Out] (-2*(2*a*d - b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(15*a*e) + (2*Sqrt
[a + c/x^2 + b/x]*x*(d + e*x)^(3/2))/(5*e) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
a^2*d^2 + b^2*e^2 - a*e*(b*d + 3*c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x
]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c
]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*a^2*e^2*Sqrt[(a*(d + e*x))/(
2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (2*Sqrt[2]*Sqrt[b^
2 - 4*a*c]*(2*a*d - b*e)*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sq
rt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x +
a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*
x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt
[b^2 - 4*a*c])*e)])/(15*a^2*e^2*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1587

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + b/x^
```

```
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p]), Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\sqrt{d+ex}\sqrt{c+bx+ax^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{5e} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{\sqrt{d+ex}(bd-2ce+(2ad-be)x)}{\sqrt{c+bx+ax^2}}\,dx}{5e\sqrt{c+bx+ax^2}}$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{5e} - \frac{\left(2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{\frac{1}{2}(ad(bd-8ce)+be(bd+ce))+(a^2d^2+b^2e^2-ae(bd+3ce))x}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{15ae\sqrt{c+bx+ax^2}}$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{5e} + \frac{\left((2ad-be)\left(ad^2-e(bd-ce)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{1}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{15ae^2\sqrt{c+bx+ax^2}} - \frac{\left(2(a^2d^2+b^2e^2-ae(bd+3ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{\sqrt{d+ex}}{\sqrt{c+bx+ax^2}}\,dx}{15ae^2\sqrt{c+bx+ax^2}}$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{5e} - \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(a^2d^2+b^2e^2-ae(bd+3ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\right.}{15a^2e^2\sqrt{\frac{a(d+ex)}{2ad-be-\sqrt{b^2-4ac}e}}\,(c+bx+ax^2)} + \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(2ad-be)\left(ad^2-e(bd-ce)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-be-\sqrt{b^2-4ac}e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\right)\text{S}}{15a^2e^2\sqrt{d+ex}\,(c+bx+ax^2)}$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{5e}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}(a^2d^2+b^2e^2-ae(bd+3ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\right.\right.}{15a^2e^2\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+bx+ax^2)}$$

$$+ \frac{2\sqrt{2}\sqrt{b^2-4ac}(2ad-be)\left(ad^2-e(bd-ce)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F}{15a^2e^2\sqrt{d+ex}(c+bx+ax^2)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.19 (sec) , antiderivative size = 1051, normalized size of antiderivative = 1.91

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx=\frac{1}{15}x\sqrt{d+ex}\sqrt{a+\frac{c+bx}{x^2}}\left(\frac{2b}{a}+\frac{2d}{e}+6x\right.$$

$$-\frac{(d+ex)\left(\frac{4e^2\sqrt{\frac{ad^2+e(-bd+ce)}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}(a^2d^2+b^2e^2-ae(bd+3ce))(c+x(b+ax))}{(d+ex)^2}-\frac{i\sqrt{2}\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)(a^2d^2+b^2e^2-ae(bd+3ce}{}\right.}{}$$

[In] Integrate[Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x],x]

[Out] (x*Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2]*((2*b)/a + (2*d)/e + 6*x - ((d + e*x)*((4*e^2*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(a^2*d^2 + b^2*e^2 - a*e*(b*d + 3*c*e))*(c + x*(b + a*x)))/(d + e*x)^2 - (I*Sqrt[2]*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(a^2*d^2 + b^2*e^2 - a*e*(b*d + 3*c*e))*Sqrt[(-2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(-d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + (I*Sqrt[2]*(b^2*e^2*(-(b*e) + Sqrt[(b^2 - 4*a

```
*c)*e^2]) + a^2*d*(-8*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]) + a*e*(2*b^2*d*e +
 4*b*c*e^2 - b*d*Sqrt[(b^2 - 4*a*c)*e^2] - 3*c*e*Sqrt[(b^2 - 4*a*c)*e^2]))*
Sqrt[(-2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*
c)*e^2]*x + b*e*(d - e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*
x))]*Sqrt[(2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*a*d*e*x + e*Sqrt[(b^2 -
4*a*c)*e^2]*x + b*e*(-d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(
d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*
d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt
[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*
x]))/(a^2*e^3*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4
*a*c)*e^2])]*(c + x*(b + a*x)))))/15
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1710 vs. 2(486) = 972.

Time = 1.67 (sec) , antiderivative size = 1711, normalized size of antiderivative = 3.11

method	result	size
risch	Expression too large to display	1711
default	Expression too large to display	4361

```
[In] int(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(3*a*e*x+a*d+b*e)*(e*x+d)^(1/2)/a/e*((a*x^2+b*x+c)/x^2)^(1/2)*x-1/15/a
/e*(2*a*b*d^2*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b
+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1
/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/
e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e
*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1
/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/
2))/a))^(1/2))+2*b^2*d*e*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1
/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a
)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/
2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^
2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1
/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a
*c+b^2)^(1/2))/a))^(1/2))+2*b*c*e^2*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((
x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^
2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a
*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d
*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*
a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/
2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))-16*a*c*d*e*(1/e*d-1/2*(b+(-4*a*c+b^2)^
(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-
```

```
b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+
1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/
(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d
-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a
)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+2*(2*a^2*d^2-2*a*b*d*e-6*a
*c*e^2+2*b^2*e^2)*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/
2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e
*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/
(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x
+c*e*x+c*d)^(1/2)*((-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)*EllipticE(((x+1/e
*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2
)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+1/2*(-b+(-4*a*c+
b^2)^(1/2))/a*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1
/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/
2))/a))^(1/2))))*((a*x^2+b*x+c)/x^2)^(1/2)*x/(a*x^2+b*x+c)*((a*x^2+b*x+c)*(
e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 490, normalized size of antiderivative = 0.89

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx$$
$$= \frac{2\left(\left(2a^3d^3-3a^2bd^2e-3(ab^2-6a^2c)de^2+(2b^3-9abc)e^3\right)\sqrt{ae}\,\text{weierstrassPInverse}\left(\frac{4(a^2d^2-abde+(b^2-3ac)e^2}{3a^2e^2}\right.\right.}{}$$

```
[In] integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*((2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a
*b*c)*e^3)*sqrt(a*e)*weierstrassPInverse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*
a*c)*e^2)/(a^2*e^2), -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)
*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(a^3*e^3), 1/3*(3*a*e*x + a*d + b*e)/(a*e))
 + 6*(a^3*d^2*e - a^2*b*d*e^2 + (a*b^2 - 3*a^2*c)*e^3)*sqrt(a*e)*weierstras
sZeta(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^2*e^2), -4/27*(2*a^3*d
^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(a^
3*e^3), weierstrassPInverse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)/(a^
2*e^2), -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b
^3 - 9*a*b*c)*e^3)/(a^3*e^3), 1/3*(3*a*e*x + a*d + b*e)/(a*e))) + 3*(3*a^3*
e^3*x^2 + (a^3*d*e^2 + a^2*b*e^3)*x)*sqrt(e*x + d)*sqrt((a*x^2 + b*x + c)/x
^2))/(a^3*e^3)
```

Sympy [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx=\int x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

```
[In] integrate(x*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
[Out] Integral(x*sqrt(d + e*x)*sqrt(a + b/x + c/x**2), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx=\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\,dx$$

```
[In] integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x, x)
```

Giac [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx=\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\,dx$$

```
[In] integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx=\int x\,\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

```
[In] int(x*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
[Out] int(x*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

3.83 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 955

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx=\frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}}{2ad-\left(b+\sqrt{b^2-\right.}\right.}}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+bx+ax^2)}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}d(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}}{\sqrt{b^2-4a}}}}{\sqrt{2}}\right.\right.}{3ae\sqrt{d+ex}(c+bx+ax^2)}$$

$$+\frac{4\sqrt{2}\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3a\sqrt{d+ex}(c+bx+ax^2)}$$

$$-\frac{\sqrt{2}c\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\operatorname{EllipticPi}\left(\right.}{\sqrt{a}(c+bx+ax^2)}$$

```
[Out] 2/3*x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)+1/3*(a*d+b*e)*x*EllipticE(1/2*((b+2
*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^
2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2
)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/a
/e/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3*d*(
```

```
a*d+b*e)*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^
(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1
/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a
*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a/e/(a*x^
2+b*x+c)/(e*x+d)^(1/2)+4/3*(b*d+c*e)*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)
^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e
*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(
1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b
^2)^(1/2))))^(1/2)/a/(a*x^2+b*x+c)/(e*x+d)^(1/2)-c*x*EllipticPi(2^(1/2)*a^(
1/2)*(e*x+d)^(1/2)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(2*a*d-b*e+e*
(-4*a*c+b^2)^(1/2))/a/d,((b-2*a*d/e-(-4*a*c+b^2)^(1/2))/(b-2*a*d/e+(-4*a*c+
b^2)^(1/2)))^(1/2))*2^(1/2)*(a+c/x^2+b/x)^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b-
(-4*a*c+b^2)^(1/2))))^(1/2)*(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*a*(
e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*x^2+b*x+c)/a^(1/2)
```

Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 955, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1463, 932, 6874, 732, 430, 948, 175, 552, 551, 857, 435}

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx$$

$$= \frac{\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right.}\right.}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(ax^2+bx+c)}$$

$$- \frac{2\sqrt{2}\sqrt{b^2-4ac}d(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2}}{\sqrt{b^2-4a}}}}{\sqrt{2}}\right.\right.}{3ae\sqrt{d+ex}(ax^2+bx+c)}$$

$$+ \frac{4\sqrt{2}\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3a\sqrt{d+ex}(ax^2+bx+c)}$$

$$- \frac{\sqrt{2}c\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\text{EllipticPi}\left(\right.}{\sqrt{a}(ax^2+bx+c)}$$

$$+ \frac{2}{3}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}x$$

```
[In] Int[Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/3 + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a
*d + b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2)
)/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 -
 4*a*c])*e)])/(3*a*e*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
]*(c + b*x + a*x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*(a*d + b*e)*Sqrt[a +
c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt
[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(
2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*a*e*Sqrt[d + e*x]*(c + b*x + a*x^2)
) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(b*d + c*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(
a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^
2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/(3*a*Sqrt[d + e*x]*(c + b*x + a*x^2)) - (Sqrt[2]*c*Sqrt[2*a
*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d +
e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d
- (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e
)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^
2 - 4*a*c])*e]], (b - Sqrt[b^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c]
 - (2*a*d)/e)])/(Sqrt[a]*(c + b*x + a*x^2))
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
+ 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
```

```
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1463

```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_
)^(n_.))^(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + b/x^n + c/x^(2*
n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p]), Int[((d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p)/x^(2*n*p), x], x] /; FreeQ[{a, b, c, d, e, n, p, q
}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] && !IntegerQ[p] && !IntegerQ[q] &&
 PosQ[n]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{\sqrt{d+ex}\sqrt{c+bx+ax^2}}{x}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}-\frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{-3cd-2(bd+ce)x-(ad+be)x^2}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{3\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\left(-\frac{2(bd+ce)}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}-\frac{3cd}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}}-\frac{(ad+be)x}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\right)dx}{3\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}+\frac{\left(cd\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{1}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{\sqrt{c+bx+ax^2}} - \frac{\left((-ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{x}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{3\sqrt{c+bx+ax^2}} + \frac{\left(2(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{1}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{3\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}$$
$$+ \frac{\left(cd\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{b-\sqrt{b^2-4ac}+2ax}\sqrt{b+\sqrt{b^2-4ac}+2ax}\right)\int \frac{1}{x\sqrt{b-\sqrt{b^2-4ac}+2ax}\sqrt{b+\sqrt{b^2-4ac}}}}{c+bx+ax^2}$$
$$- \frac{\left((-ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{\sqrt{d+ex}}{\sqrt{c+bx+ax^2}}\,dx}{3e\sqrt{c+bx+ax^2}}$$
$$+ \frac{\left(d(-ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{1}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{3e\sqrt{c+bx+ax^2}}$$
$$+ \frac{\left(4\sqrt{2}\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-be-\sqrt{b^2-4ac}e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x^2}\sqrt{1+\frac{}{2}}}\right.}{3a\sqrt{d+ex}\,(c+bx+ax^2)}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}$$
$$+ \frac{4\sqrt{2}\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3a\sqrt{d+ex}\,(c+bx+ax^2)}$$
$$- \frac{\left(2cd\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{b-\sqrt{b^2-4ac}+2ax}\sqrt{b+\sqrt{b^2-4ac}+2ax}\right)\text{Subst}\left(\int \frac{}{(d-x^2)\sqrt{b-\sqrt{b^2-4ac}-}}\right.}{c+bx+ax^2}$$
$$- \frac{\left(\sqrt{2}\sqrt{b^2-4ac}(-ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}ex^2}{2ad-be-\sqrt{b^2-4ac}e}}}{\sqrt{1-x^2}}\right.}{3ae\sqrt{\frac{a(d+ex)}{2ad-be-\sqrt{b^2-4ac}e}}\,(c+bx+ax^2)}$$
$$+ \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}d(-ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-be-\sqrt{b^2-4ac}e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x^2}\sqrt{}}\right.}{3ae\sqrt{d+ex}\,(c+bx+ax^2)}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2}{2ad-(}\right.}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+bx+ax^2)}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}d(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3ae\sqrt{d+ex}(c+bx+ax^2)}$$

$$+\frac{4\sqrt{2}\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2a}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3a\sqrt{d+ex}(c+bx+ax^2)}$$

$$-\frac{\left(2cd\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{b+\sqrt{b^2-4ac}+2ax}\sqrt{1+\frac{2a(d+ex)}{\left(b-\sqrt{b^2-4ac}-\frac{2ad}{e}\right)e}}\right)\text{Subst}\left(\int\frac{}{(d-x^2)\sqrt{b+\sqrt{b^2-4ac}-}}\right.}{c+bx+ax^2}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2}{2ad-(}\right.}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+bx+ax^2)}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}d(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3ae\sqrt{d+ex}(c+bx+ax^2)}$$

$$+\frac{4\sqrt{2}\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2a}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3a\sqrt{d+ex}(c+bx+ax^2)}$$

$$-\frac{\left(2cd\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1+\frac{2a(d+ex)}{\left(b-\sqrt{b^2-4ac}-\frac{2ad}{e}\right)e}}\sqrt{1+\frac{2a(d+ex)}{\left(b+\sqrt{b^2-4ac}-\frac{2ad}{e}\right)e}}\right)\text{Subst}\left(\int\frac{}{(d-x^2)\sqrt{1+\frac{2ax^2}{\left(b-\sqrt{b^2-4ac}\right.}}}\right.}{c+bx+ax^2}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{}{2ad\cdots}\right.}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(c+bx+ax^2\right)}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}d(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}\cdots}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3ae\sqrt{d+ex}\left(c+bx+ax^2\right)}$$

$$+\frac{4\sqrt{2}\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+\cdots}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3a\sqrt{d+ex}\left(c+bx+ax^2\right)}$$

$$-\frac{\sqrt{2}c\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\Pi\left(\cdots\right.}{\sqrt{a}\left(c+bx+ax^2\right)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.32 (sec) , antiderivative size = 1258, normalized size of antiderivative = 1.32

$$\int\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx = \frac{2}{3}x\sqrt{d+ex}\sqrt{a+\frac{c+bx}{x^2}}$$

$$+\frac{x(d+ex)^{3/2}\sqrt{a+\frac{c+bx}{x^2}}\left(\frac{4e^2(ad+be)\sqrt{\frac{ad^2+e(-bd+ce)}{-2ad+be+\sqrt{\left(b^2-4ac\right)e^2}}}(c+x(b+ax))}{(d+ex)^2}-\frac{i\sqrt{2}(ad+be)\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)\sqrt{\frac{-2ce^2+2a\cdots}{(2a\cdots}}}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x],x]
```

```
[Out] (2*x*Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2])/3 + (x*(d + e*x)^(3/2)*Sqrt[a +
 (c + b*x)/x^2]*((4*e^2*(a*d + b*e)*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d
 + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c + x*(b + a*x)))/(d + e*x)^2 - (I*Sqrt
[2]*(a*d + b*e)*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*c*e^2 + 2*
```

```
a*d*e*x + b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e
+ Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*(-d
+ e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*
a*c)*e^2])*(d + e*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c
*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d
+ b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])
/Sqrt[d + e*x] + (I*Sqrt[2]*(b*e*(-(b*e) + Sqrt[(b^2 - 4*a*c)*e^2]) + a*(3*
b*d*e - 2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]))*Sqrt[(-2*c*e^2 + 2*a*d*e*x +
b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e + Sqrt[(b^
2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*(-d + e*x) + S
qrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])
*(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*
a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sq
rt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d +
e*x] + ((6*I)*Sqrt[2]*a*c*e^2*Sqrt[(-2*c*e^2 + 2*a*d*e*x + b*e*(d - e*x) +
Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])
*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*(-d + e*x) + Sqrt[(b^2 - 4*a*c
)*e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Ell
ipticPi[(d*(2*a*d - b*e - Sqrt[(b^2 - 4*a*c)*e^2]))/(2*(a*d^2 + e*(-(b*d) +
 c*e))), I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sq
rt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*
c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/(6*a*e^
2*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]
*(c + x*(b + a*x)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3022 vs. $2(836) = 1672$.

Time = 0.45 (sec) , antiderivative size = 3023, normalized size of antiderivative = 3.17

method	result	size
default	Expression too large to display	3023

```
[In] int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((a*x^2+b*x+c)/x^2)^(1/2)*x*(e*x+d)^(1/2)*(2^(1/2)*(-(e*x+d)*a/(e*(-4*a
*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*d*a-b*e
+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^
2)^(1/2)-2*d*a+b*e))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1
/2)-2*d*a+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e)/(2*d*a-b*e+e*(-4*a
*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*d^2*e-2^(1/2)*(-(e*x+d)*a/(e*(-
4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*d*a-
b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c
+b^2)^(1/2)-2*d*a+b*e))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)
^(1/2)-2*d*a+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e)/(2*d*a-b*e+e*(-
```

4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*d*e^2-2*2^(1/2)*(-(e*x+d)*a/
(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2
*d*a-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-
4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c
+b^2)^(1/2)-2*d*a+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e)/(2*d*a-b*e
+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c*e^3+3*2^(1/2)*(-(e*x+d)
a/(e(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e
/(2*d*a-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e
*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*
a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e)/(2*d*a-
b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*d^2*e+6*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*
c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*d*a-b*e+
e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2
)^(1/2)-2*d*a+b*e))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/
2)-2*d*a+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e)/(2*d*a-b*e+e*(-4*a*
c+b^2)^(1/2)))^(1/2))*a*c*d*e^2-3*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)
-2*d*a+b*e))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*d*a-b*e+e*(-4*a*c+b^
2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*d*
a+b*e))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e
))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e)/(2*d*a-b*e+e*(-4*a*c+b^2)^(1/2)
))^(1/2))*b^2*d*e^2-2*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))
^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*d*a-b*e+e*(-4*a*c+b^2)^(1/2)))^(
1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2
)*EllipticE(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2),(-(
e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e)/(2*d*a-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a^
2*d^3-2*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*((-2*a*
x+(-4*a*c+b^2)^(1/2)-b)*e/(2*d*a-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x
+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*EllipticE(2^
(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2),(-(e*(-4*a*c+b^2)
^(1/2)-2*d*a+b*e)/(2*d*a-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*e^2+2*2^(1
/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*((-2*a*x+(-4*a*c+b^
2)^(1/2)-b)*e/(2*d*a-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2
)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*EllipticE(2^(1/2)*(-(e*x
+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*d*a
+b*e)/(2*d*a-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*d*e^2-2*2^(1/2)*(-(e*x+d
)*a/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*
e/(2*d*a-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(
e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*a/(e*(-4
*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e)/(2*d*a
-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*e^3+3*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c
+b^2)^(1/2)-2*d*a+b*e))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*d*a-b*e+e
*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)
^(1/2)-2*d*a+b*e))^(1/2)*EllipticPi(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/
2)-2*d*a+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e)/d/a,(-(e*(-4*a*c
+b^2)^(1/2)-2*d*a+b*e)/(2*d*a-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2

```
)^(1/2)*c*e^3-6*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)
*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*d*a-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(
(b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*Elli
pticPi(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e))^(1/2),-1/2*(e*
(-4*a*c+b^2)^(1/2)-2*d*a+b*e)/d/a,(-(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e)/(2*d*a
-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*e^2+3*2^(1/2)*(-(e*x+d)*a/(e*(-4*a
*c+b^2)^(1/2)-2*d*a+b*e))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*d*a-b*e
+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^
2)^(1/2)-2*d*a+b*e))^(1/2)*EllipticPi(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(
1/2)-2*d*a+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*d*a+b*e)/d/a,(-(e*(-4*a
*c+b^2)^(1/2)-2*d*a+b*e)/(2*d*a-b*e+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*e^3+2
*a^2*e^3*x^3+2*a^2*d*e^2*x^2+2*a*b*e^3*x^2+2*a*b*d*e^2*x+2*a*c*e^3*x+2*a*c*
d*e^2)/a/e^2/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx = \int \sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

```
[In] integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
[Out] Integral(sqrt(d + e*x)*sqrt(a + b/x + c/x**2), x)
```

Maxima [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx = \int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2), x)
```

Giac [F]

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx = \int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx = \int \sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

```
[In] int((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
[Out] int((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

3.84 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 929

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx = -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{3\sqrt{b^2-4ac}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+bx+ax^2)} - \frac{3\sqrt{2}\sqrt{b^2-4ac}d\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\right.}{\sqrt{d+ex}(c+bx+ax^2)} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{a\sqrt{d+ex}(c+bx+ax^2)} - \frac{(bd+ce)\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,\text{Elliptic}}{\sqrt{2}\sqrt{a}d(c+bx+ax^2)}$$

```
[Out] -(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)+3/2*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d
-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(
e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/(a*x^2+b*x+c)*2^(1/2)/(a
```

```
*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-3*d*x*EllipticF(1/2*((b+2*
a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2
)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)
*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*
d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*x^2+b*x+c)/(e*x+d)^(1/2)+2*(a*d+b*e)*
x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^
(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a/(a*x^2+b*x+c)/(e
*x+d)^(1/2)-1/2*(b*d+c*e)*x*EllipticPi(2^(1/2)*a^(1/2)*(e*x+d)^(1/2)/(2*a*d
-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(2*a*d-b*e+e*(-4*a*c+b^2)^(1/2))/a/d,(
(b-2*a*d/e-(-4*a*c+b^2)^(1/2))/(b-2*a*d/e+(-4*a*c+b^2)^(1/2)))^(1/2))*(a+c/
x^2+b/x)^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*a*
d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(
1/2))))^(1/2)/d/(a*x^2+b*x+c)*2^(1/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 929, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules

used = {1587, 930, 6874, 732, 430, 948, 175, 552, 551, 857, 435}

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx$$

$$= \frac{3\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{d+ex}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(ax^2+bx+c)}$$

$$- \frac{3\sqrt{2}\sqrt{b^2-4ac}d\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\ldots\right.}{\sqrt{d+ex}(ax^2+bx+c)}$$

$$+ \frac{2\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-\ldots}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{a\sqrt{d+ex}(ax^2+bx+c)}$$

$$- \frac{(bd+ce)\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,\text{Elliptic}\ldots}{\ldots\sqrt{2}\sqrt{a}d(ax^2+bx+c)}$$

$$- \sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}$$

```
[In] Int[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x,x]
```

```
[Out] -(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x]) + (3*Sqrt[b^2 - 4*a*c]*Sqrt[a + c/x^
2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*Ellip
ticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]
], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[2]*
Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2))
- (3*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))
/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4
*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*
a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e
)])/(Sqrt[d + e*x]*(c + b*x + a*x^2)) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a*d +
 b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4
*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*Sqrt[d + e*x]*(c + b
*x + a*x^2)) - ((b*d + c*e)*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a
+ c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*
e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticP
```

```
i[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt
[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b^2 - 4*a*c]
 - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(Sqrt[2]*Sqrt[a]*d*(c +
 b*x + a*x^2))
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
```

```
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 930

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sq
rt[a + b*x + c*x^2]/(e*(m + 1))), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)
^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*f + a*g + 2*(c*f + b
*g)*x + 3*c*g*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f
 - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ
erQ[2*m] && LtQ[m, -1]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1587

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p]), Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{\sqrt{d+ex}\sqrt{c+bx+ax^2}}{x^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{bd+ce+2(ad+be)x+3aex^2}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{2\sqrt{c+bx+ax^2}}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \left(\frac{2(ad+be)}{\sqrt{d+ex}\sqrt{c+bx+ax^2}} + \frac{bd+ce}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}} + \frac{3aex}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\right)dx}{2\sqrt{c+bx+ax^2}}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{\left(3ae\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{x}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{2\sqrt{c+bx+ax^2}} + \frac{\left((ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{1}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{\sqrt{c+bx+ax^2}} + \frac{\left((bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{2\sqrt{c+bx+ax^2}}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{\left((bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{b-\sqrt{b^2-4ac}+2ax}\sqrt{b+\sqrt{b^2-4ac}+2ax}\right)\int \frac{1}{x\sqrt{b-\sqrt{b^2-4ac}+2ax}\sqrt{b+}}}{2\left(c+bx+ax^2\right)} + \frac{\left(3a\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{\sqrt{d+ex}}{\sqrt{c+bx+ax^2}}\,dx}{2\sqrt{c+bx+ax^2}} - \frac{\left(3ad\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{1}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{2\sqrt{c+bx+ax^2}} + \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-be-\sqrt{b^2-4ac}e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-x^2}\sqrt{1+}}\right.}{a\sqrt{d+ex}\left(c+bx+ax^2\right)}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2a}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{a\sqrt{d+ex}\,(c+bx+ax^2)}$$

$$-\frac{\left((bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{b-\sqrt{b^2-4ac}+2ax}\sqrt{b+\sqrt{b^2-4ac}+2ax}\right)\text{Subst}\left(\int\frac{}{(d-x^2)\sqrt{b-\sqrt{b^2-}}}\right.}{c+bx+ax^2}$$

$$+\frac{\left(3\sqrt{b^2-4ac}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}ex^2}{2ad-be-\sqrt{b^2-4ac}e}}}{\sqrt{1-x^2}}dx,x,\frac{\sqrt{\frac{b+\sqrt{b^2-4}}{\sqrt{b^2-}}}}{\sqrt{2}}\right.}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-be-\sqrt{b^2-4ac}e}}\,(c+bx+ax^2)}$$

$$-\frac{\left(3\sqrt{2}\sqrt{b^2-4ac}d\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-be-\sqrt{b^2-4ac}e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}ex}{2ad-be-\sqrt{b^2-}}}}\right.}{\sqrt{d+ex}\,(c+bx+ax^2)}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}$$

$$+\frac{3\sqrt{b^2-4ac}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right.}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,(c+bx+ax^2)}$$

$$-\frac{3\sqrt{2}\sqrt{b^2-4ac}d\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{}{2a}\right.}{\sqrt{d+ex}\,(c+bx+ax^2)}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2a}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{a\sqrt{d+ex}\,(c+bx+ax^2)}$$

$$-\frac{\left((bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{b+\sqrt{b^2-4ac}+2ax}\sqrt{1+\frac{2a(d+ex)}{\left(b-\sqrt{b^2-4ac}-\frac{2ad}{e}\right)e}}\right)\text{Subst}\left(\int\frac{}{(d-x^2)\sqrt{b+\sqrt{b^2}}}\right.}{c+bx+ax^2}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}$$

$$+\frac{3\sqrt{b^2-4ac}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)}\right.}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+bx+ax^2)}$$

$$-\frac{3\sqrt{2}\sqrt{b^2-4ac}d\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\right.}{\sqrt{d+ex}(c+bx+ax^2)}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{a\sqrt{d+ex}(c+bx+ax^2)}$$

$$-\frac{\left((bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1+\frac{2a(d+ex)}{\left(b-\sqrt{b^2-4ac}-\frac{2ad}{e}\right)e}}\sqrt{1+\frac{2a(d+ex)}{\left(b+\sqrt{b^2-4ac}-\frac{2ad}{e}\right)e}}\right)\text{Subst}\left(\int\frac{}{(d-x^2)\sqrt{1+\frac{}{(b}}}\right.}{c+bx+ax^2}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}$$

$$+\frac{3\sqrt{b^2-4ac}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)}\right.}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+bx+ax^2)}$$

$$-\frac{3\sqrt{2}\sqrt{b^2-4ac}d\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\right.}{\sqrt{d+ex}(c+bx+ax^2)}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{a\sqrt{d+ex}(c+bx+ax^2)}$$

$$-\frac{(bd+ce)\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)}}}{\sqrt{2}\sqrt{a}d(c+bx+ax^2)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.80 (sec) , antiderivative size = 1207, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx = \frac{1}{4}\sqrt{d+ex}\sqrt{a+\frac{c+bx}{x^2}}\left(-4\right.$$

$$+\frac{x(d+ex)\left(\frac{12de^2\sqrt{\frac{ad^2+e(-bd+ce)}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}(c+x(b+ax))}{(d+ex)^2}-\frac{3i\sqrt{2}d\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)\sqrt{\frac{-2ce^2+2adex+be(d-ex)+\sqrt{(b^2-4ac)e^2}(d+e}{\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}}{}\right.}{}$$

```
[In] Integrate[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x,x]
```

```
[Out] (Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2]*(-4 + (x*(d + e*x)*((12*d*e^2*Sqrt[(
a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c + x*
(b + a*x)))/(d + e*x)^2 - ((3*I)*Sqrt[2]*d*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c
)*e^2])*Sqrt[(-2*c*e^2 + 2*a*d*e*x + b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2
]*(d + e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c
*e^2 - 2*a*d*e*x + b*e*(-d + e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((-2
*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticE[I*ArcSinh[(Sqrt
[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])
/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e +
```

```
Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + (I*Sqrt[2]*(4*a*d^2 - b*d*e - 2
*c*e^2 + 3*d*Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*c*e^2 + 2*a*d*e*x + b*e*(d -
 e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*
c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*(-d + e*x) + Sqrt[(b^2
 - 4*a*c)*e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*
x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*
e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2
- 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + (
(2*I)*Sqrt[2]*e*(b*d + c*e)*Sqrt[(-2*c*e^2 + 2*a*d*e*x + b*e*(d - e*x) + Sq
rt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(
d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*(-d + e*x) + Sqrt[(b^2 - 4*a*c)*
e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Ellip
ticPi[(d*(2*a*d - b*e - Sqrt[(b^2 - 4*a*c)*e^2]))/(2*(a*d^2 + e*(-(b*d) + c
*e))), I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt
[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)
*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/(d*e*Sqrt
[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c +
x*(b + a*x)))))/4
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 1400, normalized size of antiderivative = 1.51

method	result	size
risch	Expression too large to display	1400
default	Expression too large to display	3553

```
[In] int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -((a*x^2+b*x+c)/x^2)^(1/2)*(e*x+d)^(1/2)+(2*d*a*(1/e*d-1/2*(b+(-4*a*c+b^2)^
(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-
b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+
1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/
(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d
-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a
)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+2*b*e*(1/e*d-1/2*(b+(-4*a*
c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x
-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/
2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))
^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)
/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(
1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+3*a*e*(1/e*d-1/2*(b
+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1
/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/
a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/
```

```
2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*((-1/e*d-1/2*
(-b+(-4*a*c+b^2)^(1/2))/a)*EllipticE(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^
(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4
*a*c+b^2)^(1/2))/a))^(1/2))+1/2*(-b+(-4*a*c+b^2)^(1/2))/a*EllipticF(((x+1/e
*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2
)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)))-(b*d+c*e)*(1/e*
d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2)
)/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)
^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+
b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*e/d*E
llipticPi(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),-(-1/e*d+1
/2*(b+(-4*a*c+b^2)^(1/2))/a)*e/d,((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1
/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)))*((a*x^2+b*x+c)/x^2)^(1/2)*x/(a
*x^2+b*x+c)*((a*x^2+b*x+c)*(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx=\text{Timed out}$$

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx=\int \frac{\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

```
[In] integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2)/x,x)
[Out] Integral(sqrt(d + e*x)*sqrt(a + b/x + c/x**2)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx = \int \frac{\sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx = \int \frac{\sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx = \int \frac{\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

```
[In] int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x,x)
[Out] int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x, x)
```

3.85 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 1287

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx = -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x}$$

$$+ \frac{\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)\cdots}\right.}{4\sqrt{2}cd\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,(c+bx+ax^2)}$$

$$+ \frac{3\sqrt{b^2-4ac}e\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right), -\frac{\cdots}{2ad-\cdots}\right.}{\sqrt{2}\sqrt{d+ex}\,(c+bx+ax^2)}$$

$$- \frac{\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\cdots\right.}{2\sqrt{2}c\sqrt{d+ex}\,(c+bx+ax^2)}$$

$$- \frac{(ad+be)\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,\text{Elliptic}\cdots}{\sqrt{2}\sqrt{a}d\,(c+bx+ax^2)}$$

$$+ \frac{(bd+ce)^2\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,\text{Elliptic}\cdots}{4\sqrt{2}\sqrt{a}cd^2\,(c+bx+ax^2)}$$

```
[Out] -1/4*(b*d+c*e)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/c/d-1/2*(a+c/x^2+b/x)^(1/2
)*(e*x+d)^(1/2)/x+1/8*(b*d+c*e)*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(
-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^
(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/c/d/(a*x^2+b*x+c)*2^(1/2)/(a*(e
*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+3/2*e*x*EllipticF(1/2*((b+2*a
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)
^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(a+c/x^2
+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-
4*a*c+b^2)^(1/2))))^(1/2)/(a*x^2+b*x+c)*2^(1/2)/(e*x+d)^(1/2)-1/4*(b*d+c*e)
*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4
*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(
a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/(a*x^2+b*x+c)*2^(1/2)/(
e*x+d)^(1/2)-1/2*(a*d+b*e)*x*EllipticPi(2^(1/2)*a^(1/2)*(e*x+d)^(1/2)/(2*a*
d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(2*a*d-b*e+e*(-4*a*c+b^2)^(1/2))/a/d,
((b-2*a*d/e-(-4*a*c+b^2)^(1/2))/(b-2*a*d/e+(-4*a*c+b^2)^(1/2)))^(1/2))*(a+c
/x^2+b/x)^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*a
*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^
(1/2))))^(1/2)/d/(a*x^2+b*x+c)*2^(1/2)/a^(1/2)+1/8*(b*d+c*e)^2*x*EllipticPi
(2^(1/2)*a^(1/2)*(e*x+d)^(1/2)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(
2*a*d-b*e+e*(-4*a*c+b^2)^(1/2))/a/d,((b-2*a*d/e-(-4*a*c+b^2)^(1/2))/(b-2*a*
d/e+(-4*a*c+b^2)^(1/2)))^(1/2))*(a+c/x^2+b/x)^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e
*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2
*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/d^2/(a*x^2+b*x+c)*2^(1
/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 3.64 (sec) , antiderivative size = 1287, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules

used = {1587, 930, 6874, 732, 430, 953, 948, 175, 552, 551, 857, 435}

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx$$

$$= \frac{\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,\text{EllipticPi}\left(\frac{2ad-be+\cdots}{2\cdots}\right.}{4\sqrt{2}\sqrt{a}cd^2\left(ax^2+bx+c\right)}$$

$$+\frac{\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{d+ex}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)(bd+\cdots}{4\sqrt{2}cd\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(ax^2+bx+c\right)}$$

$$-\frac{\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{\cdots}}{2ad-(b\cdots}\right.}{2\sqrt{2}c\sqrt{d+ex}\left(ax^2+bx+c\right)}$$

$$-\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}(bd+ce)}{4cd}$$

$$+\frac{3\sqrt{b^2-4ac}e\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{\cdots}{2ad-\cdots}\right.}{\sqrt{2}\sqrt{d+ex}\left(ax^2+bx+c\right)}$$

$$-\frac{(ad+be)\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,\text{Elliptic}\cdots}{\sqrt{2}\sqrt{a}d\left(ax^2+bx+c\right)}$$

$$-\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}}{2x}$$

```
[In] Int[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x^2,x]
```

```
[Out] -1/4*((b*d + c*e)*Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/(c*d) - (Sqrt[a + c/
x^2 + b/x]*Sqrt[d + e*x])/(2*x) + (Sqrt[b^2 - 4*a*c]*(b*d + c*e)*Sqrt[a + c
/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*El
lipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(4*Sqr
t[2]*c*d*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x +
 a*x^2)) + (3*Sqrt[b^2 - 4*a*c]*e*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x)
)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 -
4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4
```

```
*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*
e)])/(Sqrt[2]*Sqrt[d + e*x]*(c + b*x + a*x^2)) - (Sqrt[b^2 - 4*a*c]*(b*d +
c*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*
a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*Sqrt[2]*c*Sqrt[d + e*
x]*(c + b*x + a*x^2)) - ((a*d + b*e)*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e
]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqr
t[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b^2
 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(Sqrt[2]*Sqrt[
a]*d*(c + b*x + a*x^2)) + ((b*d + c*e)^2*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c
])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b
^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*
e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]
*Sqrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt
[b^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(4*Sqrt[2]
*Sqrt[a]*c*d^2*(c + b*x + a*x^2))
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 930

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sq
rt[a + b*x + c*x^2]/(e*(m + 1))), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)
^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*f + a*g + 2*(c*f + b
*g)*x + 3*c*g*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f
 - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ
erQ[2*m] && LtQ[m, -1]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
```

```
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 953

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*e
*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g
)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 1587

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p]), Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{\sqrt{d+ex}\sqrt{c+bx+ax^2}}{x^3}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= -\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x}+\frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{bd+ce+2(ad+be)x+3aex^2}{x^2\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{4\sqrt{c+bx+ax^2}}$$

$$= -\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x}+\frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \left(\frac{3ae}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}+\frac{bd+ce}{x^2\sqrt{d+ex}\sqrt{c+bx+ax^2}}+\frac{2(ad+be)}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}}\right)dx}{4\sqrt{c+bx+ax^2}}$$

$$= -\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x} + \frac{\left(3ae\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{1}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{4\sqrt{c+bx+ax^2}}$$

$$+ \frac{\left((ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{2\sqrt{c+bx+ax^2}}$$

$$+ \frac{\left((bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{1}{x^2\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{4\sqrt{c+bx+ax^2}}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x}$$

$$+ \frac{\left((ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{b-\sqrt{b^2-4ac}+2ax}\sqrt{b+\sqrt{b^2-4ac}+2ax}\right)\int \frac{1}{x\sqrt{b-\sqrt{b^2-4ac}+2ax}\sqrt{b+\sqrt{}}}}{2\left(c+bx+ax^2\right)}$$

$$- \frac{\left((bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{bd+ce-aex^2}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{8cd\sqrt{c+bx+ax^2}}$$

$$+ \frac{\left(3\sqrt{b^2-4ac}e\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-be-\sqrt{b^2-4ac}e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-4ac}ex^2}{2ad-be-\sqrt{b^2-4ac}e}}}\right.}{\sqrt{2}\sqrt{d+ex}\left(c+bx+ax^2\right)}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x}$$

$$+ \frac{3\sqrt{b^2-4ac}e\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2}{2ad-(}\right.}{\sqrt{2}\sqrt{d+ex}\left(c+bx+ax^2\right)}$$

$$- \frac{\left((ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{b-\sqrt{b^2-4ac}+2ax}\sqrt{b+\sqrt{b^2-4ac}+2ax}\right)\text{Subst}\left(\int \frac{}{(d-x^2)\sqrt{b-\sqrt{b^2-}}}\right.}{c+bx+ax^2}$$

$$- \frac{\left((bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \left(\frac{bd+ce}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}} - \frac{aex}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\right)\,dx}{8cd\sqrt{c+bx+ax^2}}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x}$$

$$+ \frac{3\sqrt{b^2-4ac}e\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{}{2ad-}\right.}{\sqrt{2}\sqrt{d+ex}\,(c+bx+ax^2)}$$

$$+ \frac{\left(ae(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{x}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{8cd\sqrt{c+bx+ax^2}}$$

$$- \frac{\left((bd+ce)^2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{1}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{8cd\sqrt{c+bx+ax^2}}$$

$$- \frac{\left((ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{b+\sqrt{b^2-4ac}+2ax}\sqrt{1+\frac{2a(d+ex)}{\left(b-\sqrt{b^2-4ac}-\frac{2ad}{e}\right)e}}\right)\text{Subst}\left(\int\frac{}{(d-x^2)\sqrt{b+\sqrt{}}}\right.}{c+bx+ax^2}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x}$$

$$+ \frac{3\sqrt{b^2-4ac}e\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{}{2ad-}\right.}{\sqrt{2}\sqrt{d+ex}\,(c+bx+ax^2)}$$

$$- \frac{\left((bd+ce)^2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{b-\sqrt{b^2-4ac}+2ax}\sqrt{b+\sqrt{b^2-4ac}+2ax}\right)\int\frac{1}{x\sqrt{b-\sqrt{b^2-4ac}+2ax}\sqrt{b}}}{8cd\,(c+bx+ax^2)}$$

$$- \frac{\left(a(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{1}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{8c\sqrt{c+bx+ax^2}}$$

$$+ \frac{\left(a(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int\frac{\sqrt{d+ex}}{\sqrt{c+bx+ax^2}}\,dx}{8cd\sqrt{c+bx+ax^2}}$$

$$- \frac{\left((ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1+\frac{2a(d+ex)}{\left(b-\sqrt{b^2-4ac}-\frac{2ad}{e}\right)e}}\sqrt{1+\frac{2a(d+ex)}{\left(b+\sqrt{b^2-4ac}-\frac{2ad}{e}\right)e}}\right)\text{Subst}\left(\int\frac{}{(d-x^2)\sqrt{1+\frac{}{(b}}}\right.}{c+bx+ax^2}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x}$$

$$+ \frac{3\sqrt{b^2-4ac}e\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2}{2ad-(}\right.}{\sqrt{2}\sqrt{d+ex}\left(c+bx+ax^2\right)}$$

$$- \frac{(ad+be)\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}}{\sqrt{2}\sqrt{a}d\left(c+bx+ax^2\right)}$$

$$+ \frac{\left((bd+ce)^2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{b-\sqrt{b^2-4ac}+2ax}\sqrt{b+\sqrt{b^2-4ac}+2ax}\right)\text{Subst}\left(\int\frac{}{(d-x^2)\sqrt{b-\sqrt{b^2}}}\right.}{4cd\left(c+bx+ax^2\right)}$$

$$+ \frac{\left(\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{\sqrt{1+\frac{2\sqrt{b^2-4ac}ex^2}{2ad-be-\sqrt{b^2-4ac}e}}}{\sqrt{1-x^2}}dx, x, \right.}{4\sqrt{2}cd\sqrt{\frac{a(d+ex)}{2ad-be-\sqrt{b^2-4ac}e}}\left(c+bx+ax^2\right)}$$

$$- \frac{\left(\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-be-\sqrt{b^2-4ac}e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{2\sqrt{b^2-}}{2ad-be-}}}\right.}{2\sqrt{2}c\sqrt{d+ex}\left(c+bx+ax^2\right)}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x}$$

$$+ \frac{\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b}}{2ad-(b+}\right.}{4\sqrt{2}cd\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(c+bx+ax^2)}$$

$$+ \frac{3\sqrt{b^2-4ac}e\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{}{2ad-}\right.}{\sqrt{2}\sqrt{d+ex}(c+bx+ax^2)}$$

$$- \frac{\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{2\sqrt{2}c\sqrt{d+ex}(c+bx+ax^2)}$$

$$- \frac{(ad+be)\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)}}}{\sqrt{2}\sqrt{a}d(c+bx+ax^2)}$$

$$+ \frac{\left((bd+ce)^2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{b+\sqrt{b^2-4ac}+2ax}\sqrt{1+\frac{2a(d+ex)}{\left(b-\sqrt{b^2-4ac}-\frac{2ad}{e}\right)e}}\right)\text{Subst}\left(\int\frac{}{(d-x^2)\sqrt{b+\sqrt{}}}\right.}{4cd(c+bx+ax^2)}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x}$$

$$+\frac{\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-}}{2ad-\left(b+\sqrt{}\right.}\right.}{4\sqrt{2}cd\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(c+bx+ax^2\right)}$$

$$+\frac{3\sqrt{b^2-4ac}e\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2}{2ad-(}\right.}{\sqrt{2}\sqrt{d+ex}\left(c+bx+ax^2\right)}$$

$$-\frac{\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|\right.}{2\sqrt{2}c\sqrt{d+ex}\left(c+bx+ax^2\right)}$$

$$-\frac{(ad+be)\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}}{\sqrt{2}\sqrt{a}d\left(c+bx+ax^2\right)}$$

$$+\frac{\left((bd+ce)^2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1+\frac{2a(d+ex)}{\left(b-\sqrt{b^2-4ac}-\frac{2ad}{e}\right)e}}\sqrt{1+\frac{2a(d+ex)}{\left(b+\sqrt{b^2-4ac}-\frac{2ad}{e}\right)e}}\right)\text{Subst}\left(\int\frac{}{(d-x^2)\sqrt{1+\frac{}{(b-}}}\right.}{4cd\left(c+bx+ax^2\right)}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x}$$

$$+ \frac{\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b}}{2ad-\left(b+\right.}\right.}{4\sqrt{2}cd\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,(c+bx+ax^2)}$$

$$+ \frac{3\sqrt{b^2-4ac}e\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{}{2ad-}\right.}{\sqrt{2}\sqrt{d+ex}\,(c+bx+ax^2)}$$

$$- \frac{\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+bx+ax^2)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{b^2-4ac}+2ax}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{2\sqrt{2}c\sqrt{d+ex}\,(c+bx+ax^2)}$$

$$- \frac{(ad+be)\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)}}}{\sqrt{2}\sqrt{a}d\,(c+bx+ax^2)}$$

$$+ \frac{(bd+ce)^2\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right.}}}{4\sqrt{2}\sqrt{a}cd^2\,(c+bx+ax^2)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.61 (sec) , antiderivative size = 1392, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx = \frac{1}{16}x\sqrt{d+ex}\sqrt{a+\frac{c+bx}{x^2}}\left(-\frac{4(2cd+bdx+cex)}{cdx^2}\right.$$

$$+\frac{(d+ex)\left(\frac{4de^2(bd+ce)\sqrt{\frac{ad^2+e(-bd+ce)}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}(c+x(b+ax))}{(d+ex)^2}-\frac{i\sqrt{2}d(bd+ce)\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)\sqrt{\frac{-2ce^2+d\sqrt{(b^2-4ac)e^2}+2adex+}{\left(2ad-be+\sqrt{(b^2-4\right.}}}}{}\right.}{}$$

```
[In] Integrate[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x^2,x]
```

```
[Out] (x*Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2]*((-4*(2*c*d + b*d*x + c*e*x))/(c*d
*x^2) + ((d + e*x)*((4*d*e^2*(b*d + c*e)*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-
2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c + x*(b + a*x)))/(d + e*x)^2 - (I
*Sqrt[2]*d*(b*d + c*e)*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*c*e
^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x +
b*e*(d - e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2
*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*
x + b*e*(-d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*E
llipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sq
rt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*
c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + (I*Sqrt
[2]*(b^2*d^2*e + b*d*(-5*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]) + c*e*(4*a*d^2
```

```
+ 2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]))*Sqrt[(-2*c*e^2 + d*Sqrt[(b^2 - 4*a*
c)*e^2] + 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*x))/((2*a*d
- b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 + d*Sqrt[(b^2 -
4*a*c)*e^2] - 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(-d + e*x))/((-
2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticF[I*ArcSinh[(Sqr
t[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]
)/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e +
 Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] - ((2*I)*Sqrt[2]*e*(b^2*d^2 - 2*
b*c*d*e + c*(-4*a*d^2 + c*e^2))*Sqrt[(-2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]
+ 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*x))/((2*a*d - b*e +
Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e
^2] - 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(-d + e*x))/((-2*a*d +
b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticPi[(d*(2*a*d - b*e - Sqr
t[(b^2 - 4*a*c)*e^2]))/(2*(a*d^2 + e*(-(b*d) + c*e))), I*ArcSinh[(Sqrt[2]*S
qrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt
[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[
(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/(c*d^2*e*Sqrt[(a*d^2 + e*(-(b*d) + c
*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c + x*(b + a*x)))))/16
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 1597, normalized size of antiderivative = 1.24

method	result	size
risch	Expression too large to display	1597
default	Expression too large to display	4957

```
[In] int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(e*x+d)^(1/2)*(b*d*x+c*e*x+2*c*d)/x/c/d*((a*x^2+b*x+c)/x^2)^(1/2)+1/8/
c/d*(2*e^2*a*c*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(
b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-
1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1
/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*
e*x+c*d)^(1/2)*((-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)*EllipticE(((x+1/e*d)
/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(
1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))+1/2*(-b+(-4*a*c+b^2
)^(1/2))/a*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)
,((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))
/a))^(1/2)))+12*a*c*d*e*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/
e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)
/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2
))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2
+b*d*x+c*e*x+c*d)^(1/2)*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/
2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*
```

```
c+b^2)^(1/2))/a))^(1/2))+2*a*b*d*e*(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x
+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2
)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*
c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*
x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*((-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)*
EllipticE(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),((-1/e*d+1
/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2))
+1/2*(-b+(-4*a*c+b^2)^(1/2))/a*EllipticF(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b
^2)^(1/2))/a))^(1/2),((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b
+(-4*a*c+b^2)^(1/2))/a))^(1/2)))-2*(4*a*c*d^2-b^2*d^2+2*b*c*d*e-c^2*e^2)*(1
/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a)*((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1
/2))/a))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d-1/2*(-b+(-4*a*c+b
^2)^(1/2))/a))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/(-1/e*d+1/2*(b+(-4*a
*c+b^2)^(1/2))/a))^(1/2)/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)^(1/2)*e/
d*EllipticPi(((x+1/e*d)/(1/e*d-1/2*(b+(-4*a*c+b^2)^(1/2))/a))^(1/2),-(-1/e*
d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)*e/d,((-1/e*d+1/2*(b+(-4*a*c+b^2)^(1/2))/a)/
(-1/e*d-1/2*(-b+(-4*a*c+b^2)^(1/2))/a))^(1/2)))*((a*x^2+b*x+c)/x^2)^(1/2)*x
/(a*x^2+b*x+c)*((a*x^2+b*x+c)*(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx=\int \frac{\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x^2}\,dx$$

```
[In] integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2)/x**2,x)
[Out] Integral(sqrt(d + e*x)*sqrt(a + b/x + c/x**2)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx = \int \frac{\sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x^2}\,dx$$

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx = \int \frac{\sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x^2}\,dx$$

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx = \int \frac{\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x^2}\,dx$$

```
[In] int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x^2,x)
[Out] int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x^2, x)
```

3.86 $\int (fx)^m (d+ex^n)^q (a+cx^{2n})^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int (fx)^m (d+ex^n)^q (a+cx^{2n})^p \, dx = \text{Int}\left((fx)^m (d+ex^n)^q (a+cx^{2n})^p, x\right)$$

```
[Out] Unintegrable((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m (d+ex^n)^q (a+cx^{2n})^p \, dx = \int (fx)^m (d+ex^n)^q (a+cx^{2n})^p \, dx$$

```
[In] Int[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p,x]
```

```
[Out] Defer[Int][(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Rubi steps

$$\text{integral} = \int (fx)^m (d+ex^n)^q (a+cx^{2n})^p \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (fx)^m (d + ex^n)^q (a + cx^{2n})^p \, dx = \int (fx)^m (d + ex^n)^q (a + cx^{2n})^p \, dx$$

```
[In] Integrate[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p,x]
[Out] Integrate[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (fx)^m (d + e\,x^n)^q (a + c\,x^{2n})^p \, dx$$

```
[In] int((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x)
[Out] int((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (fx)^m (d + ex^n)^q (a + cx^{2n})^p \, dx = \int (cx^{2n} + a)^p (ex^n + d)^q (fx)^m \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m (d + ex^n)^q (a + cx^{2n})^p \, dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(d+e*x**n)**q*(a+c*x**(2*n))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (fx)^m (d + ex^n)^q (a + cx^{2n})^p \, dx = \int (cx^{2n} + a)^p (ex^n + d)^q (fx)^m \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (fx)^m (d + ex^n)^q (a + cx^{2n})^p \, dx = \int (cx^{2n} + a)^p (ex^n + d)^q (fx)^m \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 8.79 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int (fx)^m (d + ex^n)^q (a + cx^{2n})^p \, dx = \int (a + c\,x^{2\,n})^p\,(f\,x)^m\,(d + e\,x^n)^q \,dx$$

```
[In] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^q,x)
[Out] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^q, x)
```

3.87 $\int (fx)^m (d+ex^n)^3 (a+cx^{2n})^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 358

$$\int (fx)^m (d+ex^n)^3 (a+cx^{2n})^p \, dx = \frac{d^3(fx)^{1+m} (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{2n},-p,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \frac{3d^2ex^{1+n}(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m+n}{2n},-p,\frac{1+m+3n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+n} + \frac{3de^2x^{1+2n}(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m+2n}{2n},-p,\frac{1+m+4n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+2n} + \frac{e^3x^{1+3n}(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m+3n}{2n},-p,\frac{1+m+5n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+3n}$$

```
[Out] d^3*(f*x)^(1+m)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m)/n],[1+1/2*(1+m)/n]
,-c*x^(2*n)/a)/f/(1+m)/((1+c*x^(2*n)/a)^p)+3*d^2*e*x^(1+n)*(f*x)^m*(a+c*x^(
2*n))^p*hypergeom([-p, 1/2*(1+m+n)/n],[1/2*(1+m+3*n)/n],-c*x^(2*n)/a)/(1+m+
n)/((1+c*x^(2*n)/a)^p)+3*d*e^2*x^(1+2*n)*(f*x)^m*(a+c*x^(2*n))^p*hypergeom(
[-p, 1/2*(1+m+2*n)/n],[1/2*(1+m+4*n)/n],-c*x^(2*n)/a)/(1+m+2*n)/((1+c*x^(2*
n)/a)^p)+e^3*x^(1+3*n)*(f*x)^m*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m+3*n)
/n],[1/2*(1+m+5*n)/n],-c*x^(2*n)/a)/(1+m+3*n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1575, 372, 371, 20}

$$\int (fx)^m (d+ex^n)^3 (a+cx^{2n})^p \, dx$$
$$= \frac{d^3(fx)^{m+1}(a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{2n},-p,\frac{m+1}{2n}+1,-\frac{cx^{2n}}{a}\right)}{f(m+1)}$$
$$+\frac{3d^2ex^{n+1}(fx)^m(a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+n+1}{2n},-p,\frac{m+3n+1}{2n},-\frac{cx^{2n}}{a}\right)}{m+n+1}$$
$$+\frac{3de^2x^{2n+1}(fx)^m(a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+2n+1}{2n},-p,\frac{m+4n+1}{2n},-\frac{cx^{2n}}{a}\right)}{m+2n+1}$$
$$+\frac{e^3x^{3n+1}(fx)^m(a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+3n+1}{2n},-p,\frac{m+5n+1}{2n},-\frac{cx^{2n}}{a}\right)}{m+3n+1}$$

```
[In] Int[(f*x)^m*(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

```
[Out] (d^3*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1
 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) + (3
*d^2*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2
*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*
n))/a)^p) + (3*d*e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F
1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m
+ 2*n)*(1 + (c*x^(2*n))/a)^p) + (e^3*x^(1 + 3*n)*(f*x)^m*(a + c*x^(2*n))^p*
Hypergeometric2F1[(1 + m + 3*n)/(2*n), -p, (1 + m + 5*n)/(2*n), -((c*x^(2*n
))/a)])/((1 + m + 3*n)*(1 + (c*x^(2*n))/a)^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1575

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
 && (IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(d^3(fx)^m\left(a+cx^{2n}\right)^p+3d^2ex^n(fx)^m\left(a+cx^{2n}\right)^p+3de^2x^{2n}(fx)^m\left(a+cx^{2n}\right)^p\right.\\
&\qquad\left.+e^3x^{3n}(fx)^m\left(a+cx^{2n}\right)^p\right)dx\\
&= d^3\int (fx)^m\left(a+cx^{2n}\right)^p\,dx+\left(3d^2e\right)\int x^n(fx)^m\left(a+cx^{2n}\right)^p\,dx\\
&\quad+\left(3de^2\right)\int x^{2n}(fx)^m\left(a+cx^{2n}\right)^p\,dx+e^3\int x^{3n}(fx)^m\left(a+cx^{2n}\right)^p\,dx\\
&= \left(3d^2ex^{-m}(fx)^m\right)\int x^{m+n}\left(a+cx^{2n}\right)^p\,dx+\left(3de^2x^{-m}(fx)^m\right)\int x^{m+2n}\left(a+cx^{2n}\right)^p\,dx\\
&\quad+\left(e^3x^{-m}(fx)^m\right)\int x^{m+3n}\left(a+cx^{2n}\right)^p\,dx\\
&\quad+\left(d^3\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int (fx)^m\left(1+\frac{cx^{2n}}{a}\right)^p\,dx\\
&= \frac{d^3(fx)^{1+m}\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\,{}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)}\\
&\quad+\left(3d^2ex^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int x^{m+n}\left(1+\frac{cx^{2n}}{a}\right)^p\,dx\\
&\quad+\left(3de^2x^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int x^{m+2n}\left(1+\frac{cx^{2n}}{a}\right)^p\,dx\\
&\quad+\left(e^3x^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int x^{m+3n}\left(1+\frac{cx^{2n}}{a}\right)^p\,dx
\end{aligned}$$

$$= \frac{d^3(fx)^{1+m}\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \frac{3d^2ex^{1+n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1+m+n}{2n},-p;\frac{1+m+3n}{2n};-\frac{cx^{2n}}{a}\right)}{1+m+n} + \frac{3de^2x^{1+2n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1+m+2n}{2n},-p;\frac{1+m+4n}{2n};-\frac{cx^{2n}}{a}\right)}{1+m+2n} + \frac{e^3x^{1+3n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1+m+3n}{2n},-p;\frac{1+m+5n}{2n};-\frac{cx^{2n}}{a}\right)}{1+m+3n}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.70

$$\int (fx)^m\left(d+ex^n\right)^3\left(a+cx^{2n}\right)^p\,dx$$

$$= x(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(\frac{d^3\,\text{Hypergeometric2F1}\left(\frac{1+m}{2n},-p,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a}\right)}{1+m} + ex^n\left(\frac{3d^2\,\text{Hypergeometric2F1}\left(\frac{1+m+n}{2n},-p,\frac{1+m+3n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+n} + ex^n\left(\frac{3d\,\text{Hypergeometric2F1}\left(\frac{1+m+2n}{2n},-p,\frac{1+m+4n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+2n} + \frac{ex^n\,\text{Hypergeometric2F1}\left(\frac{1+m+3n}{2n},-p,\frac{1+m}{2}\right.}{1+m+3n}\right.\right.\right.$$

```
[In] Integrate[(f*x)^m*(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*((d^3*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 +
 (1 + m)/(2*n), -((c*x^(2*n))/a)])/(1 + m) + e*x^n*((3*d^2*Hypergeometric2F
1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + n
) + e*x^n*((3*d*Hypergeometric2F1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2
*n), -((c*x^(2*n))/a)])/(1 + m + 2*n) + (e*x^n*Hypergeometric2F1[(1 + m + 3
*n)/(2*n), -p, (1 + m + 5*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + 3*n)))))/(1
 + (c*x^(2*n))/a)^p
```

Maple [F]

$$\int (fx)^m (d+e x^n)^3 (a+c x^{2n})^p dx$$

```
[In] int((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x)
[Out] int((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int (fx)^m (d+ex^n)^3 (a+cx^{2n})^p dx = \int (ex^n+d)^3 (cx^{2n}+a)^p (fx)^m dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)*(c*x^(2*n) + a
)^p*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n)^3 (a+cx^{2n})^p dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(d+e*x**n)**3*(a+c*x**(2*n))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (fx)^m (d+ex^n)^3 (a+cx^{2n})^p dx = \int (ex^n+d)^3 (cx^{2n}+a)^p (fx)^m dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((e*x^n + d)^3*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (fx)^m (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{96,[1,0,6,4,0,3,5,4,1,2]%%%}+%%%{480,[1,0,6,4,0,3,4,4,1,2]%%%
}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx = \int \left(a+c\,x^{2\,n}\right)^p\,(f\,x)^m\,(d+e\,x^n)^3 \,dx$$

```
[In] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^3,x)
```

```
[Out] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^3, x)
```

3.88 $\int (fx)^m (d+ex^n)^2 (a+cx^{2n})^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 262

$$\int (fx)^m (d+ex^n)^2 (a+cx^{2n})^p \, dx = \frac{d^2(fx)^{1+m} (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{2n},-p,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \frac{2dex^{1+n}(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m+n}{2n},-p,\frac{1+m+3n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+n} + \frac{e^2x^{1+2n}(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m+2n}{2n},-p,\frac{1+m+4n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+2n}$$

```
[Out] d^2*(f*x)^(1+m)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m)/n],[1+1/2*(1+m)/n]
,-c*x^(2*n)/a)/f/(1+m)/((1+c*x^(2*n)/a)^p)+2*d*e*x^(1+n)*(f*x)^m*(a+c*x^(2*
n))^p*hypergeom([-p, 1/2*(1+m+n)/n],[1/2*(1+m+3*n)/n],-c*x^(2*n)/a)/(1+m+n)
/((1+c*x^(2*n)/a)^p)+e^2*x^(1+2*n)*(f*x)^m*(a+c*x^(2*n))^p*hypergeom([-p, 1
/2*(1+m+2*n)/n],[1/2*(1+m+4*n)/n],-c*x^(2*n)/a)/(1+m+2*n)/((1+c*x^(2*n)/a)^
p)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1575, 372, 371, 20}

$$\int (fx)^m (d+ex^n)^2 (a+cx^{2n})^p \, dx$$
$$= \frac{d^2(fx)^{m+1}(a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{2n},-p,\frac{m+1}{2n}+1,-\frac{cx^{2n}}{a}\right)}{f(m+1)}$$
$$+\frac{2dex^{n+1}(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+n+1}{2n},-p,\frac{m+3n+1}{2n},-\frac{cx^{2n}}{a}\right)}{m+n+1}$$
$$+\frac{e^2x^{2n+1}(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+2n+1}{2n},-p,\frac{m+4n+1}{2n},-\frac{cx^{2n}}{a}\right)}{m+2n+1}$$

```
[In] Int[(f*x)^m*(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

```
[Out] (d^2*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1
 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) + (2
*d*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2*n
), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*n)
)/a)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 +
 m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + 2*n)
*(1 + (c*x^(2*n))/a)^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
```

```
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1575

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
 && (IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(d^2(fx)^m \left(a+cx^{2n}\right)^p + 2dex^n(fx)^m \left(a+cx^{2n}\right)^p + e^2x^{2n}(fx)^m \left(a+cx^{2n}\right)^p\right) dx \\
&= d^2 \int (fx)^m \left(a+cx^{2n}\right)^p dx + (2de) \int x^n(fx)^m \left(a+cx^{2n}\right)^p dx + e^2 \int x^{2n}(fx)^m \left(a+cx^{2n}\right)^p dx \\
&= \left(2dex^{-m}(fx)^m\right) \int x^{m+n}\left(a+cx^{2n}\right)^p dx + \left(e^2x^{-m}(fx)^m\right) \int x^{m+2n}\left(a+cx^{2n}\right)^p dx \\
&\quad + \left(d^2\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p}\right) \int (fx)^m \left(1+\frac{cx^{2n}}{a}\right)^p dx \\
&= \frac{d^2(fx)^{1+m}\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} \\
&\quad + \left(2dex^{-m}(fx)^m\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p}\right) \int x^{m+n}\left(1+\frac{cx^{2n}}{a}\right)^p dx \\
&\quad + \left(e^2x^{-m}(fx)^m\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p}\right) \int x^{m+2n}\left(1+\frac{cx^{2n}}{a}\right)^p dx \\
&= \frac{d^2(fx)^{1+m}\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} \\
&\quad + \frac{2dex^{1+n}(fx)^m\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m+n}{2n},-p;\frac{1+m+3n}{2n};-\frac{cx^{2n}}{a}\right)}{1+m+n} \\
&\quad + \frac{e^2x^{1+2n}(fx)^m\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m+2n}{2n},-p;\frac{1+m+4n}{2n};-\frac{cx^{2n}}{a}\right)}{1+m+2n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.72

$$\int (fx)^m (d+ex^n)^2 \left(a+cx^{2n}\right)^p dx$$
$$= x(fx)^m \left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \left(\frac{d^2 \text{Hypergeometric2F1}\left(\frac{1+m}{2n},-p,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a}\right)}{1+m} + ex^n \left(\frac{2d \text{Hypergeometric2F1}\left(\frac{1+m+n}{2n},-p,\frac{1+m+3n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+n} + \frac{ex^n \text{Hypergeometric2F1}\left(\frac{1+m+2n}{2n},-p,\frac{1+m+4n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+2n}\right)\right)$$

```
[In] Integrate[(f*x)^m*(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*((d^2*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 +
 (1 + m)/(2*n), -((c*x^(2*n))/a)])/(1 + m) + e*x^n*((2*d*Hypergeometric2F1[
(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + n)
+ (e*x^n*Hypergeometric2F1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -(
(c*x^(2*n))/a)])/(1 + m + 2*n))))/(1 + (c*x^(2*n))/a)^p
```

Maple [F]

$$\int (fx)^m (d+e\,x^n)^2 \left(a+c\,x^{2n}\right)^p dx$$

```
[In] int((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x)
[Out] int((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int (fx)^m (d+ex^n)^2 \left(a+cx^{2n}\right)^p dx = \int (ex^n+d)^2 (cx^{2n}+a)^p (fx)^m dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n)^2 (a+cx^{2n})^p \, dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(d+e*x**n)**2*(a+c*x**(2*n))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (fx)^m (d+ex^n)^2 (a+cx^{2n})^p \, dx = \int (ex^n+d)^2 (cx^{2n}+a)^p (fx)^m \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((e*x^n + d)^2*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (fx)^m (d+ex^n)^2 (a+cx^{2n})^p \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{64,[1,0,4,3,0,1,4,3,1,1]%%%}+%%%{256,[1,0,4,3,0,1,3,3,1,1]%%%
}+%%%{
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n)^2 (a+cx^{2n})^p \, dx = \int (a+c\,x^{2\,n})^p\,(f\,x)^m\,(d+e\,x^n)^2 \, dx$$

```
[In] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^2,x)
[Out] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^2, x)
```

3.89 $\int (fx)^m (d+ex^n) \left(a+cx^{2n}\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 166

$$\int (fx)^m (d+ex^n) \left(a+cx^{2n}\right)^p \, dx = \frac{d(fx)^{1+m}\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2n},-p,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \frac{ex^{1+n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m+n}{2n},-p,\frac{1+m+3n}{2n},-\frac{cx^{2n}}{a}\right)}{1+m+n}$$

```
[Out] d*(f*x)^(1+m)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m)/n],[1+1/2*(1+m)/n],-
c*x^(2*n)/a)/f/(1+m)/((1+c*x^(2*n)/a)^p)+e*x^(1+n)*(f*x)^m*(a+c*x^(2*n))^p*
hypergeom([-p, 1/2*(1+m+n)/n],[1/2*(1+m+3*n)/n],-c*x^(2*n)/a)/(1+m+n)/((1+c
*x^(2*n)/a)^p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1575, 372, 371, 20}

$$\int (fx)^m (d+ex^n) \left(a+cx^{2n}\right)^p \, dx = \frac{d(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2n},-p,\frac{m+1}{2n}+1,-\frac{cx^{2n}}{a}\right)}{f(m+1)} + \frac{ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+n+1}{2n},-p,\frac{m+3n+1}{2n},-\frac{cx^{2n}}{a}\right)}{m+n+1}$$

```
[In] Int[(f*x)^m*(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

```
[Out] (d*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 +
 (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) + (e*x
^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2*n), -p,
 (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*n))/a)^p
)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1575

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
 && (IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(d(fx)^m \left(a+cx^{2n}\right)^p + ex^n(fx)^m \left(a+cx^{2n}\right)^p\right) dx \\
&= d\int (fx)^m \left(a+cx^{2n}\right)^p dx + e\int x^n(fx)^m \left(a+cx^{2n}\right)^p dx \\
&= \left(ex^{-m}(fx)^m\right)\int x^{m+n}\left(a+cx^{2n}\right)^p dx \\
&\quad + \left(d\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int (fx)^m \left(1+\frac{cx^{2n}}{a}\right)^p dx
\end{aligned}
$$

$$= \frac{d(fx)^{1+m}\left(a+cx^{2n}\right)^{p}\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_{2}F_{1}\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \left(ex^{-m}(fx)^{m}\left(a+cx^{2n}\right)^{p}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int x^{m+n}\left(1+\frac{cx^{2n}}{a}\right)^{p}dx$$

$$= \frac{d(fx)^{1+m}\left(a+cx^{2n}\right)^{p}\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_{2}F_{1}\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \frac{ex^{1+n}(fx)^{m}\left(a+cx^{2n}\right)^{p}\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_{2}F_{1}\left(\frac{1+m+n}{2n},-p;\frac{1+m+3n}{2n};-\frac{cx^{2n}}{a}\right)}{1+m+n}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.82

$$\int (fx)^{m}\left(d+ex^{n}\right)\left(a+cx^{2n}\right)^{p}dx = \frac{x(fx)^{m}\left(a+cx^{2n}\right)^{p}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(d(1+m+n)\,\text{Hypergeometric2F1}\left(\frac{1+m}{2n},-p,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a}\right)+e(1+r\right.}{(1+m)(1+m+n)}$$

```
[In] Integrate[(f*x)^m*(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*(d*(1 + m + n)*Hypergeometric2F1[(1 + m)/(2*n)
, -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)] + e*(1 + m)*x^n*Hypergeometric2F
1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)]))/((1 + m)*
(1 + m + n)*(1 + (c*x^(2*n))/a)^p)
```

Maple [F]

$$\int (fx)^{m}\left(d+e\,x^{n}\right)\left(a+c\,x^{2n}\right)^{p}dx$$

```
[In] int((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int (fx)^m (d+ex^n) \left(a+cx^{2n}\right)^p \, dx = \int (ex^n+d)(cx^{2n}+a)^p (fx)^m \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n) \left(a+cx^{2n}\right)^p \, dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(d+e*x**n)*(a+c*x**(2*n))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (fx)^m (d+ex^n) \left(a+cx^{2n}\right)^p \, dx = \int (ex^n+d)(cx^{2n}+a)^p (fx)^m \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Giac [F]

$$\int (fx)^m (d+ex^n) \left(a+cx^{2n}\right)^p \, dx = \int (ex^n+d)(cx^{2n}+a)^p (fx)^m \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n) \left(a+cx^{2n}\right)^p \, dx = \int \left(a+c\,x^{2\,n}\right)^p (f\,x)^m (d+e\,x^n) \, dx$$

```
[In] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n),x)
[Out] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n), x)
```

3.90 $\int \frac{(fx)^m(a+cx^{2n})^p}{d+ex^n}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 194

$$\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{d+ex^n}\,dx = \frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m}{2n},-p,1,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d(1+m)} - \frac{ex^{1+n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m+n}{2n},-p,1,\frac{1+m+3n}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(1+m+n)}$$

```
[Out] x*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m)/n,1,-p,1+1/2*(1+m)/n,e^2*x^(2*
n)/d^2,-c*x^(2*n)/a)/d/(1+m)/((1+c*x^(2*n)/a)^p)-e*x^(1+n)*(f*x)^m*(a+c*x^(
2*n))^p*AppellF1(1/2*(1+m+n)/n,1,-p,1/2*(1+m+3*n)/n,e^2*x^(2*n)/d^2,-c*x^(2
*n)/a)/d^2/(1+m+n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1576, 525, 524}

$$\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{d+ex^n}\,dx = \frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{m+1}{2n},-p,1,\frac{m+1}{2n}+1,-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d(m+1)} - \frac{ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{m+n+1}{2n},-p,1,\frac{m+3n+1}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(m+n+1)}$$

```
[In] Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n),x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 1, 1 + (1 + m)/(2*
n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d*(1 + m)*(1 + (c*x^(2*n))/a)^p)
 - (e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -p, 1
, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + m +
n)*(1 + (c*x^(2*n))/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1576

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(
2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/(d^2 - e^2*x^(2*n))))^(-q), x], x]
, x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !IntegerQ[p]
 && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(x^{-m}(fx)^m\right)\int\left(\frac{dx^m\left(a+cx^{2n}\right)^p}{d^2-e^2x^{2n}}+\frac{ex^{m+n}\left(a+cx^{2n}\right)^p}{-d^2+e^2x^{2n}}\right)dx\\
&= \left(dx^{-m}(fx)^m\right)\int\frac{x^m\left(a+cx^{2n}\right)^p}{d^2-e^2x^{2n}}\,dx+\left(ex^{-m}(fx)^m\right)\int\frac{x^{m+n}\left(a+cx^{2n}\right)^p}{-d^2+e^2x^{2n}}\,dx\\
&= \left(dx^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{x^m\left(1+\frac{cx^{2n}}{a}\right)^p}{d^2-e^2x^{2n}}\,dx\\
&\quad+\left(ex^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{x^{m+n}\left(1+\frac{cx^{2n}}{a}\right)^p}{-d^2+e^2x^{2n}}\,dx
\end{aligned}$$

$$= \frac{x(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} F_1\left(\frac{1+m}{2n}; -p, 1; 1+\frac{1+m}{2n}; -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d(1+m)} - \frac{ex^{1+n}(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} F_1\left(\frac{1+m+n}{2n}; -p, 1; \frac{1+m+3n}{2n}; -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^2(1+m+n)}$$

Mathematica [F]

$$\int \frac{(fx)^m (a+cx^{2n})^p}{d+ex^n} dx = \int \frac{(fx)^m (a+cx^{2n})^p}{d+ex^n} dx$$

```
[In] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n),x]
[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n), x]
```

Maple [F]

$$\int \frac{(fx)^m (a+c\,x^{2n})^p}{d+e\,x^n} dx$$

```
[In] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x)
[Out] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x)
```

Fricas [F]

$$\int \frac{(fx)^m (a+cx^{2n})^p}{d+ex^n} dx = \int \frac{(cx^{2n}+a)^p (fx)^m}{ex^n+d} dx$$

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a+cx^{2n})^p}{d+ex^n} dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(fx)^m (a + cx^{2n})^p}{d + ex^n} dx = \int \frac{(cx^{2n} + a)^p (fx)^m}{ex^n + d} dx$$

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)
```

Giac [F]

$$\int \frac{(fx)^m (a + cx^{2n})^p}{d + ex^n} dx = \int \frac{(cx^{2n} + a)^p (fx)^m}{ex^n + d} dx$$

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a + cx^{2n})^p}{d + ex^n} dx = \int \frac{(a + c\,x^{2n})^p\,(f\,x)^m}{d + e\,x^n} dx$$

```
[In] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n),x)
[Out] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n), x)
```

3.91 $\int \frac{(fx)^m(a+cx^{2n})^p}{(d+ex^n)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 302

$$\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx = \frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\text{AppellF1}\left(\frac{1+m}{2n},-p,2,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(1+m)} - \frac{2ex^{1+n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\text{AppellF1}\left(\frac{1+m+n}{2n},-p,2,\frac{1+m+3n}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(1+m+n)} + \frac{e^2x^{1+2n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\text{AppellF1}\left(\frac{1+m+2n}{2n},-p,2,\frac{1+m+4n}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(1+m+2n)}$$

```
[Out] x*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m)/n,2,-p,1+1/2*(1+m)/n,e^2*x^(2*
n)/d^2,-c*x^(2*n)/a)/d^2/(1+m)/((1+c*x^(2*n)/a)^p)-2*e*x^(1+n)*(f*x)^m*(a+c
*x^(2*n))^p*AppellF1(1/2*(1+m+n)/n,2,-p,1/2*(1+m+3*n)/n,e^2*x^(2*n)/d^2,-c*
x^(2*n)/a)/d^3/(1+m+n)/((1+c*x^(2*n)/a)^p)+e^2*x^(1+2*n)*(f*x)^m*(a+c*x^(2*
n))^p*AppellF1(1/2*(1+m+2*n)/n,2,-p,1/2*(1+m+4*n)/n,e^2*x^(2*n)/d^2,-c*x^(2
*n)/a)/d^4/(1+m+2*n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1576, 525, 524}

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^2} dx$$
$$= \frac{x(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+1}{2n},-p,2,\frac{m+1}{2n}+1,-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(m+1)}$$
$$+\frac{e^2x^{2n+1}(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+2n+1}{2n},-p,2,\frac{m+4n+1}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(m+2n+1)}$$
$$-\frac{2ex^{n+1}(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+n+1}{2n},-p,2,\frac{m+3n+1}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(m+n+1)}$$

```
[In] Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 2, 1 + (1 + m)/(2*
n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + m)*(1 + (c*x^(2*n))/a)^
p) - (2*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -
p, 2, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 +
m + n)*(1 + (c*x^(2*n))/a)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*
AppellF1[(1 + m + 2*n)/(2*n), -p, 2, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a),
 (e^2*x^(2*n))/d^2])/(d^4*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1576

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(
```

```
2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/(d^2 - e^2*x^(2*n))))^(-q), x], x]
, x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] && !IntegerQ[p]
 && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(x^{-m}(fx)^m\right)\int\left(\frac{d^2x^m(a+cx^{2n})^p}{(d^2-e^2x^{2n})^2}-\frac{2dex^{m+n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^2}+\frac{e^2x^{m+2n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^2}\right)dx\\
&= \left(d^2x^{-m}(fx)^m\right)\int\frac{x^m(a+cx^{2n})^p}{(d^2-e^2x^{2n})^2}\,dx-\left(2dex^{-m}(fx)^m\right)\int\frac{x^{m+n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^2}\,dx\\
&\quad+\left(e^2x^{-m}(fx)^m\right)\int\frac{x^{m+2n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^2}\,dx\\
&= \left(d^2x^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{x^m\left(1+\frac{cx^{2n}}{a}\right)^p}{(d^2-e^2x^{2n})^2}\,dx\\
&\quad-\left(2dex^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{x^{m+n}\left(1+\frac{cx^{2n}}{a}\right)^p}{(-d^2+e^2x^{2n})^2}\,dx\\
&\quad+\left(e^2x^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{x^{m+2n}\left(1+\frac{cx^{2n}}{a}\right)^p}{(-d^2+e^2x^{2n})^2}\,dx\\
&= \frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1+m}{2n};-p,2;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(1+m)}\\
&\quad-\frac{2ex^{1+n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1+m+n}{2n};-p,2;\frac{1+m+3n}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(1+m+n)}\\
&\quad+\frac{e^2x^{1+2n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1+m+2n}{2n};-p,2;\frac{1+m+4n}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(1+m+2n)}
\end{aligned}$$

Mathematica [F]

$$\int\frac{(fx)^m\left(a+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx=\int\frac{(fx)^m\left(a+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$$

```
[In] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2,x]
[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

Maple [F]

$$\int \frac{(fx)^m (a+c\,x^{2n})^p}{(d+e\,x^n)^2}dx$$

```
[In] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x)
[Out] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(cx^{2n}+a)^p (fx)^m}{(ex^n+d)^2}\,dx$$

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(e^2*x^(2*n) + 2*d*e*x^n + d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(cx^{2n}+a)^p (fx)^m}{(ex^n+d)^2}\,dx$$

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

Giac [F]

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(cx^{2n}+a)^p (fx)^m}{(ex^n+d)^2}\,dx$$

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(a+c\,x^{2n})^p\,(f\,x)^m}{(d+e\,x^n)^2}\,dx$$

```
[In] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^2,x)
[Out] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^2, x)
```

3.92 $\int \frac{(fx)^m(a+cx^{2n})^p}{(d+ex^n)^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 412

$$\int \frac{(fx)^m(a+cx^{2n})^p}{(d+ex^n)^3} dx = \frac{x(fx)^m(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\text{AppellF1}\left(\frac{1+m}{2n},-p,3,1+\frac{1+m}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(1+m)} - \frac{3ex^{1+n}(fx)^m(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\text{AppellF1}\left(\frac{1+m+n}{2n},-p,3,\frac{1+m+3n}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(1+m+n)} + \frac{3e^2x^{1+2n}(fx)^m(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\text{AppellF1}\left(\frac{1+m+2n}{2n},-p,3,\frac{1+m+4n}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^5(1+m+2n)} - \frac{e^3x^{1+3n}(fx)^m(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\text{AppellF1}\left(\frac{1+m+3n}{2n},-p,3,\frac{1+m+5n}{2n},-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^6(1+m+3n)}$$

```
[Out] x*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m)/n,3,-p,1+1/2*(1+m)/n,e^2*x^(2*
n)/d^2,-c*x^(2*n)/a)/d^3/(1+m)/((1+c*x^(2*n)/a)^p)-3*e*x^(1+n)*(f*x)^m*(a+c
*x^(2*n))^p*AppellF1(1/2*(1+m+n)/n,3,-p,1/2*(1+m+3*n)/n,e^2*x^(2*n)/d^2,-c*
x^(2*n)/a)/d^4/(1+m+n)/((1+c*x^(2*n)/a)^p)+3*e^2*x^(1+2*n)*(f*x)^m*(a+c*x^(
2*n))^p*AppellF1(1/2*(1+m+2*n)/n,3,-p,1/2*(1+m+4*n)/n,e^2*x^(2*n)/d^2,-c*x^
(2*n)/a)/d^5/(1+m+2*n)/((1+c*x^(2*n)/a)^p)-e^3*x^(1+3*n)*(f*x)^m*(a+c*x^(2*
n))^p*AppellF1(1/2*(1+m+3*n)/n,3,-p,1/2*(1+m+5*n)/n,e^2*x^(2*n)/d^2,-c*x^(2
*n)/a)/d^6/(1+m+3*n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1576, 525, 524}

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^3} dx$$
$$= -\frac{e^3x^{3n+1}(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+3n+1}{2n}, -p, 3, \frac{m+5n+1}{2n}, -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^6(m+3n+1)}$$
$$+ \frac{3e^2x^{2n+1}(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+2n+1}{2n}, -p, 3, \frac{m+4n+1}{2n}, -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^5(m+2n+1)}$$
$$- \frac{3ex^{n+1}(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+n+1}{2n}, -p, 3, \frac{m+3n+1}{2n}, -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^4(m+n+1)}$$
$$+ \frac{x(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+1}{2n}, -p, 3, \frac{m+1}{2n}+1, -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^3(m+1)}$$

```
[In] Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 3, 1 + (1 + m)/(2*
n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + m)*(1 + (c*x^(2*n))/a)^
p) - (3*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -
p, 3, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 +
m + n)*(1 + (c*x^(2*n))/a)^p) + (3*e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^
p*AppellF1[(1 + m + 2*n)/(2*n), -p, 3, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a
), (e^2*x^(2*n))/d^2])/(d^5*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p) - (e^3*x^(
1 + 3*n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + 3*n)/(2*n), -p, 3, (1
+ m + 5*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^6*(1 + m + 3*n)*
(1 + (c*x^(2*n))/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1576

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(
2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/(d^2 - e^2*x^(2*n))))^(-q), x], x]
, x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !IntegerQ[p]
 && ILtQ[q, 0]
```

Rubi steps

$$\text{integral} = \left(x^{-m}(fx)^m\right)\int\left(\frac{d^3x^m(a+cx^{2n})^p}{(d^2-e^2x^{2n})^3}+\frac{3d^2ex^{m+n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}-\frac{3de^2x^{m+2n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}+\frac{e^3x^{m+3n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}\right)dx$$

$$= \left(d^3x^{-m}(fx)^m\right)\int\frac{x^m(a+cx^{2n})^p}{(d^2-e^2x^{2n})^3}\,dx+\left(3d^2ex^{-m}(fx)^m\right)\int\frac{x^{m+n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}\,dx - \left(3de^2x^{-m}(fx)^m\right)\int\frac{x^{m+2n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}\,dx+\left(e^3x^{-m}(fx)^m\right)\int\frac{x^{m+3n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3}\,dx$$

$$= \left(d^3x^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{x^m\left(1+\frac{cx^{2n}}{a}\right)^p}{(d^2-e^2x^{2n})^3}\,dx + \left(3d^2ex^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{x^{m+n}\left(1+\frac{cx^{2n}}{a}\right)^p}{(-d^2+e^2x^{2n})^3}\,dx - \left(3de^2x^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{x^{m+2n}\left(1+\frac{cx^{2n}}{a}\right)^p}{(-d^2+e^2x^{2n})^3}\,dx + \left(e^3x^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{x^{m+3n}\left(1+\frac{cx^{2n}}{a}\right)^p}{(-d^2+e^2x^{2n})^3}\,dx$$

$$= \frac{x(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} F_1\left(\frac{1+m}{2n}; -p, 3; 1+\frac{1+m}{2n}; -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^3(1+m)}$$
$$- \frac{3ex^{1+n}(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} F_1\left(\frac{1+m+n}{2n}; -p, 3; \frac{1+m+3n}{2n}; -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^4(1+m+n)}$$
$$+ \frac{3e^2x^{1+2n}(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} F_1\left(\frac{1+m+2n}{2n}; -p, 3; \frac{1+m+4n}{2n}; -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^5(1+m+2n)}$$
$$- \frac{e^3x^{1+3n}(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} F_1\left(\frac{1+m+3n}{2n}; -p, 3; \frac{1+m+5n}{2n}; -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^6(1+m+3n)}$$

Mathematica [F]

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^3} dx = \int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^3} dx$$

```
[In] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3,x]
[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3, x]
```

Maple [F]

$$\int \frac{(fx)^m (a+c\,x^{2n})^p}{(d+e\,x^n)^3} dx$$

```
[In] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x)
[Out] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^3} dx = \int \frac{(cx^{2n}+a)^p (fx)^m}{(ex^n+d)^3} dx$$

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e
*x^n + d^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^3}\, dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^3}\, dx = \int \frac{(cx^{2n} + a)^p (fx)^m}{(ex^n + d)^3}\, dx$$

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^3, x)
```

Giac [F]

$$\int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^3}\, dx = \int \frac{(cx^{2n} + a)^p (fx)^m}{(ex^n + d)^3}\, dx$$

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^3}\, dx = \int \frac{(a + c\,x^{2\,n})^p\,(f\,x)^m}{(d + e\,x^n)^3}\, dx$$

```
[In] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^3,x)
[Out] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^3, x)
```

3.93 $\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 16

$$\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(a+bx+cx^2\right)^{14}$$

```
[Out] 1/14*(c*x^2+b*x+a)^14
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {643}

$$\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(a+bx+cx^2\right)^{14}$$

```
[In] Int[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]
```

```
[Out] (a + b*x + c*x^2)^14/14
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{14}\left(a+bx+cx^2\right)^{14}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(16) = 32.

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 12.56

$$\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx=\frac{1}{14}x(b+cx)\left(14a^{13}+91a^{12}x(b+cx)+364a^{11}x^2(b+cx)^2+1001a^{10}x^3(b+cx)^3+2002a^9x^4(b+cx)^4+3003a^8x^5(b+cx)^5+3432a^7x^6(b+cx)^6+3003a^6x^7(b+cx)^7+2002a^5x^8(b+cx)^8+1001a^4x^9(b+cx)^9+364a^3x^{10}(b+cx)^{10}+91a^2x^{11}(b+cx)^{11}+14ax^{12}(b+cx)^{12}+x^{13}(b+cx)^{13}\right)$$

```
[In] Integrate[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]
```

```
[Out] (x*(b + c*x)*(14*a^13 + 91*a^12*x*(b + c*x) + 364*a^11*x^2*(b + c*x)^2 + 10
01*a^10*x^3*(b + c*x)^3 + 2002*a^9*x^4*(b + c*x)^4 + 3003*a^8*x^5*(b + c*x)
^5 + 3432*a^7*x^6*(b + c*x)^6 + 3003*a^6*x^7*(b + c*x)^7 + 2002*a^5*x^8*(b
+ c*x)^8 + 1001*a^4*x^9*(b + c*x)^9 + 364*a^3*x^10*(b + c*x)^10 + 91*a^2*x^
11*(b + c*x)^11 + 14*a*x^12*(b + c*x)^12 + x^13*(b + c*x)^13))/14
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\left(c\,x^{2}+b x+a\right)^{14}}{14}$	15
norman	Expression too large to display	1224
gosper	Expression too large to display	1447
parallelrisch	Expression too large to display	1447
risch	Expression too large to display	1452

```
[In] int((2*c*x+b)*(c*x^2+b*x+a)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*(c*x^2+b*x+a)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1234 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 1234, normalized size of antiderivative = 77.12

$$\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="fricas")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 1/2*(13*b^2*c^12 + 2*a*c^13)*x^26 + 13*(2*b^
3*c^11 + a*b*c^12)*x^25 + 13/2*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^12)*x^2
4 + 13*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^23 + 13/2*(33*b^6*c^8
+ 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^22 + 143/7*(12*b^7*c^7 +
63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^21 + 143/2*(3*b^8*c^6 + 24
*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^20 + 143*(b^9*c^
5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^19 + 14
3/2*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7 + 45*a^4*b^
2*c^8 + 2*a^5*c^9)*x^18 + 13*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*a^2*b^7*c^5 +
 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^17 + 13/2*(b^12*c^2 +
44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 + 792
*a^5*b^2*c^7 + 33*a^6*c^8)*x^16 + (b^13*c + 78*a*b^11*c^2 + 1430*a^2*b^9*c^
3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c^6 + 1716*a^6*b*c
^7)*x^15 + a^13*b*x + 1/14*(b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 + 60060
*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6
+ 3432*a^7*c^7)*x^14 + (a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2 + 8580*a^
4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^13 +
13/2*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c^3 + 2310*
a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^12 + 13*(2*a^3*b^11 + 55*a^4*
b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 99*a^8*b*c^5)
*x^11 + 143/2*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a^7*b^4*c^3 +
 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^10 + 143*(a^5*b^9 + 12*a^6*b^7*c + 36*a^7*b^
5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^9 + 143/2*(3*a^6*b^8 + 24*a^7*b^6*c
 + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^8 + 143/7*(12*a^7*b^7 + 63
*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^7 + 13/2*(33*a^8*b^6 + 110*a
^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)*x^6 + 13*(11*a^9*b^5 + 22*a^10*b^3
*c + 6*a^11*b*c^2)*x^5 + 13/2*(11*a^10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^4
+ 13*(2*a^11*b^3 + a^12*b*c)*x^3 + 1/2*(13*a^12*b^2 + 2*a^13*c)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1326 vs. 2(12) = 24.

Time = 0.16 (sec) , antiderivative size = 1326, normalized size of antiderivative = 82.88

$$\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate((2*c*x+b)*(c*x**2+b*x+a)**13,x)
```

```
[Out] a**13*b*x + b*c**13*x**27 + c**14*x**28/14 + x**26*(a*c**13 + 13*b**2*c**12
/2) + x**25*(13*a*b*c**12 + 26*b**3*c**11) + x**24*(13*a**2*c**12/2 + 78*a*
b**2*c**11 + 143*b**4*c**10/2) + x**23*(78*a**2*b*c**11 + 286*a*b**3*c**10
+ 143*b**5*c**9) + x**22*(26*a**3*c**11 + 429*a**2*b**2*c**10 + 715*a*b**4*
c**9 + 429*b**6*c**8/2) + x**21*(286*a**3*b*c**10 + 1430*a**2*b**3*c**9 + 1
287*a*b**5*c**8 + 1716*b**7*c**7/7) + x**20*(143*a**4*c**10/2 + 1430*a**3*b
**2*c**9 + 6435*a**2*b**4*c**8/2 + 1716*a*b**6*c**7 + 429*b**8*c**6/2) + x*
*19*(715*a**4*b*c**9 + 4290*a**3*b**3*c**8 + 5148*a**2*b**5*c**7 + 1716*a*b
**7*c**6 + 143*b**9*c**5) + x**18*(143*a**5*c**9 + 6435*a**4*b**2*c**8/2 +
8580*a**3*b**4*c**7 + 6006*a**2*b**6*c**6 + 1287*a*b**8*c**5 + 143*b**10*c*
*4/2) + x**17*(1287*a**5*b*c**8 + 8580*a**4*b**3*c**7 + 12012*a**3*b**5*c**
6 + 5148*a**2*b**7*c**5 + 715*a*b**9*c**4 + 26*b**11*c**3) + x**16*(429*a**
6*c**8/2 + 5148*a**5*b**2*c**7 + 15015*a**4*b**4*c**6 + 12012*a**3*b**6*c**
5 + 6435*a**2*b**8*c**4/2 + 286*a*b**10*c**3 + 13*b**12*c**2/2) + x**15*(17
16*a**6*b*c**7 + 12012*a**5*b**3*c**6 + 18018*a**4*b**5*c**5 + 8580*a**3*b*
*7*c**4 + 1430*a**2*b**9*c**3 + 78*a*b**11*c**2 + b**13*c) + x**14*(1716*a*
*7*c**7/7 + 6006*a**6*b**2*c**6 + 18018*a**5*b**4*c**5 + 15015*a**4*b**6*c*
*4 + 4290*a**3*b**8*c**3 + 429*a**2*b**10*c**2 + 13*a*b**12*c + b**14/14) +
 x**13*(1716*a**7*b*c**6 + 12012*a**6*b**3*c**5 + 18018*a**5*b**5*c**4 + 85
80*a**4*b**7*c**3 + 1430*a**3*b**9*c**2 + 78*a**2*b**11*c + a*b**13) + x**1
2*(429*a**8*c**6/2 + 5148*a**7*b**2*c**5 + 15015*a**6*b**4*c**4 + 12012*a**
5*b**6*c**3 + 6435*a**4*b**8*c**2/2 + 286*a**3*b**10*c + 13*a**2*b**12/2) +
 x**11*(1287*a**8*b*c**5 + 8580*a**7*b**3*c**4 + 12012*a**6*b**5*c**3 + 514
8*a**5*b**7*c**2 + 715*a**4*b**9*c + 26*a**3*b**11) + x**10*(143*a**9*c**5
+ 6435*a**8*b**2*c**4/2 + 8580*a**7*b**4*c**3 + 6006*a**6*b**6*c**2 + 1287*
a**5*b**8*c + 143*a**4*b**10/2) + x**9*(715*a**9*b*c**4 + 4290*a**8*b**3*c*
*3 + 5148*a**7*b**5*c**2 + 1716*a**6*b**7*c + 143*a**5*b**9) + x**8*(143*a*
*10*c**4/2 + 1430*a**9*b**2*c**3 + 6435*a**8*b**4*c**2/2 + 1716*a**7*b**6*c
 + 429*a**6*b**8/2) + x**7*(286*a**10*b*c**3 + 1430*a**9*b**3*c**2 + 1287*a
**8*b**5*c + 1716*a**7*b**7/7) + x**6*(26*a**11*c**3 + 429*a**10*b**2*c**2
+ 715*a**9*b**4*c + 429*a**8*b**6/2) + x**5*(78*a**11*b*c**2 + 286*a**10*b*
*3*c + 143*a**9*b**5) + x**4*(13*a**12*c**2/2 + 78*a**11*b**2*c + 143*a**10
*b**4/2) + x**3*(13*a**12*b*c + 26*a**11*b**3) + x**2*(a**13*c + 13*a**12*b
**2/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(cx^2+bx+a\right)^{14}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x + a)^14
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(14) = 28.

Time = 0.32 (sec) , antiderivative size = 216, normalized size of antiderivative = 13.50

$$\begin{aligned}\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx=&\frac{1}{14}\left(cx^2+bx\right)^{14}+\left(cx^2+bx\right)^{13}a+\frac{13}{2}\left(cx^2+bx\right)^{12}a^2\\&+26\left(cx^2+bx\right)^{11}a^3+\frac{143}{2}\left(cx^2+bx\right)^{10}a^4\\&+143\left(cx^2+bx\right)^{9}a^5+\frac{429}{2}\left(cx^2+bx\right)^{8}a^6\\&+\frac{1716}{7}\left(cx^2+bx\right)^{7}a^7+\frac{429}{2}\left(cx^2+bx\right)^{6}a^8\\&+143\left(cx^2+bx\right)^{5}a^9+\frac{143}{2}\left(cx^2+bx\right)^{4}a^{10}\\&+26\left(cx^2+bx\right)^{3}a^{11}+\frac{13}{2}\left(cx^2+bx\right)^{2}a^{12}+\left(cx^2+bx\right)a^{13}\end{aligned}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="giac")
```

```
[Out] 1/14*(c*x^2 + b*x)^14 + (c*x^2 + b*x)^13*a + 13/2*(c*x^2 + b*x)^12*a^2 + 26
*(c*x^2 + b*x)^11*a^3 + 143/2*(c*x^2 + b*x)^10*a^4 + 143*(c*x^2 + b*x)^9*a^
5 + 429/2*(c*x^2 + b*x)^8*a^6 + 1716/7*(c*x^2 + b*x)^7*a^7 + 429/2*(c*x^2 +
 b*x)^6*a^8 + 143*(c*x^2 + b*x)^5*a^9 + 143/2*(c*x^2 + b*x)^4*a^10 + 26*(c*
x^2 + b*x)^3*a^11 + 13/2*(c*x^2 + b*x)^2*a^12 + (c*x^2 + b*x)*a^13
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 1203, normalized size of antiderivative = 75.19

$$\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx = x^{12}\left(\frac{429\,a^8\,c^6}{2}+5148\,a^7\,b^2\,c^5+15015\,a^6\,b^4\,c^4+12012\,a^5\,b^6\,c^3+\frac{6435\,a^4\,b^8\,c^2}{2}+286\,a^3\,b^{10}\,c+\frac{13\,a^2\,b^{12}}{2}\right)+x^{16}\left(\frac{429\,a^6\,c^8}{2}+5148\,a^5\,b^2\,c^7+15015\,a^4\,b^4\,c^6+12012\,a^3\,b^6\,c^5+\frac{6435\,a^2\,b^8\,c^4}{2}+286\,a\,b^{10}\,c^3+\frac{13\,b^{12}\,c^2}{2}\right)+x^{13}\left(1716\,a^7\,b\,c^6+12012\,a^6\,b^3\,c^5+18018\,a^5\,b^5\,c^4+8580\,a^4\,b^7\,c^3+1430\,a^3\,b^9\,c^2+78\,a^2\,b^{11}\,c+a\,b^{13}\right)+x^{15}\left(1716\,a^6\,b\,c^7+12012\,a^5\,b^3\,c^6+18018\,a^4\,b^5\,c^5+8580\,a^3\,b^7\,c^4+1430\,a^2\,b^9\,c^3+78\,a\,b^{11}\,c^2+b^{13}\,c\right)+x^6\left(26\,a^{11}\,c^3+429\,a^{10}\,b^2\,c^2+715\,a^9\,b^4\,c+\frac{429\,a^8\,b^6}{2}\right)+x^{22}\left(26\,a^3\,c^{11}+429\,a^2\,b^2\,c^{10}+715\,a\,b^4\,c^9+\frac{429\,b^6\,c^8}{2}\right)+x^{10}\left(143\,a^9\,c^5+\frac{6435\,a^8\,b^2\,c^4}{2}+8580\,a^7\,b^4\,c^3+6006\,a^6\,b^6\,c^2+1287\,a^5\,b^8\,c+\frac{143\,a^4\,b^{10}}{2}\right)+x^{18}\left(143\,a^5\,c^9+\frac{6435\,a^4\,b^2\,c^8}{2}+8580\,a^3\,b^4\,c^7+6006\,a^2\,b^6\,c^6+1287\,a\,b^8\,c^5+\frac{143\,b^{10}\,c^4}{2}\right)+x^{14}\left(\frac{1716\,a^7\,c^7}{7}+6006\,a^6\,b^2\,c^6+18018\,a^5\,b^4\,c^5+15015\,a^4\,b^6\,c^4+4290\,a^3\,b^8\,c^3+429\,a^2\,b^{10}\,c^2+13\,a\,b^{12}\,c+\frac{b^{14}}{14}\right)+x^8\left(\frac{143\,a^{10}\,c^4}{2}+1430\,a^9\,b^2\,c^3+\frac{6435\,a^8\,b^4\,c^2}{2}+1716\,a^7\,b^6\,c+\frac{429\,a^6\,b^8}{2}\right)+x^{20}\left(\frac{143\,a^4\,c^{10}}{2}+1430\,a^3\,b^2\,c^9+\frac{6435\,a^2\,b^4\,c^8}{2}+1716\,a\,b^6\,c^7+\frac{429\,b^8\,c^6}{2}\right)+\frac{c^{14}\,x^{28}}{14}+x^2\left(c\,a^{13}+\frac{13\,a^{12}\,b^2}{2}\right)+\frac{13\,a^{10}\,x^4\left(a^2\,c^2+12\,a\,b^2\,c+11\,b^4\right)}{2}+\frac{13\,c^{10}\,x^{24}\left(a^2\,c^2+12\,a\,b^2\,c+11\,b^4\right)}{2}+b\,c^{13}\,x^{27}+\frac{c^{12}\,x^{26}\left(13\,b^2+2\,a\,c\right)}{2}+a^{13}\,b\,x+\frac{143\,a^7\,b\,x^7\left(14\,a^3\,c^3+70\,a^2\,b^2\,c^2+63\,a\,b^4\,c+12\,b^6\right)}{7}$$

```
[In] int((b + 2*c*x)*(a + b*x + c*x^2)^13,x)

[Out] x^12*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 + 286*a^3*b^10*c + (6435*a^4*b^8*c^
2)/2 + 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 + 5148*a^7*b^2*c^5) + x^16*((4
29*a^6*c^8)/2 + (13*b^12*c^2)/2 + 286*a*b^10*c^3 + (6435*a^2*b^8*c^4)/2 + 1
2012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 + 5148*a^5*b^2*c^7) + x^13*(a*b^13 + 7
8*a^2*b^11*c + 1716*a^7*b*c^6 + 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018
*a^5*b^5*c^4 + 12012*a^6*b^3*c^5) + x^15*(b^13*c + 78*a*b^11*c^2 + 1716*a^6
*b*c^7 + 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^
5*b^3*c^6) + x^6*((429*a^8*b^6)/2 + 26*a^11*c^3 + 715*a^9*b^4*c + 429*a^10*
b^2*c^2) + x^22*(26*a^3*c^11 + (429*b^6*c^8)/2 + 715*a*b^4*c^9 + 429*a^2*b^
2*c^10) + x^10*((143*a^4*b^10)/2 + 143*a^9*c^5 + 1287*a^5*b^8*c + 6006*a^6*
b^6*c^2 + 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2) + x^18*(143*a^5*c^9 + (1
43*b^10*c^4)/2 + 1287*a*b^8*c^5 + 6006*a^2*b^6*c^6 + 8580*a^3*b^4*c^7 + (64
35*a^4*b^2*c^8)/2) + x^14*(b^14/14 + (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 +
4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^4 + 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6
 + 13*a*b^12*c) + x^8*((429*a^6*b^8)/2 + (143*a^10*c^4)/2 + 1716*a^7*b^6*c
+ (6435*a^8*b^4*c^2)/2 + 1430*a^9*b^2*c^3) + x^20*((143*a^4*c^10)/2 + (429*
b^8*c^6)/2 + 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 + 1430*a^3*b^2*c^9) + (c
^14*x^28)/14 + x^2*(a^13*c + (13*a^12*b^2)/2) + (13*a^10*x^4*(11*b^4 + a^2*
c^2 + 12*a*b^2*c))/2 + (13*c^10*x^24*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/2 + b
*c^13*x^27 + (c^12*x^26*(2*a*c + 13*b^2))/2 + a^13*b*x + (143*a^7*b*x^7*(12
*b^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/7 + (143*b*c^7*x^21*(12*b
^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/7 + 143*a^5*b*x^9*(b^8 + 5*
a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c) + 143*b*c^5*x^19*(b
^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c) + 13*a^3*b*x
^11*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*
c^4 + 55*a*b^8*c) + 13*b*c^3*x^17*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 +
924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c) + 13*a^9*b*x^5*(11*b^4 + 6*
a^2*c^2 + 22*a*b^2*c) + 13*b*c^9*x^23*(11*b^4 + 6*a^2*c^2 + 22*a*b^2*c) + 1
3*a^11*b*x^3*(a*c + 2*b^2) + 13*b*c^11*x^25*(a*c + 2*b^2)
```

3.94 $\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^{13}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 18

$$\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{13}\,dx=\frac{1}{28}\left(a+bx^2+cx^4\right)^{14}$$

```
[Out] 1/28*(c*x^4+b*x^2+a)^14
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1261, 643}

$$\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{13}\,dx=\frac{1}{28}\left(a+bx^2+cx^4\right)^{14}$$

```
[In] Int[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (a + b*x^2 + c*x^4)^14/28
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx,x,x^2\right)\\
&= \frac{1}{28}\left(a+bx^2+cx^4\right)^{14}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(18) = 36.

Time = 0.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 12.94

$$
\begin{aligned}
\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{13}\,dx = \frac{1}{28}x^2\left(b+cx^2\right)\Big(14a^{13}+91a^{12}x^2\left(b+cx^2\right)\\
+364a^{11}x^4\left(b+cx^2\right)^2+1001a^{10}x^6\left(b+cx^2\right)^3\\
+2002a^9x^8\left(b+cx^2\right)^4+3003a^8x^{10}\left(b+cx^2\right)^5\\
+3432a^7x^{12}\left(b+cx^2\right)^6+3003a^6x^{14}\left(b+cx^2\right)^7\\
+2002a^5x^{16}\left(b+cx^2\right)^8+1001a^4x^{18}\left(b+cx^2\right)^9\\
+364a^3x^{20}\left(b+cx^2\right)^{10}+91a^2x^{22}\left(b+cx^2\right)^{11}\\
+14ax^{24}\left(b+cx^2\right)^{12}+x^{26}\left(b+cx^2\right)^{13}\Big)
\end{aligned}
$$

```
[In] Integrate[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (x^2*(b + c*x^2)*(14*a^13 + 91*a^12*x^2*(b + c*x^2) + 364*a^11*x^4*(b + c*x
^2)^2 + 1001*a^10*x^6*(b + c*x^2)^3 + 2002*a^9*x^8*(b + c*x^2)^4 + 3003*a^8
*x^10*(b + c*x^2)^5 + 3432*a^7*x^12*(b + c*x^2)^6 + 3003*a^6*x^14*(b + c*x^
2)^7 + 2002*a^5*x^16*(b + c*x^2)^8 + 1001*a^4*x^18*(b + c*x^2)^9 + 364*a^3*
x^20*(b + c*x^2)^10 + 91*a^2*x^22*(b + c*x^2)^11 + 14*a*x^24*(b + c*x^2)^12
 + x^26*(b + c*x^2)^13))/28
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{14}}{28}$	17
gosper	Expression too large to display	1455
parallelrisch	Expression too large to display	1455
risch	Expression too large to display	1460

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*(c*x^4+b*x^2+a)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1240 vs. 2(16) = 32.

Time = 0.26 (sec) , antiderivative size = 1240, normalized size of antiderivative = 68.89

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^{13}\,dx=\text{Too large to display}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="fricas")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 + 2*a*c^13)*x^52 + 13/2
*(2*b^3*c^11 + a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^1
2)*x^48 + 13/2*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(33*
b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^44 + 143/14*(12*b
^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^42 + 143/4*(3*b^8
*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^40 + 14
3/2*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^38 + 143/4*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^34 + 13/4
*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c + 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 +
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 + 3432*a^7*c^7)*x^28 + 1/2*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^26 + 13/4*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^24 + 13/2*(2*a^3*b^
11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 9
9*a^8*b*c^5)*x^22 + 143/4*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^20 + 143/2*(a^5*b^9 + 12*a^6*b^7
*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^18 + 143/4*(3*a^6*b^8
 + 24*a^7*b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^16 + 1/2*a^
13*b*x^2 + 143/14*(12*a^7*b^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c
^3)*x^14 + 13/4*(33*a^8*b^6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)
*x^12 + 13/2*(11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^10 + 13/4*(11*a^
10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^8 + 13/2*(2*a^11*b^3 + a^12*b*c)*x^6 +
 1/4*(13*a^12*b^2 + 2*a^13*c)*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1384 vs. $2(14) = 28$.

Time = 0.14 (sec) , antiderivative size = 1384, normalized size of antiderivative = 76.89

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**13,x)
```

```
[Out] a**13*b*x**2/2 + b*c**13*x**54/2 + c**14*x**56/28 + x**52*(a*c**13/2 + 13*b
**2*c**12/4) + x**50*(13*a*b*c**12/2 + 13*b**3*c**11) + x**48*(13*a**2*c**1
2/4 + 39*a*b**2*c**11 + 143*b**4*c**10/4) + x**46*(39*a**2*b*c**11 + 143*a*
b**3*c**10 + 143*b**5*c**9/2) + x**44*(13*a**3*c**11 + 429*a**2*b**2*c**10/
2 + 715*a*b**4*c**9/2 + 429*b**6*c**8/4) + x**42*(143*a**3*b*c**10 + 715*a*
*2*b**3*c**9 + 1287*a*b**5*c**8/2 + 858*b**7*c**7/7) + x**40*(143*a**4*c**1
0/4 + 715*a**3*b**2*c**9 + 6435*a**2*b**4*c**8/4 + 858*a*b**6*c**7 + 429*b*
*8*c**6/4) + x**38*(715*a**4*b*c**9/2 + 2145*a**3*b**3*c**8 + 2574*a**2*b**
5*c**7 + 858*a*b**7*c**6 + 143*b**9*c**5/2) + x**36*(143*a**5*c**9/2 + 6435
*a**4*b**2*c**8/4 + 4290*a**3*b**4*c**7 + 3003*a**2*b**6*c**6 + 1287*a*b**8
*c**5/2 + 143*b**10*c**4/4) + x**34*(1287*a**5*b*c**8/2 + 4290*a**4*b**3*c*
*7 + 6006*a**3*b**5*c**6 + 2574*a**2*b**7*c**5 + 715*a*b**9*c**4/2 + 13*b**
11*c**3) + x**32*(429*a**6*c**8/4 + 2574*a**5*b**2*c**7 + 15015*a**4*b**4*c
**6/2 + 6006*a**3*b**6*c**5 + 6435*a**2*b**8*c**4/4 + 143*a*b**10*c**3 + 13
*b**12*c**2/4) + x**30*(858*a**6*b*c**7 + 6006*a**5*b**3*c**6 + 9009*a**4*b
**5*c**5 + 4290*a**3*b**7*c**4 + 715*a**2*b**9*c**3 + 39*a*b**11*c**2 + b**
13*c/2) + x**28*(858*a**7*c**7/7 + 3003*a**6*b**2*c**6 + 9009*a**5*b**4*c**
5 + 15015*a**4*b**6*c**4/2 + 2145*a**3*b**8*c**3 + 429*a**2*b**10*c**2/2 +
13*a*b**12*c/2 + b**14/28) + x**26*(858*a**7*b*c**6 + 6006*a**6*b**3*c**5 +
 9009*a**5*b**5*c**4 + 4290*a**4*b**7*c**3 + 715*a**3*b**9*c**2 + 39*a**2*b
**11*c + a*b**13/2) + x**24*(429*a**8*c**6/4 + 2574*a**7*b**2*c**5 + 15015*
a**6*b**4*c**4/2 + 6006*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/4 + 143*a**3*b
**10*c + 13*a**2*b**12/4) + x**22*(1287*a**8*b*c**5/2 + 4290*a**7*b**3*c**4
 + 6006*a**6*b**5*c**3 + 2574*a**5*b**7*c**2 + 715*a**4*b**9*c/2 + 13*a**3*
b**11) + x**20*(143*a**9*c**5/2 + 6435*a**8*b**2*c**4/4 + 4290*a**7*b**4*c*
*3 + 3003*a**6*b**6*c**2 + 1287*a**5*b**8*c/2 + 143*a**4*b**10/4) + x**18*(
715*a**9*b*c**4/2 + 2145*a**8*b**3*c**3 + 2574*a**7*b**5*c**2 + 858*a**6*b*
*7*c + 143*a**5*b**9/2) + x**16*(143*a**10*c**4/4 + 715*a**9*b**2*c**3 + 64
35*a**8*b**4*c**2/4 + 858*a**7*b**6*c + 429*a**6*b**8/4) + x**14*(143*a**10
*b*c**3 + 715*a**9*b**3*c**2 + 1287*a**8*b**5*c/2 + 858*a**7*b**7/7) + x**1
2*(13*a**11*c**3 + 429*a**10*b**2*c**2/2 + 715*a**9*b**4*c/2 + 429*a**8*b**
6/4) + x**10*(39*a**11*b*c**2 + 143*a**10*b**3*c + 143*a**9*b**5/2) + x**8*
(13*a**12*c**2/4 + 39*a**11*b**2*c + 143*a**10*b**4/4) + x**6*(13*a**12*b*c
/2 + 13*a**11*b**3) + x**4*(a**13*c/2 + 13*a**12*b**2/4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1240 vs. 2(16) = 32.

Time = 0.21 (sec) , antiderivative size = 1240, normalized size of antiderivative = 68.89

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 + 2*a*c^13)*x^52 + 13/2
*(2*b^3*c^11 + a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^1
2)*x^48 + 13/2*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(33*
b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^44 + 143/14*(12*b
^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^42 + 143/4*(3*b^8
*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^40 + 14
3/2*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^38 + 143/4*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7
 + 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^34 + 13/4
*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c + 78*a*b^11*c^2
 + 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 +
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 + 3432*a^7*c^7)*x^28 + 1/2*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^26 + 13/4*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^24 + 13/2*(2*a^3*b^
11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 9
9*a^8*b*c^5)*x^22 + 143/4*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^20 + 143/2*(a^5*b^9 + 12*a^6*b^7
*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^18 + 143/4*(3*a^6*b^8
 + 24*a^7*b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^16 + 1/2*a^
13*b*x^2 + 143/14*(12*a^7*b^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c
^3)*x^14 + 13/4*(33*a^8*b^6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)
*x^12 + 13/2*(11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^10 + 13/4*(11*a^
10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^8 + 13/2*(2*a^11*b^3 + a^12*b*c)*x^6 +
 1/4*(13*a^12*b^2 + 2*a^13*c)*x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(16) = 32.

Time = 0.32 (sec) , antiderivative size = 246, normalized size of antiderivative = 13.67

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^{13}\,dx = \frac{1}{28}\left(cx^4+bx^2\right)^{14}+\frac{1}{2}\left(cx^4+bx^2\right)^{13}a+\frac{13}{4}\left(cx^4+bx^2\right)^{12}a^2+13\left(cx^4+bx^2\right)^{11}a^3+\frac{143}{4}\left(cx^4+bx^2\right)^{10}a^4+\frac{143}{2}\left(cx^4+bx^2\right)^{9}a^5+\frac{429}{4}\left(cx^4+bx^2\right)^{8}a^6+\frac{858}{7}\left(cx^4+bx^2\right)^{7}a^7+\frac{429}{4}\left(cx^4+bx^2\right)^{6}a^8+\frac{143}{2}\left(cx^4+bx^2\right)^{5}a^9+\frac{143}{4}\left(cx^4+bx^2\right)^{4}a^{10}+13\left(cx^4+bx^2\right)^{3}a^{11}+\frac{13}{4}\left(cx^4+bx^2\right)^{2}a^{12}+\frac{1}{2}\left(cx^4+bx^2\right)a^{13}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="giac")
```

```
[Out] 1/28*(c*x^4 + b*x^2)^14 + 1/2*(c*x^4 + b*x^2)^13*a + 13/4*(c*x^4 + b*x^2)^1
2*a^2 + 13*(c*x^4 + b*x^2)^11*a^3 + 143/4*(c*x^4 + b*x^2)^10*a^4 + 143/2*(c
*x^4 + b*x^2)^9*a^5 + 429/4*(c*x^4 + b*x^2)^8*a^6 + 858/7*(c*x^4 + b*x^2)^7
*a^7 + 429/4*(c*x^4 + b*x^2)^6*a^8 + 143/2*(c*x^4 + b*x^2)^5*a^9 + 143/4*(c
*x^4 + b*x^2)^4*a^10 + 13*(c*x^4 + b*x^2)^3*a^11 + 13/4*(c*x^4 + b*x^2)^2*a
^12 + 1/2*(c*x^4 + b*x^2)*a^13
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 1210, normalized size of antiderivative = 67.22

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^{13}\,dx = \text{Too large to display}$$

```
[In] int(x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x)
```

```
[Out] x^24*((13*a^2*b^12)/4 + (429*a^8*c^6)/4 + 143*a^3*b^10*c + (6435*a^4*b^8*c^
2)/4 + 6006*a^5*b^6*c^3 + (15015*a^6*b^4*c^4)/2 + 2574*a^7*b^2*c^5) + x^32*
((429*a^6*c^8)/4 + (13*b^12*c^2)/4 + 143*a*b^10*c^3 + (6435*a^2*b^8*c^4)/4
+ 6006*a^3*b^6*c^5 + (15015*a^4*b^4*c^6)/2 + 2574*a^5*b^2*c^7) + x^26*((a*b
^13)/2 + 39*a^2*b^11*c + 858*a^7*b*c^6 + 715*a^3*b^9*c^2 + 4290*a^4*b^7*c^3
 + 9009*a^5*b^5*c^4 + 6006*a^6*b^3*c^5) + x^30*((b^13*c)/2 + 39*a*b^11*c^2
+ 858*a^6*b*c^7 + 715*a^2*b^9*c^3 + 4290*a^3*b^7*c^4 + 9009*a^4*b^5*c^5 + 6
006*a^5*b^3*c^6) + x^12*((429*a^8*b^6)/4 + 13*a^11*c^3 + (715*a^9*b^4*c)/2
```

```
+ (429*a^10*b^2*c^2)/2) + x^44*(13*a^3*c^11 + (429*b^6*c^8)/4 + (715*a*b^4*
c^9)/2 + (429*a^2*b^2*c^10)/2) + x^20*((143*a^4*b^10)/4 + (143*a^9*c^5)/2 +
 (1287*a^5*b^8*c)/2 + 3003*a^6*b^6*c^2 + 4290*a^7*b^4*c^3 + (6435*a^8*b^2*c
^4)/4) + x^36*((143*a^5*c^9)/2 + (143*b^10*c^4)/4 + (1287*a*b^8*c^5)/2 + 30
03*a^2*b^6*c^6 + 4290*a^3*b^4*c^7 + (6435*a^4*b^2*c^8)/4) + x^28*(b^14/28 +
 (858*a^7*c^7)/7 + (429*a^2*b^10*c^2)/2 + 2145*a^3*b^8*c^3 + (15015*a^4*b^6
*c^4)/2 + 9009*a^5*b^4*c^5 + 3003*a^6*b^2*c^6 + (13*a*b^12*c)/2) + x^16*((4
29*a^6*b^8)/4 + (143*a^10*c^4)/4 + 858*a^7*b^6*c + (6435*a^8*b^4*c^2)/4 + 7
15*a^9*b^2*c^3) + x^40*((143*a^4*c^10)/4 + (429*b^8*c^6)/4 + 858*a*b^6*c^7
+ (6435*a^2*b^4*c^8)/4 + 715*a^3*b^2*c^9) + (c^14*x^56)/28 + x^4*((a^13*c)/
2 + (13*a^12*b^2)/4) + (13*a^10*x^8*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/4 + (1
3*c^10*x^48*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/4 + (a^13*b*x^2)/2 + (b*c^13*x
^54)/2 + (c^12*x^52*(2*a*c + 13*b^2))/4 + (143*a^7*b*x^14*(12*b^6 + 14*a^3*
c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/14 + (143*b*c^7*x^42*(12*b^6 + 14*a^3*c
^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/14 + (143*a^5*b*x^18*(b^8 + 5*a^4*c^4 +
36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/2 + (143*b*c^5*x^38*(b^8 + 5
*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/2 + (13*a^3*b*x^2
2*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 + 55*a*b^8*c))/2 + (13*b*c^3*x^34*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2
+ 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/2 + (13*a^9*b*x^10*(11*b
^4 + 6*a^2*c^2 + 22*a*b^2*c))/2 + (13*b*c^9*x^46*(11*b^4 + 6*a^2*c^2 + 22*a
*b^2*c))/2 + (13*a^11*b*x^6*(a*c + 2*b^2))/2 + (13*b*c^11*x^50*(a*c + 2*b^2
))/2
```

3.95 $\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^{13}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 18

$$\int x^2\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{13}\,dx=\frac{1}{42}\left(a+bx^3+cx^6\right)^{14}$$

```
[Out] 1/42*(c*x^6+b*x^3+a)^14
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1482, 643}

$$\int x^2\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{13}\,dx=\frac{1}{42}\left(a+bx^3+cx^6\right)^{14}$$

```
[In] Int[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (a + b*x^3 + c*x^6)^14/42
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
```

```
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx,x,x^3\right)$$
$$= \frac{1}{42}\left(a+bx^3+cx^6\right)^{14}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. $2(18) = 36$.

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 12.94

$$\int x^2\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{13}\,dx = \frac{1}{42}x^3\left(b+cx^3\right)\left(14a^{13}+91a^{12}x^3\left(b+cx^3\right)\right.$$
$$+364a^{11}x^6\left(b+cx^3\right)^2+1001a^{10}x^9\left(b+cx^3\right)^3$$
$$+2002a^9x^{12}\left(b+cx^3\right)^4+3003a^8x^{15}\left(b+cx^3\right)^5$$
$$+3432a^7x^{18}\left(b+cx^3\right)^6+3003a^6x^{21}\left(b+cx^3\right)^7$$
$$+2002a^5x^{24}\left(b+cx^3\right)^8+1001a^4x^{27}\left(b+cx^3\right)^9$$
$$+364a^3x^{30}\left(b+cx^3\right)^{10}+91a^2x^{33}\left(b+cx^3\right)^{11}$$
$$\left.+14ax^{36}\left(b+cx^3\right)^{12}+x^{39}\left(b+cx^3\right)^{13}\right)$$

```
[In] Integrate[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^3*(b + c*x^3)*(14*a^13 + 91*a^12*x^3*(b + c*x^3) + 364*a^11*x^6*(b + c*x
^3)^2 + 1001*a^10*x^9*(b + c*x^3)^3 + 2002*a^9*x^12*(b + c*x^3)^4 + 3003*a^
8*x^15*(b + c*x^3)^5 + 3432*a^7*x^18*(b + c*x^3)^6 + 3003*a^6*x^21*(b + c*x
^3)^7 + 2002*a^5*x^24*(b + c*x^3)^8 + 1001*a^4*x^27*(b + c*x^3)^9 + 364*a^3
*x^30*(b + c*x^3)^10 + 91*a^2*x^33*(b + c*x^3)^11 + 14*a*x^36*(b + c*x^3)^1
2 + x^39*(b + c*x^3)^13))/42
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(c x^6+b x^3+a)^{14}}{42}$	17
gosper	Expression too large to display	1455
parallelrisch	Expression too large to display	1455
risch	Expression too large to display	1460

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x,method=_RETURNVERBOSE)
[Out] 1/42*(c*x^6+b*x^3+a)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1240 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 1240, normalized size of antiderivative = 68.89

$$\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="fricas")
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 + 2*a*c^13)*x^78 + 13/3
*(2*b^3*c^11 + a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^1
2)*x^72 + 13/3*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(33*
b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^66 + 143/21*(12*b
^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^63 + 143/6*(3*b^8
*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^60 + 14
3/3*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^57 + 143/6*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^51 + 13/6
*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c + 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 +
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 + 3432*a^7*c^7)*x^42 + 1/3*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^39 + 13/6*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^36 + 13/3*(2*a^3*b^
11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 9
9*a^8*b*c^5)*x^33 + 143/6*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^30 + 143/3*(a^5*b^9 + 12*a^6*b^7
*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^27 + 143/6*(3*a^6*b^8
```

```
 + 24*a^7*b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^24 + 143/21
*(12*a^7*b^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^21 + 13/6*(
33*a^8*b^6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)*x^18 + 1/3*a^13*
b*x^3 + 13/3*(11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^15 + 13/6*(11*a^
10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^12 + 13/3*(2*a^11*b^3 + a^12*b*c)*x^9
+ 1/6*(13*a^12*b^2 + 2*a^13*c)*x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1394 vs. $2(14) = 28$.

Time = 0.15 (sec) , antiderivative size = 1394, normalized size of antiderivative = 77.44

$$\int x^2\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{13}\,dx=\text{Too large to display}$$

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**13,x)
```

```
[Out] a**13*b*x**3/3 + b*c**13*x**81/3 + c**14*x**84/42 + x**78*(a*c**13/3 + 13*b
**2*c**12/6) + x**75*(13*a*b*c**12/3 + 26*b**3*c**11/3) + x**72*(13*a**2*c*
*12/6 + 26*a*b**2*c**11 + 143*b**4*c**10/6) + x**69*(26*a**2*b*c**11 + 286*
a*b**3*c**10/3 + 143*b**5*c**9/3) + x**66*(26*a**3*c**11/3 + 143*a**2*b**2*
c**10 + 715*a*b**4*c**9/3 + 143*b**6*c**8/2) + x**63*(286*a**3*b*c**10/3 +
1430*a**2*b**3*c**9/3 + 429*a*b**5*c**8 + 572*b**7*c**7/7) + x**60*(143*a**
4*c**10/6 + 1430*a**3*b**2*c**9/3 + 2145*a**2*b**4*c**8/2 + 572*a*b**6*c**7
 + 143*b**8*c**6/2) + x**57*(715*a**4*b*c**9/3 + 1430*a**3*b**3*c**8 + 1716
*a**2*b**5*c**7 + 572*a*b**7*c**6 + 143*b**9*c**5/3) + x**54*(143*a**5*c**9
/3 + 2145*a**4*b**2*c**8/2 + 2860*a**3*b**4*c**7 + 2002*a**2*b**6*c**6 + 42
9*a*b**8*c**5 + 143*b**10*c**4/6) + x**51*(429*a**5*b*c**8 + 2860*a**4*b**3
*c**7 + 4004*a**3*b**5*c**6 + 1716*a**2*b**7*c**5 + 715*a*b**9*c**4/3 + 26*
b**11*c**3/3) + x**48*(143*a**6*c**8/2 + 1716*a**5*b**2*c**7 + 5005*a**4*b*
*4*c**6 + 4004*a**3*b**6*c**5 + 2145*a**2*b**8*c**4/2 + 286*a*b**10*c**3/3
+ 13*b**12*c**2/6) + x**45*(572*a**6*b*c**7 + 4004*a**5*b**3*c**6 + 6006*a*
*4*b**5*c**5 + 2860*a**3*b**7*c**4 + 1430*a**2*b**9*c**3/3 + 26*a*b**11*c**
2 + b**13*c/3) + x**42*(572*a**7*c**7/7 + 2002*a**6*b**2*c**6 + 6006*a**5*b
**4*c**5 + 5005*a**4*b**6*c**4 + 1430*a**3*b**8*c**3 + 143*a**2*b**10*c**2
+ 13*a*b**12*c/3 + b**14/42) + x**39*(572*a**7*b*c**6 + 4004*a**6*b**3*c**5
 + 6006*a**5*b**5*c**4 + 2860*a**4*b**7*c**3 + 1430*a**3*b**9*c**2/3 + 26*a
**2*b**11*c + a*b**13/3) + x**36*(143*a**8*c**6/2 + 1716*a**7*b**2*c**5 + 5
005*a**6*b**4*c**4 + 4004*a**5*b**6*c**3 + 2145*a**4*b**8*c**2/2 + 286*a**3
*b**10*c/3 + 13*a**2*b**12/6) + x**33*(429*a**8*b*c**5 + 2860*a**7*b**3*c**
4 + 4004*a**6*b**5*c**3 + 1716*a**5*b**7*c**2 + 715*a**4*b**9*c/3 + 26*a**3
*b**11/3) + x**30*(143*a**9*c**5/3 + 2145*a**8*b**2*c**4/2 + 2860*a**7*b**4
*c**3 + 2002*a**6*b**6*c**2 + 429*a**5*b**8*c + 143*a**4*b**10/6) + x**27*(
715*a**9*b*c**4/3 + 1430*a**8*b**3*c**3 + 1716*a**7*b**5*c**2 + 572*a**6*b*
*7*c + 143*a**5*b**9/3) + x**24*(143*a**10*c**4/6 + 1430*a**9*b**2*c**3/3 +
```

```
 2145*a**8*b**4*c**2/2 + 572*a**7*b**6*c + 143*a**6*b**8/2) + x**21*(286*a*
*10*b*c**3/3 + 1430*a**9*b**3*c**2/3 + 429*a**8*b**5*c + 572*a**7*b**7/7) +
 x**18*(26*a**11*c**3/3 + 143*a**10*b**2*c**2 + 715*a**9*b**4*c/3 + 143*a**
8*b**6/2) + x**15*(26*a**11*b*c**2 + 286*a**10*b**3*c/3 + 143*a**9*b**5/3)
+ x**12*(13*a**12*c**2/6 + 26*a**11*b**2*c + 143*a**10*b**4/6) + x**9*(13*a
**12*b*c/3 + 26*a**11*b**3/3) + x**6*(a**13*c/3 + 13*a**12*b**2/6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1240 vs. $2(16) = 32$.

Time = 0.20 (sec) , antiderivative size = 1240, normalized size of antiderivative = 68.89

$$\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 + 2*a*c^13)*x^78 + 13/3
*(2*b^3*c^11 + a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^1
2)*x^72 + 13/3*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(33*
b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^66 + 143/21*(12*b
^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^63 + 143/6*(3*b^8
*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^60 + 14
3/3*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^57 + 143/6*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^51 + 13/6
*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c + 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 +
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 + 3432*a^7*c^7)*x^42 + 1/3*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^39 + 13/6*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^36 + 13/3*(2*a^3*b^
11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 9
9*a^8*b*c^5)*x^33 + 143/6*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^30 + 143/3*(a^5*b^9 + 12*a^6*b^7
*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^27 + 143/6*(3*a^6*b^8
 + 24*a^7*b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^24 + 143/21
*(12*a^7*b^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^21 + 13/6*(
33*a^8*b^6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)*x^18 + 1/3*a^13*
b*x^3 + 13/3*(11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^15 + 13/6*(11*a^
10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^12 + 13/3*(2*a^11*b^3 + a^12*b*c)*x^9
+ 1/6*(13*a^12*b^2 + 2*a^13*c)*x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(16) = 32$.

Time = 0.32 (sec) , antiderivative size = 246, normalized size of antiderivative = 13.67

$$\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^{13}\,dx = \frac{1}{42}\left(cx^6+bx^3\right)^{14}+\frac{1}{3}\left(cx^6+bx^3\right)^{13}a+\frac{13}{6}\left(cx^6+bx^3\right)^{12}a^2+\frac{26}{3}\left(cx^6+bx^3\right)^{11}a^3+\frac{143}{6}\left(cx^6+bx^3\right)^{10}a^4+\frac{143}{3}\left(cx^6+bx^3\right)^{9}a^5+\frac{143}{2}\left(cx^6+bx^3\right)^{8}a^6+\frac{572}{7}\left(cx^6+bx^3\right)^{7}a^7+\frac{143}{2}\left(cx^6+bx^3\right)^{6}a^8+\frac{143}{3}\left(cx^6+bx^3\right)^{5}a^9+\frac{143}{6}\left(cx^6+bx^3\right)^{4}a^{10}+\frac{26}{3}\left(cx^6+bx^3\right)^{3}a^{11}+\frac{13}{6}\left(cx^6+bx^3\right)^{2}a^{12}+\frac{1}{3}\left(cx^6+bx^3\right)a^{13}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="giac")
```

```
[Out] 1/42*(c*x^6 + b*x^3)^14 + 1/3*(c*x^6 + b*x^3)^13*a + 13/6*(c*x^6 + b*x^3)^1
2*a^2 + 26/3*(c*x^6 + b*x^3)^11*a^3 + 143/6*(c*x^6 + b*x^3)^10*a^4 + 143/3*
(c*x^6 + b*x^3)^9*a^5 + 143/2*(c*x^6 + b*x^3)^8*a^6 + 572/7*(c*x^6 + b*x^3)
^7*a^7 + 143/2*(c*x^6 + b*x^3)^6*a^8 + 143/3*(c*x^6 + b*x^3)^5*a^9 + 143/6*
(c*x^6 + b*x^3)^4*a^10 + 26/3*(c*x^6 + b*x^3)^3*a^11 + 13/6*(c*x^6 + b*x^3)
^2*a^12 + 1/3*(c*x^6 + b*x^3)*a^13
```

Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 1210, normalized size of antiderivative = 67.22

$$\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^{13}\,dx = \text{Too large to display}$$

```
[In] int(x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x)
```

```
[Out] x^36*((13*a^2*b^12)/6 + (143*a^8*c^6)/2 + (286*a^3*b^10*c)/3 + (2145*a^4*b^
8*c^2)/2 + 4004*a^5*b^6*c^3 + 5005*a^6*b^4*c^4 + 1716*a^7*b^2*c^5) + x^48*(
(143*a^6*c^8)/2 + (13*b^12*c^2)/6 + (286*a*b^10*c^3)/3 + (2145*a^2*b^8*c^4)
/2 + 4004*a^3*b^6*c^5 + 5005*a^4*b^4*c^6 + 1716*a^5*b^2*c^7) + x^39*((a*b^1
3)/3 + 26*a^2*b^11*c + 572*a^7*b*c^6 + (1430*a^3*b^9*c^2)/3 + 2860*a^4*b^7*
c^3 + 6006*a^5*b^5*c^4 + 4004*a^6*b^3*c^5) + x^45*((b^13*c)/3 + 26*a*b^11*c
^2 + 572*a^6*b*c^7 + (1430*a^2*b^9*c^3)/3 + 2860*a^3*b^7*c^4 + 6006*a^4*b^5
*c^5 + 4004*a^5*b^3*c^6) + x^18*((143*a^8*b^6)/2 + (26*a^11*c^3)/3 + (715*a
```

```
^9*b^4*c)/3 + 143*a^10*b^2*c^2) + x^66*((26*a^3*c^11)/3 + (143*b^6*c^8)/2 +
 (715*a*b^4*c^9)/3 + 143*a^2*b^2*c^10) + x^30*((143*a^4*b^10)/6 + (143*a^9*
c^5)/3 + 429*a^5*b^8*c + 2002*a^6*b^6*c^2 + 2860*a^7*b^4*c^3 + (2145*a^8*b^
2*c^4)/2) + x^54*((143*a^5*c^9)/3 + (143*b^10*c^4)/6 + 429*a*b^8*c^5 + 2002
*a^2*b^6*c^6 + 2860*a^3*b^4*c^7 + (2145*a^4*b^2*c^8)/2) + x^42*(b^14/42 + (
572*a^7*c^7)/7 + 143*a^2*b^10*c^2 + 1430*a^3*b^8*c^3 + 5005*a^4*b^6*c^4 + 6
006*a^5*b^4*c^5 + 2002*a^6*b^2*c^6 + (13*a*b^12*c)/3) + x^24*((143*a^6*b^8)
/2 + (143*a^10*c^4)/6 + 572*a^7*b^6*c + (2145*a^8*b^4*c^2)/2 + (1430*a^9*b^
2*c^3)/3) + x^60*((143*a^4*c^10)/6 + (143*b^8*c^6)/2 + 572*a*b^6*c^7 + (214
5*a^2*b^4*c^8)/2 + (1430*a^3*b^2*c^9)/3) + (c^14*x^84)/42 + x^6*((a^13*c)/3
 + (13*a^12*b^2)/6) + (13*a^10*x^12*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/6 + (1
3*c^10*x^72*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/6 + (a^13*b*x^3)/3 + (b*c^13*x
^81)/3 + (c^12*x^78*(2*a*c + 13*b^2))/6 + (143*a^7*b*x^21*(12*b^6 + 14*a^3*
c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/21 + (143*b*c^7*x^63*(12*b^6 + 14*a^3*c
^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/21 + (143*a^5*b*x^27*(b^8 + 5*a^4*c^4 +
36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/3 + (143*b*c^5*x^57*(b^8 + 5
*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/3 + (13*a^3*b*x^3
3*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 + 55*a*b^8*c))/3 + (13*b*c^3*x^51*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2
+ 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/3 + (13*a^9*b*x^15*(11*b
^4 + 6*a^2*c^2 + 22*a*b^2*c))/3 + (13*b*c^9*x^69*(11*b^4 + 6*a^2*c^2 + 22*a
*b^2*c))/3 + (13*a^11*b*x^9*(a*c + 2*b^2))/3 + (13*b*c^11*x^75*(a*c + 2*b^2
))/3
```

3.96 $\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 23

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx=\frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}$$

```
[Out] 1/14*(a+b*x^n+c*x^(2*n))^14/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1482, 643}

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx=\frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}$$

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (a + b*x^n + c*x^(2*n))^14/(14*n)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
```

```
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx,x,x^n\right)}{n}$$
$$= \frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx = \frac{(a+x^n(b+cx^n))^{14}}{14n}$$

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x]
[Out] (a + x^n*(b + c*x^n))^14/(14*n)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2041 vs. 2(21) = 42.

Time = 0.02 (sec) , antiderivative size = 2042, normalized size of antiderivative = 88.78

Expression too large to display

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x)
[Out] 286*a^10*b/n*(x^n)^7*c^3+1430*a^9*b^3/n*(x^n)^7*c^2+1287*a^8*b^5/n*(x^n)^7*
c+429*a^10/n*(x^n)^6*b^2*c^2+429/2*a^6/n*(x^n)^8*b^8+143/2*a^10/n*(x^n)^8*c
^4+26*a^11*b^3/n*(x^n)^3+c^13/n*(x^n)^26*a+13/2*c^12/n*(x^n)^26*b^2+13/2*a^
12/n*(x^n)^2*b^2+13/2*c^12/n*(x^n)^24*a^2+143/2*c^10/n*(x^n)^24*b^4+143/2*c
^10/n*(x^n)^20*a^4+429/2*c^6/n*(x^n)^20*b^8+26*a^11/n*(x^n)^6*c^3+429/2*a^8
/n*(x^n)^6*b^6+13/2*a^12/n*(x^n)^4*c^2+143/2*a^10/n*(x^n)^4*b^4+1/14*c^14/n
*(x^n)^28+1/14/n*(x^n)^14*b^14+429/2*a^8/n*(x^n)^12*c^6+13/2*a^2/n*(x^n)^12
*b^12+143*a^9/n*(x^n)^10*c^5+143/2*a^4/n*(x^n)^10*b^10+a^13/n*(x^n)^2*c+715
*b*c^9/n*(x^n)^19*a^4+4290*b^3*c^8/n*(x^n)^19*a^3+5148*b^5*c^7/n*(x^n)^19*a
^2+1716*b^7*c^6/n*(x^n)^19*a+1716/7/n*(x^n)^14*a^7*c^7+143*a^5*b^9/n*(x^n)^
9+b^13*c/n*(x^n)^15+26*c^11*b^3/n*(x^n)^25+143*c^9/n*(x^n)^18*a^5+143/2*c^4
/n*(x^n)^18*b^10+1716/7*b^7*c^7/n*(x^n)^21+26*c^11/n*(x^n)^22*a^3+429/2*c^8
/n*(x^n)^22*b^6+a*b^13/n*(x^n)^13+b*a^13/n*x^n+5148*a^7/n*(x^n)^12*b^2*c^5+
15015*a^6/n*(x^n)^12*b^4*c^4+12012*a^5/n*(x^n)^12*b^6*c^3+6435/2*a^4/n*(x^n
)^12*b^8*c^2+286*a^3/n*(x^n)^12*b^10*c+1716/7*a^7*b^7/n*(x^n)^7+143*c^9*b^5
/n*(x^n)^23+429/2*c^8/n*(x^n)^16*a^6+13/2*c^2/n*(x^n)^16*b^12+b*c^13/n*(x^n
```

```
)^27+26*b^11*c^3/n*(x^n)^17+26*a^3*b^11/n*(x^n)^11+143*b^9*c^5/n*(x^n)^19+1
43*a^9*b^5/n*(x^n)^5+6435/2*a^8/n*(x^n)^10*b^2*c^4+8580*a^7/n*(x^n)^10*b^4*
c^3+6006*a^6/n*(x^n)^10*b^6*c^2+1287*a^5/n*(x^n)^10*b^8*c+6006/n*(x^n)^14*a
^6*b^2*c^6+18018/n*(x^n)^14*a^5*b^4*c^5+15015/n*(x^n)^14*a^4*b^6*c^4+4290/n
*(x^n)^14*a^3*b^8*c^3+429/n*(x^n)^14*a^2*b^10*c^2+13/n*(x^n)^14*a*b^12*c+14
30*c^9/n*(x^n)^20*a^3*b^2+1716*b*c^7/n*(x^n)^15*a^6+12012*b^3*c^6/n*(x^n)^1
5*a^5+18018*b^5*c^5/n*(x^n)^15*a^4+8580*b^7*c^4/n*(x^n)^15*a^3+1430*b^9*c^3
/n*(x^n)^15*a^2+78*b^11*c^2/n*(x^n)^15*a+1430*b^3*c^9/n*(x^n)^21*a^2+1287*b
^5*c^8/n*(x^n)^21*a+78*c^11*b/n*(x^n)^23*a^2+286*c^10*b^3/n*(x^n)^23*a+5148
*b^7*c^5/n*(x^n)^17*a^2+715*b^9*c^4/n*(x^n)^17*a+1287*a^8*b/n*(x^n)^11*c^5+
8580*a^7*b^3/n*(x^n)^11*c^4+12012*a^6*b^5/n*(x^n)^11*c^3+5148*a^5*b^7/n*(x^
n)^11*c^2+715*a^4*b^9/n*(x^n)^11*c+6435/2*c^8/n*(x^n)^20*a^2*b^4+1716*c^7/n
*(x^n)^20*a*b^6+6435/2*c^8/n*(x^n)^18*a^4*b^2+8580*c^7/n*(x^n)^18*a^3*b^4+6
006*c^6/n*(x^n)^18*a^2*b^6+1287*c^5/n*(x^n)^18*a*b^8+286*b*c^10/n*(x^n)^21*
a^3+78*a^11*b/n*(x^n)^5*c^2+286*a^10*b^3/n*(x^n)^5*c+13*a^12*b/n*(x^n)^3*c+
715*a^9*b/n*(x^n)^9*c^4+4290*a^8*b^3/n*(x^n)^9*c^3+5148*a^7*b^5/n*(x^n)^9*c
^2+1716*a^6*b^7/n*(x^n)^9*c+1430*a^9/n*(x^n)^8*b^2*c^3+6435/2*a^8/n*(x^n)^8
*b^4*c^2+1716*a^7/n*(x^n)^8*b^6*c+78*c^11/n*(x^n)^24*a*b^2+429*c^10/n*(x^n)
^22*a^2*b^2+715*c^9/n*(x^n)^22*a*b^4+715*a^9/n*(x^n)^6*b^4*c+78*a^11/n*(x^n
)^4*b^2*c+13*c^12*b/n*(x^n)^25*a+1287*b*c^8/n*(x^n)^17*a^5+8580*b^3*c^7/n*(
x^n)^17*a^4+12012*b^5*c^6/n*(x^n)^17*a^3+1716*a^7*b/n*(x^n)^13*c^6+12012*a^
6*b^3/n*(x^n)^13*c^5+18018*a^5*b^5/n*(x^n)^13*c^4+8580*a^4*b^7/n*(x^n)^13*c
^3+1430*a^3*b^9/n*(x^n)^13*c^2+78*a^2*b^11/n*(x^n)^13*c+5148*c^7/n*(x^n)^16
*a^5*b^2+15015*c^6/n*(x^n)^16*a^4*b^4+12012*c^5/n*(x^n)^16*a^3*b^6+6435/2*c
^4/n*(x^n)^16*a^2*b^8+286*c^3/n*(x^n)^16*a*b^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1297 vs. $2(21) = 42$.

Time = 0.29 (sec) , antiderivative size = 1297, normalized size of antiderivative = 56.39

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="fricas"
)
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 14*a^13*b*x^n + 7*(13*b^2*c^12 +
 2*a*c^13)*x^(26*n) + 182*(2*b^3*c^11 + a*b*c^12)*x^(25*n) + 91*(11*b^4*c^1
0 + 12*a*b^2*c^11 + a^2*c^12)*x^(24*n) + 182*(11*b^5*c^9 + 22*a*b^3*c^10 +
6*a^2*b*c^11)*x^(23*n) + 91*(33*b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 +
 4*a^3*c^11)*x^(22*n) + 286*(12*b^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 1
4*a^3*b*c^10)*x^(21*n) + 1001*(3*b^8*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 +
20*a^3*b^2*c^9 + a^4*c^10)*x^(20*n) + 2002*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2
*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^(19*n) + 1001*(b^10*c^4 + 18*a*b
```

```
^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7 + 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^(
18*n) + 182*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*a^2*b^7*c^5 + 924*a^3*b^5*c^6
+ 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^(17*n) + 91*(b^12*c^2 + 44*a*b^10*c^3 +
 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 + 792*a^5*b^2*c^7 +
33*a^6*c^8)*x^(16*n) + 14*(b^13*c + 78*a*b^11*c^2 + 1430*a^2*b^9*c^3 + 8580
*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^(1
5*n) + (b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 + 60060*a^3*b^8*c^3 + 21021
0*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6 + 3432*a^7*c^7)*x^(1
4*n) + 14*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 1
8018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^(13*n) + 91*(a^2*b
^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4
 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^(12*n) + 182*(2*a^3*b^11 + 55*a^4*b^9*c
+ 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 99*a^8*b*c^5)*x^(11
*n) + 1001*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a^7*b^4*c^3 + 45
*a^8*b^2*c^4 + 2*a^9*c^5)*x^(10*n) + 2002*(a^5*b^9 + 12*a^6*b^7*c + 36*a^7*
b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^(9*n) + 1001*(3*a^6*b^8 + 24*a^7*
b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^(8*n) + 286*(12*a^7*b
^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^(7*n) + 91*(33*a^8*b^
6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)*x^(6*n) + 182*(11*a^9*b^5
 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^(5*n) + 91*(11*a^10*b^4 + 12*a^11*b^2*c
+ a^12*c^2)*x^(4*n) + 182*(2*a^11*b^3 + a^12*b*c)*x^(3*n) + 7*(13*a^12*b^2
+ 2*a^13*c)*x^(2*n))/n
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx=\text{Timed out}$$

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**13,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2041 vs. $2(21) = 42$.

Time = 0.27 (sec) , antiderivative size = 2041, normalized size of antiderivative = 88.74

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx=\text{Too large to display}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="maxima"
)
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + a*c^1
3*x^(26*n)/n + 26*b^3*c^11*x^(25*n)/n + 13*a*b*c^12*x^(25*n)/n + 143/2*b^4*
c^10*x^(24*n)/n + 78*a*b^2*c^11*x^(24*n)/n + 13/2*a^2*c^12*x^(24*n)/n + 143
*b^5*c^9*x^(23*n)/n + 286*a*b^3*c^10*x^(23*n)/n + 78*a^2*b*c^11*x^(23*n)/n
+ 429/2*b^6*c^8*x^(22*n)/n + 715*a*b^4*c^9*x^(22*n)/n + 429*a^2*b^2*c^10*x^
(22*n)/n + 26*a^3*c^11*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 1287*a*b^5*
c^8*x^(21*n)/n + 1430*a^2*b^3*c^9*x^(21*n)/n + 286*a^3*b*c^10*x^(21*n)/n +
429/2*b^8*c^6*x^(20*n)/n + 1716*a*b^6*c^7*x^(20*n)/n + 6435/2*a^2*b^4*c^8*x
^(20*n)/n + 1430*a^3*b^2*c^9*x^(20*n)/n + 143/2*a^4*c^10*x^(20*n)/n + 143*b
^9*c^5*x^(19*n)/n + 1716*a*b^7*c^6*x^(19*n)/n + 5148*a^2*b^5*c^7*x^(19*n)/n
 + 4290*a^3*b^3*c^8*x^(19*n)/n + 715*a^4*b*c^9*x^(19*n)/n + 143/2*b^10*c^4*
x^(18*n)/n + 1287*a*b^8*c^5*x^(18*n)/n + 6006*a^2*b^6*c^6*x^(18*n)/n + 8580
*a^3*b^4*c^7*x^(18*n)/n + 6435/2*a^4*b^2*c^8*x^(18*n)/n + 143*a^5*c^9*x^(18
*n)/n + 26*b^11*c^3*x^(17*n)/n + 715*a*b^9*c^4*x^(17*n)/n + 5148*a^2*b^7*c^
5*x^(17*n)/n + 12012*a^3*b^5*c^6*x^(17*n)/n + 8580*a^4*b^3*c^7*x^(17*n)/n +
 1287*a^5*b*c^8*x^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n + 286*a*b^10*c^3*x^(1
6*n)/n + 6435/2*a^2*b^8*c^4*x^(16*n)/n + 12012*a^3*b^6*c^5*x^(16*n)/n + 150
15*a^4*b^4*c^6*x^(16*n)/n + 5148*a^5*b^2*c^7*x^(16*n)/n + 429/2*a^6*c^8*x^(
16*n)/n + b^13*c*x^(15*n)/n + 78*a*b^11*c^2*x^(15*n)/n + 1430*a^2*b^9*c^3*x
^(15*n)/n + 8580*a^3*b^7*c^4*x^(15*n)/n + 18018*a^4*b^5*c^5*x^(15*n)/n + 12
012*a^5*b^3*c^6*x^(15*n)/n + 1716*a^6*b*c^7*x^(15*n)/n + 1/14*b^14*x^(14*n)
/n + 13*a*b^12*c*x^(14*n)/n + 429*a^2*b^10*c^2*x^(14*n)/n + 4290*a^3*b^8*c^
3*x^(14*n)/n + 15015*a^4*b^6*c^4*x^(14*n)/n + 18018*a^5*b^4*c^5*x^(14*n)/n
+ 6006*a^6*b^2*c^6*x^(14*n)/n + 1716/7*a^7*c^7*x^(14*n)/n + a*b^13*x^(13*n)
/n + 78*a^2*b^11*c*x^(13*n)/n + 1430*a^3*b^9*c^2*x^(13*n)/n + 8580*a^4*b^7*
c^3*x^(13*n)/n + 18018*a^5*b^5*c^4*x^(13*n)/n + 12012*a^6*b^3*c^5*x^(13*n)/
n + 1716*a^7*b*c^6*x^(13*n)/n + 13/2*a^2*b^12*x^(12*n)/n + 286*a^3*b^10*c*x
^(12*n)/n + 6435/2*a^4*b^8*c^2*x^(12*n)/n + 12012*a^5*b^6*c^3*x^(12*n)/n +
15015*a^6*b^4*c^4*x^(12*n)/n + 5148*a^7*b^2*c^5*x^(12*n)/n + 429/2*a^8*c^6*
x^(12*n)/n + 26*a^3*b^11*x^(11*n)/n + 715*a^4*b^9*c*x^(11*n)/n + 5148*a^5*b
^7*c^2*x^(11*n)/n + 12012*a^6*b^5*c^3*x^(11*n)/n + 8580*a^7*b^3*c^4*x^(11*n
)/n + 1287*a^8*b*c^5*x^(11*n)/n + 143/2*a^4*b^10*x^(10*n)/n + 1287*a^5*b^8*
c*x^(10*n)/n + 6006*a^6*b^6*c^2*x^(10*n)/n + 8580*a^7*b^4*c^3*x^(10*n)/n +
6435/2*a^8*b^2*c^4*x^(10*n)/n + 143*a^9*c^5*x^(10*n)/n + 143*a^5*b^9*x^(9*n
)/n + 1716*a^6*b^7*c*x^(9*n)/n + 5148*a^7*b^5*c^2*x^(9*n)/n + 4290*a^8*b^3*
c^3*x^(9*n)/n + 715*a^9*b*c^4*x^(9*n)/n + 429/2*a^6*b^8*x^(8*n)/n + 1716*a^
7*b^6*c*x^(8*n)/n + 6435/2*a^8*b^4*c^2*x^(8*n)/n + 1430*a^9*b^2*c^3*x^(8*n)
/n + 143/2*a^10*c^4*x^(8*n)/n + 1716/7*a^7*b^7*x^(7*n)/n + 1287*a^8*b^5*c*x
^(7*n)/n + 1430*a^9*b^3*c^2*x^(7*n)/n + 286*a^10*b*c^3*x^(7*n)/n + 429/2*a^
8*b^6*x^(6*n)/n + 715*a^9*b^4*c*x^(6*n)/n + 429*a^10*b^2*c^2*x^(6*n)/n + 26
*a^11*c^3*x^(6*n)/n + 143*a^9*b^5*x^(5*n)/n + 286*a^10*b^3*c*x^(5*n)/n + 78
*a^11*b*c^2*x^(5*n)/n + 143/2*a^10*b^4*x^(4*n)/n + 78*a^11*b^2*c*x^(4*n)/n
+ 13/2*a^12*c^2*x^(4*n)/n + 26*a^11*b^3*x^(3*n)/n + 13*a^12*b*c*x^(3*n)/n +
 13/2*a^12*b^2*x^(2*n)/n + a^13*c*x^(2*n)/n + a^13*b*x^n/n
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1693 vs. $2(21) = 42$.

Time = 0.38 (sec) , antiderivative size = 1693, normalized size of antiderivative = 73.61

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 14*a*c^13
*x^(26*n) + 364*b^3*c^11*x^(25*n) + 182*a*b*c^12*x^(25*n) + 1001*b^4*c^10*x
^(24*n) + 1092*a*b^2*c^11*x^(24*n) + 91*a^2*c^12*x^(24*n) + 2002*b^5*c^9*x^
(23*n) + 4004*a*b^3*c^10*x^(23*n) + 1092*a^2*b*c^11*x^(23*n) + 3003*b^6*c^8
*x^(22*n) + 10010*a*b^4*c^9*x^(22*n) + 6006*a^2*b^2*c^10*x^(22*n) + 364*a^3
*c^11*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 18018*a*b^5*c^8*x^(21*n) + 20020*a
^2*b^3*c^9*x^(21*n) + 4004*a^3*b*c^10*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 24
024*a*b^6*c^7*x^(20*n) + 45045*a^2*b^4*c^8*x^(20*n) + 20020*a^3*b^2*c^9*x^(
20*n) + 1001*a^4*c^10*x^(20*n) + 2002*b^9*c^5*x^(19*n) + 24024*a*b^7*c^6*x^
(19*n) + 72072*a^2*b^5*c^7*x^(19*n) + 60060*a^3*b^3*c^8*x^(19*n) + 10010*a^
4*b*c^9*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 18018*a*b^8*c^5*x^(18*n) + 8408
4*a^2*b^6*c^6*x^(18*n) + 120120*a^3*b^4*c^7*x^(18*n) + 45045*a^4*b^2*c^8*x^
(18*n) + 2002*a^5*c^9*x^(18*n) + 364*b^11*c^3*x^(17*n) + 10010*a*b^9*c^4*x^
(17*n) + 72072*a^2*b^7*c^5*x^(17*n) + 168168*a^3*b^5*c^6*x^(17*n) + 120120*
a^4*b^3*c^7*x^(17*n) + 18018*a^5*b*c^8*x^(17*n) + 91*b^12*c^2*x^(16*n) + 40
04*a*b^10*c^3*x^(16*n) + 45045*a^2*b^8*c^4*x^(16*n) + 168168*a^3*b^6*c^5*x^
(16*n) + 210210*a^4*b^4*c^6*x^(16*n) + 72072*a^5*b^2*c^7*x^(16*n) + 3003*a^
6*c^8*x^(16*n) + 14*b^13*c*x^(15*n) + 1092*a*b^11*c^2*x^(15*n) + 20020*a^2*
b^9*c^3*x^(15*n) + 120120*a^3*b^7*c^4*x^(15*n) + 252252*a^4*b^5*c^5*x^(15*n
) + 168168*a^5*b^3*c^6*x^(15*n) + 24024*a^6*b*c^7*x^(15*n) + b^14*x^(14*n)
+ 182*a*b^12*c*x^(14*n) + 6006*a^2*b^10*c^2*x^(14*n) + 60060*a^3*b^8*c^3*x^
(14*n) + 210210*a^4*b^6*c^4*x^(14*n) + 252252*a^5*b^4*c^5*x^(14*n) + 84084*
a^6*b^2*c^6*x^(14*n) + 3432*a^7*c^7*x^(14*n) + 14*a*b^13*x^(13*n) + 1092*a^
2*b^11*c*x^(13*n) + 20020*a^3*b^9*c^2*x^(13*n) + 120120*a^4*b^7*c^3*x^(13*n
) + 252252*a^5*b^5*c^4*x^(13*n) + 168168*a^6*b^3*c^5*x^(13*n) + 24024*a^7*b
*c^6*x^(13*n) + 91*a^2*b^12*x^(12*n) + 4004*a^3*b^10*c*x^(12*n) + 45045*a^4
*b^8*c^2*x^(12*n) + 168168*a^5*b^6*c^3*x^(12*n) + 210210*a^6*b^4*c^4*x^(12*
n) + 72072*a^7*b^2*c^5*x^(12*n) + 3003*a^8*c^6*x^(12*n) + 364*a^3*b^11*x^(1
1*n) + 10010*a^4*b^9*c*x^(11*n) + 72072*a^5*b^7*c^2*x^(11*n) + 168168*a^6*b
^5*c^3*x^(11*n) + 120120*a^7*b^3*c^4*x^(11*n) + 18018*a^8*b*c^5*x^(11*n) +
1001*a^4*b^10*x^(10*n) + 18018*a^5*b^8*c*x^(10*n) + 84084*a^6*b^6*c^2*x^(10
*n) + 120120*a^7*b^4*c^3*x^(10*n) + 45045*a^8*b^2*c^4*x^(10*n) + 2002*a^9*c
^5*x^(10*n) + 2002*a^5*b^9*x^(9*n) + 24024*a^6*b^7*c*x^(9*n) + 72072*a^7*b^
5*c^2*x^(9*n) + 60060*a^8*b^3*c^3*x^(9*n) + 10010*a^9*b*c^4*x^(9*n) + 3003*
a^6*b^8*x^(8*n) + 24024*a^7*b^6*c*x^(8*n) + 45045*a^8*b^4*c^2*x^(8*n) + 200
```

```
20*a^9*b^2*c^3*x^(8*n) + 1001*a^10*c^4*x^(8*n) + 3432*a^7*b^7*x^(7*n) + 180
18*a^8*b^5*c*x^(7*n) + 20020*a^9*b^3*c^2*x^(7*n) + 4004*a^10*b*c^3*x^(7*n)
+ 3003*a^8*b^6*x^(6*n) + 10010*a^9*b^4*c*x^(6*n) + 6006*a^10*b^2*c^2*x^(6*n
) + 364*a^11*c^3*x^(6*n) + 2002*a^9*b^5*x^(5*n) + 4004*a^10*b^3*c*x^(5*n) +
 1092*a^11*b*c^2*x^(5*n) + 1001*a^10*b^4*x^(4*n) + 1092*a^11*b^2*c*x^(4*n)
+ 91*a^12*c^2*x^(4*n) + 364*a^11*b^3*x^(3*n) + 182*a^12*b*c*x^(3*n) + 91*a^
12*b^2*x^(2*n) + 14*a^13*c*x^(2*n) + 14*a^13*b*x^n)/n
```

Mupad [B] (verification not implemented)

Time = 11.02 (sec) , antiderivative size = 1395, normalized size of antiderivative = 60.65

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx=\text{Too large to display}$$

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x)
```

```
[Out] x^(n - 1)*((x^(11*n + 1)*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 + 286*a^3*b^10*
c + (6435*a^4*b^8*c^2)/2 + 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 + 5148*a^7
*b^2*c^5))/n + (x^(15*n + 1)*((429*a^6*c^8)/2 + (13*b^12*c^2)/2 + 286*a*b^1
0*c^3 + (6435*a^2*b^8*c^4)/2 + 12012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 + 5148
*a^5*b^2*c^7))/n + (x^(12*n + 1)*(a*b^13 + 78*a^2*b^11*c + 1716*a^7*b*c^6 +
 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^
5))/n + (x^(14*n + 1)*(b^13*c + 78*a*b^11*c^2 + 1716*a^6*b*c^7 + 1430*a^2*b
^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c^6))/n + (x^
(5*n + 1)*((429*a^8*b^6)/2 + 26*a^11*c^3 + 715*a^9*b^4*c + 429*a^10*b^2*c^2
))/n + (x^(21*n + 1)*(26*a^3*c^11 + (429*b^6*c^8)/2 + 715*a*b^4*c^9 + 429*a
^2*b^2*c^10))/n + (x^(9*n + 1)*((143*a^4*b^10)/2 + 143*a^9*c^5 + 1287*a^5*b
^8*c + 6006*a^6*b^6*c^2 + 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2))/n + (x^
(17*n + 1)*(143*a^5*c^9 + (143*b^10*c^4)/2 + 1287*a*b^8*c^5 + 6006*a^2*b^6*
c^6 + 8580*a^3*b^4*c^7 + (6435*a^4*b^2*c^8)/2))/n + (x^(13*n + 1)*(b^14/14
+ (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 + 4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^
4 + 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6 + 13*a*b^12*c))/n + (x^(7*n + 1)*(
(429*a^6*b^8)/2 + (143*a^10*c^4)/2 + 1716*a^7*b^6*c + (6435*a^8*b^4*c^2)/2
+ 1430*a^9*b^2*c^3))/n + (x^(19*n + 1)*((143*a^4*c^10)/2 + (429*b^8*c^6)/2
+ 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 + 1430*a^3*b^2*c^9))/n + (c^14*x^(2
7*n + 1))/(14*n) + (a^12*x^(n + 1)*(a*c + (13*b^2)/2))/n + (a^10*x^(3*n + 1
)*((143*b^4)/2 + (13*a^2*c^2)/2 + 78*a*b^2*c))/n + (c^10*x^(23*n + 1)*((143
*b^4)/2 + (13*a^2*c^2)/2 + 78*a*b^2*c))/n + (b*c^13*x^(26*n + 1))/n + (c^12
*x^(25*n + 1)*(a*c + (13*b^2)/2))/n + (a^13*b*x)/n + (143*a^7*b*x^(6*n + 1)
*(12*b^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/(7*n) + (143*b*c^7*x^
(20*n + 1)*(12*b^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/(7*n) + (14
3*a^5*b*x^(8*n + 1)*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12
*a*b^6*c))/n + (143*b*c^5*x^(18*n + 1)*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 +
30*a^3*b^2*c^3 + 12*a*b^6*c))/n + (13*a^3*b*x^(10*n + 1)*(2*b^10 + 99*a^5*c
```

```
^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/n +
 (13*b*c^3*x^(16*n + 1)*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^
4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/n + (13*a^9*b*x^(4*n + 1)*(11*b^4 +
6*a^2*c^2 + 22*a*b^2*c))/n + (13*b*c^9*x^(22*n + 1)*(11*b^4 + 6*a^2*c^2 + 2
2*a*b^2*c))/n + (13*a^11*b*x^(2*n + 1)*(a*c + 2*b^2))/n + (13*b*c^11*x^(24*
n + 1)*(a*c + 2*b^2))/n)
```

3.97 $\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 18

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

```
[Out] 1/14*(-c*x^2-b*x+a)^14
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}=0.048$, Rules used = $\{643\}$

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

```
[In] Int[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]
```

```
[Out] (a - b*x - c*x^2)^14/14
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 201 vs. $2(18) = 36$.

Time = 0.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 11.17

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx = \frac{1}{14}x(b+cx)\left(-14a^{13}+91a^{12}x(b+cx)-364a^{11}x^2(b+cx)^2 + 1001a^{10}x^3(b+cx)^3-2002a^9x^4(b+cx)^4 + 3003a^8x^5(b+cx)^5-3432a^7x^6(b+cx)^6 + 3003a^6x^7(b+cx)^7-2002a^5x^8(b+cx)^8 + 1001a^4x^9(b+cx)^9-364a^3x^{10}(b+cx)^{10} + 91a^2x^{11}(b+cx)^{11}-14ax^{12}(b+cx)^{12}+x^{13}(b+cx)^{13}\right)$$

```
[In] Integrate[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]
```

```
[Out] (x*(b + c*x)*(-14*a^13 + 91*a^12*x*(b + c*x) - 364*a^11*x^2*(b + c*x)^2 + 1
001*a^10*x^3*(b + c*x)^3 - 2002*a^9*x^4*(b + c*x)^4 + 3003*a^8*x^5*(b + c*x
)^5 - 3432*a^7*x^6*(b + c*x)^6 + 3003*a^6*x^7*(b + c*x)^7 - 2002*a^5*x^8*(b
 + c*x)^8 + 1001*a^4*x^9*(b + c*x)^9 - 364*a^3*x^10*(b + c*x)^10 + 91*a^2*x
^11*(b + c*x)^11 - 14*a*x^12*(b + c*x)^12 + x^13*(b + c*x)^13))/14
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\left(c\,x^{2}+b x-a\right)^{14}}{14}$	17
norman	Expression too large to display	1228
gosper	Expression too large to display	1451
parallelrisch	Expression too large to display	1451
risch	Expression too large to display	1456

```
[In] int((2*c*x+b)*(c*x^2+b*x-a)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*(c*x^2+b*x-a)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1238 vs. $2(16) = 32$.

Time = 0.26 (sec) , antiderivative size = 1238, normalized size of antiderivative = 68.78

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="fricas")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 1/2*(13*b^2*c^12 - 2*a*c^13)*x^26 + 13*(2*b^
3*c^11 - a*b*c^12)*x^25 + 13/2*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^12)*x^2
4 + 13*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^23 + 13/2*(33*b^6*c^8
- 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^22 + 143/7*(12*b^7*c^7 -
63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^21 + 143/2*(3*b^8*c^6 - 24
*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^20 + 143*(b^9*c^
5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^19 + 14
3/2*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7 + 45*a^4*b^
2*c^8 - 2*a^5*c^9)*x^18 + 13*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*a^2*b^7*c^5 -
 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^17 + 13/2*(b^12*c^2 -
44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 - 792
*a^5*b^2*c^7 + 33*a^6*c^8)*x^16 + (b^13*c - 78*a*b^11*c^2 + 1430*a^2*b^9*c^
3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c^6 + 1716*a^6*b*c
^7)*x^15 - a^13*b*x + 1/14*(b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 - 60060
*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6
- 3432*a^7*c^7)*x^14 - (a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2 - 8580*a^
4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^13 +
13/2*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c^3 + 2310*
a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^12 - 13*(2*a^3*b^11 - 55*a^4*
b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 99*a^8*b*c^5)
*x^11 + 143/2*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a^7*b^4*c^3 +
 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^10 - 143*(a^5*b^9 - 12*a^6*b^7*c + 36*a^7*b^
5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^9 + 143/2*(3*a^6*b^8 - 24*a^7*b^6*c
 + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^8 - 143/7*(12*a^7*b^7 - 63
*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^7 + 13/2*(33*a^8*b^6 - 110*a
^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)*x^6 - 13*(11*a^9*b^5 - 22*a^10*b^3
*c + 6*a^11*b*c^2)*x^5 + 13/2*(11*a^10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^4
- 13*(2*a^11*b^3 - a^12*b*c)*x^3 + 1/2*(13*a^12*b^2 - 2*a^13*c)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1326 vs. $2(12) = 24$.

Time = 0.15 (sec) , antiderivative size = 1326, normalized size of antiderivative = 73.67

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate((2*c*x+b)*(c*x**2+b*x-a)**13,x)
```

```
[Out] -a**13*b*x + b*c**13*x**27 + c**14*x**28/14 + x**26*(-a*c**13 + 13*b**2*c**
12/2) + x**25*(-13*a*b*c**12 + 26*b**3*c**11) + x**24*(13*a**2*c**12/2 - 78
*a*b**2*c**11 + 143*b**4*c**10/2) + x**23*(78*a**2*b*c**11 - 286*a*b**3*c**
10 + 143*b**5*c**9) + x**22*(-26*a**3*c**11 + 429*a**2*b**2*c**10 - 715*a*b
**4*c**9 + 429*b**6*c**8/2) + x**21*(-286*a**3*b*c**10 + 1430*a**2*b**3*c**
9 - 1287*a*b**5*c**8 + 1716*b**7*c**7/7) + x**20*(143*a**4*c**10/2 - 1430*a
**3*b**2*c**9 + 6435*a**2*b**4*c**8/2 - 1716*a*b**6*c**7 + 429*b**8*c**6/2)
 + x**19*(715*a**4*b*c**9 - 4290*a**3*b**3*c**8 + 5148*a**2*b**5*c**7 - 171
6*a*b**7*c**6 + 143*b**9*c**5) + x**18*(-143*a**5*c**9 + 6435*a**4*b**2*c**
8/2 - 8580*a**3*b**4*c**7 + 6006*a**2*b**6*c**6 - 1287*a*b**8*c**5 + 143*b*
*10*c**4/2) + x**17*(-1287*a**5*b*c**8 + 8580*a**4*b**3*c**7 - 12012*a**3*b
**5*c**6 + 5148*a**2*b**7*c**5 - 715*a*b**9*c**4 + 26*b**11*c**3) + x**16*(
429*a**6*c**8/2 - 5148*a**5*b**2*c**7 + 15015*a**4*b**4*c**6 - 12012*a**3*b
**6*c**5 + 6435*a**2*b**8*c**4/2 - 286*a*b**10*c**3 + 13*b**12*c**2/2) + x*
*15*(1716*a**6*b*c**7 - 12012*a**5*b**3*c**6 + 18018*a**4*b**5*c**5 - 8580*
a**3*b**7*c**4 + 1430*a**2*b**9*c**3 - 78*a*b**11*c**2 + b**13*c) + x**14*(
-1716*a**7*c**7/7 + 6006*a**6*b**2*c**6 - 18018*a**5*b**4*c**5 + 15015*a**4
*b**6*c**4 - 4290*a**3*b**8*c**3 + 429*a**2*b**10*c**2 - 13*a*b**12*c + b**
14/14) + x**13*(-1716*a**7*b*c**6 + 12012*a**6*b**3*c**5 - 18018*a**5*b**5*
c**4 + 8580*a**4*b**7*c**3 - 1430*a**3*b**9*c**2 + 78*a**2*b**11*c - a*b**1
3) + x**12*(429*a**8*c**6/2 - 5148*a**7*b**2*c**5 + 15015*a**6*b**4*c**4 -
12012*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/2 - 286*a**3*b**10*c + 13*a**2*b
**12/2) + x**11*(1287*a**8*b*c**5 - 8580*a**7*b**3*c**4 + 12012*a**6*b**5*c
**3 - 5148*a**5*b**7*c**2 + 715*a**4*b**9*c - 26*a**3*b**11) + x**10*(-143*
a**9*c**5 + 6435*a**8*b**2*c**4/2 - 8580*a**7*b**4*c**3 + 6006*a**6*b**6*c*
*2 - 1287*a**5*b**8*c + 143*a**4*b**10/2) + x**9*(-715*a**9*b*c**4 + 4290*a
**8*b**3*c**3 - 5148*a**7*b**5*c**2 + 1716*a**6*b**7*c - 143*a**5*b**9) + x
**8*(143*a**10*c**4/2 - 1430*a**9*b**2*c**3 + 6435*a**8*b**4*c**2/2 - 1716*
a**7*b**6*c + 429*a**6*b**8/2) + x**7*(286*a**10*b*c**3 - 1430*a**9*b**3*c*
*2 + 1287*a**8*b**5*c - 1716*a**7*b**7/7) + x**6*(-26*a**11*c**3 + 429*a**1
0*b**2*c**2 - 715*a**9*b**4*c + 429*a**8*b**6/2) + x**5*(-78*a**11*b*c**2 +
 286*a**10*b**3*c - 143*a**9*b**5) + x**4*(13*a**12*c**2/2 - 78*a**11*b**2*
c + 143*a**10*b**4/2) + x**3*(13*a**12*b*c - 26*a**11*b**3) + x**2*(-a**13*
c + 13*a**12*b**2/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(cx^2+bx-a\right)^{14}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="maxima")
[Out] 1/14*(c*x^2 + b*x - a)^14
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(16) = 32.

Time = 0.31 (sec) , antiderivative size = 218, normalized size of antiderivative = 12.11

$$\begin{aligned}\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx&=\frac{1}{14}\left(cx^2+bx\right)^{14}-\left(cx^2+bx\right)^{13}a\\&\quad+\frac{13}{2}\left(cx^2+bx\right)^{12}a^2-26\left(cx^2+bx\right)^{11}a^3\\&\quad+\frac{143}{2}\left(cx^2+bx\right)^{10}a^4-143\left(cx^2+bx\right)^{9}a^5\\&\quad+\frac{429}{2}\left(cx^2+bx\right)^{8}a^6-\frac{1716}{7}\left(cx^2+bx\right)^{7}a^7\\&\quad+\frac{429}{2}\left(cx^2+bx\right)^{6}a^8-143\left(cx^2+bx\right)^{5}a^9\\&\quad+\frac{143}{2}\left(cx^2+bx\right)^{4}a^{10}-26\left(cx^2+bx\right)^{3}a^{11}\\&\quad+\frac{13}{2}\left(cx^2+bx\right)^{2}a^{12}-\left(cx^2+bx\right)a^{13}\end{aligned}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="giac")
[Out] 1/14*(c*x^2 + b*x)^14 - (c*x^2 + b*x)^13*a + 13/2*(c*x^2 + b*x)^12*a^2 - 26
*(c*x^2 + b*x)^11*a^3 + 143/2*(c*x^2 + b*x)^10*a^4 - 143*(c*x^2 + b*x)^9*a^
5 + 429/2*(c*x^2 + b*x)^8*a^6 - 1716/7*(c*x^2 + b*x)^7*a^7 + 429/2*(c*x^2 +
 b*x)^6*a^8 - 143*(c*x^2 + b*x)^5*a^9 + 143/2*(c*x^2 + b*x)^4*a^10 - 26*(c*
x^2 + b*x)^3*a^11 + 13/2*(c*x^2 + b*x)^2*a^12 - (c*x^2 + b*x)*a^13
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 1208, normalized size of antiderivative = 67.11

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx = x^{12}\left(\frac{429\,a^8\,c^6}{2}-5148\,a^7\,b^2\,c^5+15015\,a^6\,b^4\,c^4-12012\,a^5\,b^6\,c^3+\frac{6435\,a^4\,b^8\,c^2}{2}-286\,a^3\,b^{10}\,c+\frac{13\,a^2\,b^{12}}{2}\right)+x^{16}\left(\frac{429\,a^6\,c^8}{2}-5148\,a^5\,b^2\,c^7+15015\,a^4\,b^4\,c^6-12012\,a^3\,b^6\,c^5+\frac{6435\,a^2\,b^8\,c^4}{2}-286\,a\,b^{10}\,c^3+\frac{13\,b^{12}\,c^2}{2}\right)-x^{13}\left(1716\,a^7\,b\,c^6-12012\,a^6\,b^3\,c^5+18018\,a^5\,b^5\,c^4-8580\,a^4\,b^7\,c^3+1430\,a^3\,b^9\,c^2-78\,a^2\,b^{11}\,c+a\,b^{13}\right)+x^{15}\left(1716\,a^6\,b\,c^7-12012\,a^5\,b^3\,c^6+18018\,a^4\,b^5\,c^5-8580\,a^3\,b^7\,c^4+1430\,a^2\,b^9\,c^3-78\,a\,b^{11}\,c^2+b^{13}\,c\right)+x^6\left(-26\,a^{11}\,c^3+429\,a^{10}\,b^2\,c^2-715\,a^9\,b^4\,c+\frac{429\,a^8\,b^6}{2}\right)-x^{22}\left(26\,a^3\,c^{11}-429\,a^2\,b^2\,c^{10}+715\,a\,b^4\,c^9-\frac{429\,b^6\,c^8}{2}\right)+x^{10}\left(-143\,a^9\,c^5+\frac{6435\,a^8\,b^2\,c^4}{2}-8580\,a^7\,b^4\,c^3+6006\,a^6\,b^6\,c^2-1287\,a^5\,b^8\,c+\frac{143\,a^4\,b^{10}}{2}\right)-x^{18}\left(143\,a^5\,c^9-\frac{6435\,a^4\,b^2\,c^8}{2}+8580\,a^3\,b^4\,c^7-6006\,a^2\,b^6\,c^6+1287\,a\,b^8\,c^5-\frac{143\,b^{10}\,c^4}{2}\right)+x^{14}\left(-\frac{1716\,a^7\,c^7}{7}+6006\,a^6\,b^2\,c^6-18018\,a^5\,b^4\,c^5+15015\,a^4\,b^6\,c^4-4290\,a^3\,b^8\,c^3+429\,a^2\,b^{10}\,c^2-13\,a\,b^{12}\,c+\frac{b^{14}}{14}\right)+x^8\left(\frac{143\,a^{10}\,c^4}{2}-1430\,a^9\,b^2\,c^3+\frac{6435\,a^8\,b^4\,c^2}{2}-1716\,a^7\,b^6\,c+\frac{429\,a^6\,b^8}{2}\right)+x^{20}\left(\frac{143\,a^4\,c^{10}}{2}-1430\,a^3\,b^2\,c^9+\frac{6435\,a^2\,b^4\,c^8}{2}-1716\,a\,b^6\,c^7+\frac{429\,b^8\,c^6}{2}\right)+\frac{c^{14}\,x^{28}}{14}-x^2\left(a^{13}\,c-\frac{13\,a^{12}\,b^2}{2}\right)+\frac{13\,a^{10}\,x^4\,\left(a^2\,c^2-12\,a\,b^2\,c+11\,b^4\right)}{2}+\frac{13\,c^{10}\,x^{24}\,\left(a^2\,c^2-12\,a\,b^2\,c+11\,b^4\right)}{2}+b\,c^{13}\,x^{27}-\frac{c^{12}\,x^{26}\,\left(2\,a\,c-13\,b^2\right)}{2}-a^{13}\,b\,x$$

```
[In] int((b + 2*c*x)*(b*x - a + c*x^2)^13,x)

[Out] x^12*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 - 286*a^3*b^10*c + (6435*a^4*b^8*c^
2)/2 - 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 - 5148*a^7*b^2*c^5) + x^16*((4
29*a^6*c^8)/2 + (13*b^12*c^2)/2 - 286*a*b^10*c^3 + (6435*a^2*b^8*c^4)/2 - 1
2012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 - 5148*a^5*b^2*c^7) - x^13*(a*b^13 - 7
8*a^2*b^11*c + 1716*a^7*b*c^6 + 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018
*a^5*b^5*c^4 - 12012*a^6*b^3*c^5) + x^15*(b^13*c - 78*a*b^11*c^2 + 1716*a^6
*b*c^7 + 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^
5*b^3*c^6) + x^6*((429*a^8*b^6)/2 - 26*a^11*c^3 - 715*a^9*b^4*c + 429*a^10*
b^2*c^2) - x^22*(26*a^3*c^11 - (429*b^6*c^8)/2 + 715*a*b^4*c^9 - 429*a^2*b^
2*c^10) + x^10*((143*a^4*b^10)/2 - 143*a^9*c^5 - 1287*a^5*b^8*c + 6006*a^6*
b^6*c^2 - 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2) - x^18*(143*a^5*c^9 - (1
43*b^10*c^4)/2 + 1287*a*b^8*c^5 - 6006*a^2*b^6*c^6 + 8580*a^3*b^4*c^7 - (64
35*a^4*b^2*c^8)/2) + x^14*(b^14/14 - (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 -
4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^4 - 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6
 - 13*a*b^12*c) + x^8*((429*a^6*b^8)/2 + (143*a^10*c^4)/2 - 1716*a^7*b^6*c
+ (6435*a^8*b^4*c^2)/2 - 1430*a^9*b^2*c^3) + x^20*((143*a^4*c^10)/2 + (429*
b^8*c^6)/2 - 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 - 1430*a^3*b^2*c^9) + (c
^14*x^28)/14 - x^2*(a^13*c - (13*a^12*b^2)/2) + (13*a^10*x^4*(11*b^4 + a^2*
c^2 - 12*a*b^2*c))/2 + (13*c^10*x^24*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/2 + b
*c^13*x^27 - (c^12*x^26*(2*a*c - 13*b^2))/2 - a^13*b*x - (143*a^7*b*x^7*(12
*b^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/7 + (143*b*c^7*x^21*(12*b
^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/7 - 143*a^5*b*x^9*(b^8 + 5*
a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c) + 143*b*c^5*x^19*(b
^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c) - 13*a^3*b*x
^11*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*
c^4 - 55*a*b^8*c) + 13*b*c^3*x^17*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 -
924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c) - 13*a^9*b*x^5*(11*b^4 + 6*
a^2*c^2 - 22*a*b^2*c) + 13*b*c^9*x^23*(11*b^4 + 6*a^2*c^2 - 22*a*b^2*c) + 1
3*a^11*b*x^3*(a*c - 2*b^2) - 13*b*c^11*x^25*(a*c - 2*b^2)
```

3.98 $\int x(b+2cx^2)(-a+bx^2+cx^4)^{13}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 20

$$\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^{13}\,dx=\frac{1}{28}\left(a-bx^2-cx^4\right)^{14}$$

```
[Out] 1/28*(-c*x^4-b*x^2+a)^14
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1261, 643}

$$\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^{13}\,dx=\frac{1}{28}\left(a-bx^2-cx^4\right)^{14}$$

```
[In] Int[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (a - b*x^2 - c*x^4)^14/28
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx, x, x^2\right)\\ &= \frac{1}{28}\left(a-bx^2-cx^4\right)^{14}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(20) = 40.

Time = 0.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 11.65

$$\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^{13}\,dx = \frac{1}{28}x^2\left(b+cx^2\right)\left(-14a^{13}+91a^{12}x^2\left(b+cx^2\right) - 364a^{11}x^4\left(b+cx^2\right)^2+1001a^{10}x^6\left(b+cx^2\right)^3 - 2002a^9x^8\left(b+cx^2\right)^4+3003a^8x^{10}\left(b+cx^2\right)^5 - 3432a^7x^{12}\left(b+cx^2\right)^6+3003a^6x^{14}\left(b+cx^2\right)^7 - 2002a^5x^{16}\left(b+cx^2\right)^8+1001a^4x^{18}\left(b+cx^2\right)^9 - 364a^3x^{20}\left(b+cx^2\right)^{10}+91a^2x^{22}\left(b+cx^2\right)^{11} - 14ax^{24}\left(b+cx^2\right)^{12}+x^{26}\left(b+cx^2\right)^{13}\right)$$

```
[In] Integrate[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (x^2*(b + c*x^2)*(-14*a^13 + 91*a^12*x^2*(b + c*x^2) - 364*a^11*x^4*(b + c*
x^2)^2 + 1001*a^10*x^6*(b + c*x^2)^3 - 2002*a^9*x^8*(b + c*x^2)^4 + 3003*a^
8*x^10*(b + c*x^2)^5 - 3432*a^7*x^12*(b + c*x^2)^6 + 3003*a^6*x^14*(b + c*x
^2)^7 - 2002*a^5*x^16*(b + c*x^2)^8 + 1001*a^4*x^18*(b + c*x^2)^9 - 364*a^3
*x^20*(b + c*x^2)^10 + 91*a^2*x^22*(b + c*x^2)^11 - 14*a*x^24*(b + c*x^2)^1
2 + x^26*(b + c*x^2)^13))/28
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\left(c\,x^{4}+b\,x^{2}-a\right)^{14}}{28}$	19
parallelrisch	Expression too large to display	1455
gosper	Expression too large to display	1457
risch	Expression too large to display	1460

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*(c*x^4+b*x^2-a)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. $2(18) = 36$.

Time = 0.26 (sec) , antiderivative size = 1242, normalized size of antiderivative = 62.10

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="fricas")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 - 2*a*c^13)*x^52 + 13/2
*(2*b^3*c^11 - a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^1
2)*x^48 + 13/2*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(33*
b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^44 + 143/14*(12*b
^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^42 + 143/4*(3*b^8
*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^40 + 14
3/2*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^38 + 143/4*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^34 + 13/4
*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c - 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 -
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 - 3432*a^7*c^7)*x^28 - 1/2*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^26 + 13/4*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^24 - 13/2*(2*a^3*b^
11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 9
9*a^8*b*c^5)*x^22 + 143/4*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^20 - 143/2*(a^5*b^9 - 12*a^6*b^7
*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^18 + 143/4*(3*a^6*b^8
 - 24*a^7*b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^16 - 1/2*a^
13*b*x^2 - 143/14*(12*a^7*b^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c
^3)*x^14 + 13/4*(33*a^8*b^6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)
*x^12 - 13/2*(11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^10 + 13/4*(11*a^
10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^8 - 13/2*(2*a^11*b^3 - a^12*b*c)*x^6 +
 1/4*(13*a^12*b^2 - 2*a^13*c)*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1384 vs. 2(14) = 28.

Time = 0.14 (sec) , antiderivative size = 1384, normalized size of antiderivative = 69.20

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**13,x)
```

```
[Out] -a**13*b*x**2/2 + b*c**13*x**54/2 + c**14*x**56/28 + x**52*(-a*c**13/2 + 13
*b**2*c**12/4) + x**50*(-13*a*b*c**12/2 + 13*b**3*c**11) + x**48*(13*a**2*c
**12/4 - 39*a*b**2*c**11 + 143*b**4*c**10/4) + x**46*(39*a**2*b*c**11 - 143
*a*b**3*c**10 + 143*b**5*c**9/2) + x**44*(-13*a**3*c**11 + 429*a**2*b**2*c*
*10/2 - 715*a*b**4*c**9/2 + 429*b**6*c**8/4) + x**42*(-143*a**3*b*c**10 + 7
15*a**2*b**3*c**9 - 1287*a*b**5*c**8/2 + 858*b**7*c**7/7) + x**40*(143*a**4
*c**10/4 - 715*a**3*b**2*c**9 + 6435*a**2*b**4*c**8/4 - 858*a*b**6*c**7 + 4
29*b**8*c**6/4) + x**38*(715*a**4*b*c**9/2 - 2145*a**3*b**3*c**8 + 2574*a**
2*b**5*c**7 - 858*a*b**7*c**6 + 143*b**9*c**5/2) + x**36*(-143*a**5*c**9/2
+ 6435*a**4*b**2*c**8/4 - 4290*a**3*b**4*c**7 + 3003*a**2*b**6*c**6 - 1287*
a*b**8*c**5/2 + 143*b**10*c**4/4) + x**34*(-1287*a**5*b*c**8/2 + 4290*a**4*
b**3*c**7 - 6006*a**3*b**5*c**6 + 2574*a**2*b**7*c**5 - 715*a*b**9*c**4/2 +
 13*b**11*c**3) + x**32*(429*a**6*c**8/4 - 2574*a**5*b**2*c**7 + 15015*a**4
*b**4*c**6/2 - 6006*a**3*b**6*c**5 + 6435*a**2*b**8*c**4/4 - 143*a*b**10*c*
*3 + 13*b**12*c**2/4) + x**30*(858*a**6*b*c**7 - 6006*a**5*b**3*c**6 + 9009
*a**4*b**5*c**5 - 4290*a**3*b**7*c**4 + 715*a**2*b**9*c**3 - 39*a*b**11*c**
2 + b**13*c/2) + x**28*(-858*a**7*c**7/7 + 3003*a**6*b**2*c**6 - 9009*a**5*
b**4*c**5 + 15015*a**4*b**6*c**4/2 - 2145*a**3*b**8*c**3 + 429*a**2*b**10*c
**2/2 - 13*a*b**12*c/2 + b**14/28) + x**26*(-858*a**7*b*c**6 + 6006*a**6*b*
*3*c**5 - 9009*a**5*b**5*c**4 + 4290*a**4*b**7*c**3 - 715*a**3*b**9*c**2 +
39*a**2*b**11*c - a*b**13/2) + x**24*(429*a**8*c**6/4 - 2574*a**7*b**2*c**5
 + 15015*a**6*b**4*c**4/2 - 6006*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/4 - 1
43*a**3*b**10*c + 13*a**2*b**12/4) + x**22*(1287*a**8*b*c**5/2 - 4290*a**7*
b**3*c**4 + 6006*a**6*b**5*c**3 - 2574*a**5*b**7*c**2 + 715*a**4*b**9*c/2 -
 13*a**3*b**11) + x**20*(-143*a**9*c**5/2 + 6435*a**8*b**2*c**4/4 - 4290*a*
*7*b**4*c**3 + 3003*a**6*b**6*c**2 - 1287*a**5*b**8*c/2 + 143*a**4*b**10/4)
 + x**18*(-715*a**9*b*c**4/2 + 2145*a**8*b**3*c**3 - 2574*a**7*b**5*c**2 +
858*a**6*b**7*c - 143*a**5*b**9/2) + x**16*(143*a**10*c**4/4 - 715*a**9*b**
2*c**3 + 6435*a**8*b**4*c**2/4 - 858*a**7*b**6*c + 429*a**6*b**8/4) + x**14
*(143*a**10*b*c**3 - 715*a**9*b**3*c**2 + 1287*a**8*b**5*c/2 - 858*a**7*b**
7/7) + x**12*(-13*a**11*c**3 + 429*a**10*b**2*c**2/2 - 715*a**9*b**4*c/2 +
429*a**8*b**6/4) + x**10*(-39*a**11*b*c**2 + 143*a**10*b**3*c - 143*a**9*b*
*5/2) + x**8*(13*a**12*c**2/4 - 39*a**11*b**2*c + 143*a**10*b**4/4) + x**6*
(13*a**12*b*c/2 - 13*a**11*b**3) + x**4*(-a**13*c/2 + 13*a**12*b**2/4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. $2(18) = 36$.

Time = 0.20 (sec) , antiderivative size = 1242, normalized size of antiderivative = 62.10

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 - 2*a*c^13)*x^52 + 13/2
*(2*b^3*c^11 - a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^1
2)*x^48 + 13/2*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(33*
b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^44 + 143/14*(12*b
^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^42 + 143/4*(3*b^8
*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^40 + 14
3/2*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^38 + 143/4*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7
 + 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^34 + 13/4
*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c - 78*a*b^11*c^2
 + 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 -
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 - 3432*a^7*c^7)*x^28 - 1/2*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^26 + 13/4*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^24 - 13/2*(2*a^3*b^
11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 9
9*a^8*b*c^5)*x^22 + 143/4*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^20 - 143/2*(a^5*b^9 - 12*a^6*b^7
*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^18 + 143/4*(3*a^6*b^8
 - 24*a^7*b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^16 - 1/2*a^
13*b*x^2 - 143/14*(12*a^7*b^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c
^3)*x^14 + 13/4*(33*a^8*b^6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)
*x^12 - 13/2*(11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^10 + 13/4*(11*a^
10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^8 - 13/2*(2*a^11*b^3 - a^12*b*c)*x^6 +
 1/4*(13*a^12*b^2 - 2*a^13*c)*x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(18) = 36$.

Time = 0.32 (sec) , antiderivative size = 246, normalized size of antiderivative = 12.30

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^{13}\,dx = \frac{1}{28}\left(cx^4+bx^2\right)^{14} - \frac{1}{2}\left(cx^4+bx^2\right)^{13}a + \frac{13}{4}\left(cx^4+bx^2\right)^{12}a^2 - 13\left(cx^4+bx^2\right)^{11}a^3 + \frac{143}{4}\left(cx^4+bx^2\right)^{10}a^4 - \frac{143}{2}\left(cx^4+bx^2\right)^{9}a^5 + \frac{429}{4}\left(cx^4+bx^2\right)^{8}a^6 - \frac{858}{7}\left(cx^4+bx^2\right)^{7}a^7 + \frac{429}{4}\left(cx^4+bx^2\right)^{6}a^8 - \frac{143}{2}\left(cx^4+bx^2\right)^{5}a^9 + \frac{143}{4}\left(cx^4+bx^2\right)^{4}a^{10} - 13\left(cx^4+bx^2\right)^{3}a^{11} + \frac{13}{4}\left(cx^4+bx^2\right)^{2}a^{12} - \frac{1}{2}\left(cx^4+bx^2\right)a^{13}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="giac")
```

```
[Out] 1/28*(c*x^4 + b*x^2)^14 - 1/2*(c*x^4 + b*x^2)^13*a + 13/4*(c*x^4 + b*x^2)^1
2*a^2 - 13*(c*x^4 + b*x^2)^11*a^3 + 143/4*(c*x^4 + b*x^2)^10*a^4 - 143/2*(c
*x^4 + b*x^2)^9*a^5 + 429/4*(c*x^4 + b*x^2)^8*a^6 - 858/7*(c*x^4 + b*x^2)^7
*a^7 + 429/4*(c*x^4 + b*x^2)^6*a^8 - 143/2*(c*x^4 + b*x^2)^5*a^9 + 143/4*(c
*x^4 + b*x^2)^4*a^10 - 13*(c*x^4 + b*x^2)^3*a^11 + 13/4*(c*x^4 + b*x^2)^2*a
^12 - 1/2*(c*x^4 + b*x^2)*a^13
```

Mupad [B] (verification not implemented)

Time = 9.46 (sec) , antiderivative size = 1214, normalized size of antiderivative = 60.70

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^{13}\,dx = \text{Too large to display}$$

```
[In] int(x*(b + 2*c*x^2)*(b*x^2 - a + c*x^4)^13,x)
```

```
[Out] x^24*((13*a^2*b^12)/4 + (429*a^8*c^6)/4 - 143*a^3*b^10*c + (6435*a^4*b^8*c^
2)/4 - 6006*a^5*b^6*c^3 + (15015*a^6*b^4*c^4)/2 - 2574*a^7*b^2*c^5) + x^32*
((429*a^6*c^8)/4 + (13*b^12*c^2)/4 - 143*a*b^10*c^3 + (6435*a^2*b^8*c^4)/4
- 6006*a^3*b^6*c^5 + (15015*a^4*b^4*c^6)/2 - 2574*a^5*b^2*c^7) - x^26*((a*b
^13)/2 - 39*a^2*b^11*c + 858*a^7*b*c^6 + 715*a^3*b^9*c^2 - 4290*a^4*b^7*c^3
 + 9009*a^5*b^5*c^4 - 6006*a^6*b^3*c^5) + x^30*((b^13*c)/2 - 39*a*b^11*c^2
+ 858*a^6*b*c^7 + 715*a^2*b^9*c^3 - 4290*a^3*b^7*c^4 + 9009*a^4*b^5*c^5 - 6
006*a^5*b^3*c^6) + x^12*((429*a^8*b^6)/4 - 13*a^11*c^3 - (715*a^9*b^4*c)/2
```

```
+ (429*a^10*b^2*c^2)/2) - x^44*(13*a^3*c^11 - (429*b^6*c^8)/4 + (715*a*b^4*
c^9)/2 - (429*a^2*b^2*c^10)/2) + x^20*((143*a^4*b^10)/4 - (143*a^9*c^5)/2 -
 (1287*a^5*b^8*c)/2 + 3003*a^6*b^6*c^2 - 4290*a^7*b^4*c^3 + (6435*a^8*b^2*c
^4)/4) - x^36*((143*a^5*c^9)/2 - (143*b^10*c^4)/4 + (1287*a*b^8*c^5)/2 - 30
03*a^2*b^6*c^6 + 4290*a^3*b^4*c^7 - (6435*a^4*b^2*c^8)/4) + x^28*(b^14/28 -
 (858*a^7*c^7)/7 + (429*a^2*b^10*c^2)/2 - 2145*a^3*b^8*c^3 + (15015*a^4*b^6
*c^4)/2 - 9009*a^5*b^4*c^5 + 3003*a^6*b^2*c^6 - (13*a*b^12*c)/2) + x^16*((4
29*a^6*b^8)/4 + (143*a^10*c^4)/4 - 858*a^7*b^6*c + (6435*a^8*b^4*c^2)/4 - 7
15*a^9*b^2*c^3) + x^40*((143*a^4*c^10)/4 + (429*b^8*c^6)/4 - 858*a*b^6*c^7
+ (6435*a^2*b^4*c^8)/4 - 715*a^3*b^2*c^9) + (c^14*x^56)/28 - x^4*((a^13*c)/
2 - (13*a^12*b^2)/4) + (13*a^10*x^8*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/4 + (1
3*c^10*x^48*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/4 - (a^13*b*x^2)/2 + (b*c^13*x
^54)/2 - (c^12*x^52*(2*a*c - 13*b^2))/4 - (143*a^7*b*x^14*(12*b^6 - 14*a^3*
c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/14 + (143*b*c^7*x^42*(12*b^6 - 14*a^3*c
^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/14 - (143*a^5*b*x^18*(b^8 + 5*a^4*c^4 +
36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/2 + (143*b*c^5*x^38*(b^8 + 5
*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/2 - (13*a^3*b*x^2
2*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 - 55*a*b^8*c))/2 + (13*b*c^3*x^34*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2
- 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/2 - (13*a^9*b*x^10*(11*b
^4 + 6*a^2*c^2 - 22*a*b^2*c))/2 + (13*b*c^9*x^46*(11*b^4 + 6*a^2*c^2 - 22*a
*b^2*c))/2 + (13*a^11*b*x^6*(a*c - 2*b^2))/2 - (13*b*c^11*x^50*(a*c - 2*b^2
))/2
```

3.99 $\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^{13}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 20

$$\int x^2\left(b+2cx^3\right)\left(-a+bx^3+cx^6\right)^{13}\,dx=\frac{1}{42}\left(a-bx^3-cx^6\right)^{14}$$

```
[Out] 1/42*(-c*x^6-b*x^3+a)^14
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1482, 643}

$$\int x^2\left(b+2cx^3\right)\left(-a+bx^3+cx^6\right)^{13}\,dx=\frac{1}{42}\left(a-bx^3-cx^6\right)^{14}$$

```
[In] Int[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (a - b*x^3 - c*x^6)^14/42
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
```

```
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx, x, x^3\right)\\ &= \frac{1}{42}\left(a-bx^3-cx^6\right)^{14}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 233 vs. $2(20) = 40$.

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 11.65

$$\begin{aligned}\int x^2\left(b+2cx^3\right)\left(-a+bx^3+cx^6\right)^{13}\,dx = \frac{1}{42}x^3\left(b+cx^3\right)\Big(&-14a^{13}+91a^{12}x^3\left(b+cx^3\right)\\ &-364a^{11}x^6\left(b+cx^3\right)^2+1001a^{10}x^9\left(b+cx^3\right)^3\\ &-2002a^9x^{12}\left(b+cx^3\right)^4+3003a^8x^{15}\left(b+cx^3\right)^5\\ &-3432a^7x^{18}\left(b+cx^3\right)^6+3003a^6x^{21}\left(b+cx^3\right)^7\\ &-2002a^5x^{24}\left(b+cx^3\right)^8+1001a^4x^{27}\left(b+cx^3\right)^9\\ &-364a^3x^{30}\left(b+cx^3\right)^{10}+91a^2x^{33}\left(b+cx^3\right)^{11}\\ &-14ax^{36}\left(b+cx^3\right)^{12}+x^{39}\left(b+cx^3\right)^{13}\Big)\end{aligned}$$

```
[In] Integrate[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^3*(b + c*x^3)*(-14*a^13 + 91*a^12*x^3*(b + c*x^3) - 364*a^11*x^6*(b + c*
x^3)^2 + 1001*a^10*x^9*(b + c*x^3)^3 - 2002*a^9*x^12*(b + c*x^3)^4 + 3003*a
^8*x^15*(b + c*x^3)^5 - 3432*a^7*x^18*(b + c*x^3)^6 + 3003*a^6*x^21*(b + c*
x^3)^7 - 2002*a^5*x^24*(b + c*x^3)^8 + 1001*a^4*x^27*(b + c*x^3)^9 - 364*a^
3*x^30*(b + c*x^3)^10 + 91*a^2*x^33*(b + c*x^3)^11 - 14*a*x^36*(b + c*x^3)^
12 + x^39*(b + c*x^3)^13))/42
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{(c\,x^6+b\,x^3-a)^{14}}{42}$	19
parallelrisch	Expression too large to display	1455
gosper	Expression too large to display	1457
risch	Expression too large to display	1460

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/42*(c*x^6+b*x^3-a)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 1242, normalized size of antiderivative = 62.10

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="fricas")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 - 2*a*c^13)*x^78 + 13/3
*(2*b^3*c^11 - a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^1
2)*x^72 + 13/3*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(33*
b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^66 + 143/21*(12*b
^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^63 + 143/6*(3*b^8
*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^60 + 14
3/3*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^57 + 143/6*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^51 + 13/6
*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c - 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 -
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 - 3432*a^7*c^7)*x^42 - 1/3*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^39 + 13/6*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^36 - 13/3*(2*a^3*b^
11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 9
9*a^8*b*c^5)*x^33 + 143/6*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^30 - 143/3*(a^5*b^9 - 12*a^6*b^7
*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^27 + 143/6*(3*a^6*b^8
```

```
 - 24*a^7*b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^24 - 143/21
*(12*a^7*b^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^21 + 13/6*(
33*a^8*b^6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)*x^18 - 1/3*a^13*
b*x^3 - 13/3*(11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^15 + 13/6*(11*a^
10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^12 - 13/3*(2*a^11*b^3 - a^12*b*c)*x^9
+ 1/6*(13*a^12*b^2 - 2*a^13*c)*x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1394 vs. $2(14) = 28$.

Time = 0.14 (sec) , antiderivative size = 1394, normalized size of antiderivative = 69.70

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**13,x)
```

```
[Out] -a**13*b*x**3/3 + b*c**13*x**81/3 + c**14*x**84/42 + x**78*(-a*c**13/3 + 13
*b**2*c**12/6) + x**75*(-13*a*b*c**12/3 + 26*b**3*c**11/3) + x**72*(13*a**2
*c**12/6 - 26*a*b**2*c**11 + 143*b**4*c**10/6) + x**69*(26*a**2*b*c**11 - 2
86*a*b**3*c**10/3 + 143*b**5*c**9/3) + x**66*(-26*a**3*c**11/3 + 143*a**2*b
**2*c**10 - 715*a*b**4*c**9/3 + 143*b**6*c**8/2) + x**63*(-286*a**3*b*c**10
/3 + 1430*a**2*b**3*c**9/3 - 429*a*b**5*c**8 + 572*b**7*c**7/7) + x**60*(14
3*a**4*c**10/6 - 1430*a**3*b**2*c**9/3 + 2145*a**2*b**4*c**8/2 - 572*a*b**6
*c**7 + 143*b**8*c**6/2) + x**57*(715*a**4*b*c**9/3 - 1430*a**3*b**3*c**8 +
 1716*a**2*b**5*c**7 - 572*a*b**7*c**6 + 143*b**9*c**5/3) + x**54*(-143*a**
5*c**9/3 + 2145*a**4*b**2*c**8/2 - 2860*a**3*b**4*c**7 + 2002*a**2*b**6*c**
6 - 429*a*b**8*c**5 + 143*b**10*c**4/6) + x**51*(-429*a**5*b*c**8 + 2860*a*
*4*b**3*c**7 - 4004*a**3*b**5*c**6 + 1716*a**2*b**7*c**5 - 715*a*b**9*c**4/
3 + 26*b**11*c**3/3) + x**48*(143*a**6*c**8/2 - 1716*a**5*b**2*c**7 + 5005*
a**4*b**4*c**6 - 4004*a**3*b**6*c**5 + 2145*a**2*b**8*c**4/2 - 286*a*b**10*
c**3/3 + 13*b**12*c**2/6) + x**45*(572*a**6*b*c**7 - 4004*a**5*b**3*c**6 +
6006*a**4*b**5*c**5 - 2860*a**3*b**7*c**4 + 1430*a**2*b**9*c**3/3 - 26*a*b*
*11*c**2 + b**13*c/3) + x**42*(-572*a**7*c**7/7 + 2002*a**6*b**2*c**6 - 600
6*a**5*b**4*c**5 + 5005*a**4*b**6*c**4 - 1430*a**3*b**8*c**3 + 143*a**2*b**
10*c**2 - 13*a*b**12*c/3 + b**14/42) + x**39*(-572*a**7*b*c**6 + 4004*a**6*
b**3*c**5 - 6006*a**5*b**5*c**4 + 2860*a**4*b**7*c**3 - 1430*a**3*b**9*c**2
/3 + 26*a**2*b**11*c - a*b**13/3) + x**36*(143*a**8*c**6/2 - 1716*a**7*b**2
*c**5 + 5005*a**6*b**4*c**4 - 4004*a**5*b**6*c**3 + 2145*a**4*b**8*c**2/2 -
 286*a**3*b**10*c/3 + 13*a**2*b**12/6) + x**33*(429*a**8*b*c**5 - 2860*a**7
*b**3*c**4 + 4004*a**6*b**5*c**3 - 1716*a**5*b**7*c**2 + 715*a**4*b**9*c/3
- 26*a**3*b**11/3) + x**30*(-143*a**9*c**5/3 + 2145*a**8*b**2*c**4/2 - 2860
*a**7*b**4*c**3 + 2002*a**6*b**6*c**2 - 429*a**5*b**8*c + 143*a**4*b**10/6)
 + x**27*(-715*a**9*b*c**4/3 + 1430*a**8*b**3*c**3 - 1716*a**7*b**5*c**2 +
572*a**6*b**7*c - 143*a**5*b**9/3) + x**24*(143*a**10*c**4/6 - 1430*a**9*b*
```

```
*2*c**3/3 + 2145*a**8*b**4*c**2/2 - 572*a**7*b**6*c + 143*a**6*b**8/2) + x*
*21*(286*a**10*b*c**3/3 - 1430*a**9*b**3*c**2/3 + 429*a**8*b**5*c - 572*a**
7*b**7/7) + x**18*(-26*a**11*c**3/3 + 143*a**10*b**2*c**2 - 715*a**9*b**4*c
/3 + 143*a**8*b**6/2) + x**15*(-26*a**11*b*c**2 + 286*a**10*b**3*c/3 - 143*
a**9*b**5/3) + x**12*(13*a**12*c**2/6 - 26*a**11*b**2*c + 143*a**10*b**4/6)
 + x**9*(13*a**12*b*c/3 - 26*a**11*b**3/3) + x**6*(-a**13*c/3 + 13*a**12*b*
*2/6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. 2(18) = 36.

Time = 0.21 (sec) , antiderivative size = 1242, normalized size of antiderivative = 62.10

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 - 2*a*c^13)*x^78 + 13/3
*(2*b^3*c^11 - a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^1
2)*x^72 + 13/3*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(33*
b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^66 + 143/21*(12*b
^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^63 + 143/6*(3*b^8
*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^60 + 14
3/3*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^57 + 143/6*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^51 + 13/6
*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c - 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 -
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 - 3432*a^7*c^7)*x^42 - 1/3*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^39 + 13/6*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^36 - 13/3*(2*a^3*b^
11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 9
9*a^8*b*c^5)*x^33 + 143/6*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^30 - 143/3*(a^5*b^9 - 12*a^6*b^7
*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^27 + 143/6*(3*a^6*b^8
 - 24*a^7*b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^24 - 143/21
*(12*a^7*b^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^21 + 13/6*(
33*a^8*b^6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)*x^18 - 1/3*a^13*
b*x^3 - 13/3*(11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^15 + 13/6*(11*a^
```

```
10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^12 - 13/3*(2*a^11*b^3 - a^12*b*c)*x^9
+ 1/6*(13*a^12*b^2 - 2*a^13*c)*x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(18) = 36$.

Time = 0.32 (sec) , antiderivative size = 246, normalized size of antiderivative = 12.30

$$\begin{aligned}\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^{13}\,dx &= \frac{1}{42}\left(cx^6+bx^3\right)^{14}-\frac{1}{3}\left(cx^6+bx^3\right)^{13}a\\&\quad+\frac{13}{6}\left(cx^6+bx^3\right)^{12}a^2-\frac{26}{3}\left(cx^6+bx^3\right)^{11}a^3\\&\quad+\frac{143}{6}\left(cx^6+bx^3\right)^{10}a^4-\frac{143}{3}\left(cx^6+bx^3\right)^{9}a^5\\&\quad+\frac{143}{2}\left(cx^6+bx^3\right)^{8}a^6-\frac{572}{7}\left(cx^6+bx^3\right)^{7}a^7\\&\quad+\frac{143}{2}\left(cx^6+bx^3\right)^{6}a^8-\frac{143}{3}\left(cx^6+bx^3\right)^{5}a^9\\&\quad+\frac{143}{6}\left(cx^6+bx^3\right)^{4}a^{10}-\frac{26}{3}\left(cx^6+bx^3\right)^{3}a^{11}\\&\quad+\frac{13}{6}\left(cx^6+bx^3\right)^{2}a^{12}-\frac{1}{3}\left(cx^6+bx^3\right)a^{13}\end{aligned}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="giac")
```

```
[Out] 1/42*(c*x^6 + b*x^3)^14 - 1/3*(c*x^6 + b*x^3)^13*a + 13/6*(c*x^6 + b*x^3)^1
2*a^2 - 26/3*(c*x^6 + b*x^3)^11*a^3 + 143/6*(c*x^6 + b*x^3)^10*a^4 - 143/3*
(c*x^6 + b*x^3)^9*a^5 + 143/2*(c*x^6 + b*x^3)^8*a^6 - 572/7*(c*x^6 + b*x^3)
^7*a^7 + 143/2*(c*x^6 + b*x^3)^6*a^8 - 143/3*(c*x^6 + b*x^3)^5*a^9 + 143/6*
(c*x^6 + b*x^3)^4*a^10 - 26/3*(c*x^6 + b*x^3)^3*a^11 + 13/6*(c*x^6 + b*x^3)
^2*a^12 - 1/3*(c*x^6 + b*x^3)*a^13
```

Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 1214, normalized size of antiderivative = 60.70

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^{13}\,dx = \text{Too large to display}$$

```
[In] int(x^2*(b + 2*c*x^3)*(b*x^3 - a + c*x^6)^13,x)
```

```
[Out] x^36*((13*a^2*b^12)/6 + (143*a^8*c^6)/2 - (286*a^3*b^10*c)/3 + (2145*a^4*b^
8*c^2)/2 - 4004*a^5*b^6*c^3 + 5005*a^6*b^4*c^4 - 1716*a^7*b^2*c^5) + x^48*(
(143*a^6*c^8)/2 + (13*b^12*c^2)/6 - (286*a*b^10*c^3)/3 + (2145*a^2*b^8*c^4)
/2 - 4004*a^3*b^6*c^5 + 5005*a^4*b^4*c^6 - 1716*a^5*b^2*c^7) - x^39*((a*b^1
3)/3 - 26*a^2*b^11*c + 572*a^7*b*c^6 + (1430*a^3*b^9*c^2)/3 - 2860*a^4*b^7*
```

```
c^3 + 6006*a^5*b^5*c^4 - 4004*a^6*b^3*c^5) + x^45*((b^13*c)/3 - 26*a*b^11*c
^2 + 572*a^6*b*c^7 + (1430*a^2*b^9*c^3)/3 - 2860*a^3*b^7*c^4 + 6006*a^4*b^5
*c^5 - 4004*a^5*b^3*c^6) + x^18*((143*a^8*b^6)/2 - (26*a^11*c^3)/3 - (715*a
^9*b^4*c)/3 + 143*a^10*b^2*c^2) - x^66*((26*a^3*c^11)/3 - (143*b^6*c^8)/2 +
 (715*a*b^4*c^9)/3 - 143*a^2*b^2*c^10) + x^30*((143*a^4*b^10)/6 - (143*a^9*
c^5)/3 - 429*a^5*b^8*c + 2002*a^6*b^6*c^2 - 2860*a^7*b^4*c^3 + (2145*a^8*b^
2*c^4)/2) - x^54*((143*a^5*c^9)/3 - (143*b^10*c^4)/6 + 429*a*b^8*c^5 - 2002
*a^2*b^6*c^6 + 2860*a^3*b^4*c^7 - (2145*a^4*b^2*c^8)/2) + x^42*(b^14/42 - (
572*a^7*c^7)/7 + 143*a^2*b^10*c^2 - 1430*a^3*b^8*c^3 + 5005*a^4*b^6*c^4 - 6
006*a^5*b^4*c^5 + 2002*a^6*b^2*c^6 - (13*a*b^12*c)/3) + x^24*((143*a^6*b^8)
/2 + (143*a^10*c^4)/6 - 572*a^7*b^6*c + (2145*a^8*b^4*c^2)/2 - (1430*a^9*b^
2*c^3)/3) + x^60*((143*a^4*c^10)/6 + (143*b^8*c^6)/2 - 572*a*b^6*c^7 + (214
5*a^2*b^4*c^8)/2 - (1430*a^3*b^2*c^9)/3) + (c^14*x^84)/42 - x^6*((a^13*c)/3
 - (13*a^12*b^2)/6) + (13*a^10*x^12*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/6 + (1
3*c^10*x^72*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/6 - (a^13*b*x^3)/3 + (b*c^13*x
^81)/3 - (c^12*x^78*(2*a*c - 13*b^2))/6 - (143*a^7*b*x^21*(12*b^6 - 14*a^3*
c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/21 + (143*b*c^7*x^63*(12*b^6 - 14*a^3*c
^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/21 - (143*a^5*b*x^27*(b^8 + 5*a^4*c^4 +
36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/3 + (143*b*c^5*x^57*(b^8 + 5
*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/3 - (13*a^3*b*x^3
3*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 - 55*a*b^8*c))/3 + (13*b*c^3*x^51*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2
- 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/3 - (13*a^9*b*x^15*(11*b
^4 + 6*a^2*c^2 - 22*a*b^2*c))/3 + (13*b*c^9*x^69*(11*b^4 + 6*a^2*c^2 - 22*a
*b^2*c))/3 + (13*a^11*b*x^9*(a*c - 2*b^2))/3 - (13*b*c^11*x^75*(a*c - 2*b^2
))/3
```

3.100 $\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 25

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx=\frac{(a-bx^n-cx^{2n})^{14}}{14n}$$

```
[Out] 1/14*(a-b*x^n-c*x^(2*n))^14/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1482, 643}

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx=\frac{(a-bx^n-cx^{2n})^{14}}{14n}$$

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (a - b*x^n - c*x^(2*n))^14/(14*n)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
```

```
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx,x,x^n\right)}{n}$$
$$= \frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx = \frac{(-a+x^n(b+cx^n))^{14}}{14n}$$

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]
[Out] (-a + x^n*(b + c*x^n))^14/(14*n)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2045 vs. $2(23) = 46$.

Time = 0.02 (sec) , antiderivative size = 2046, normalized size of antiderivative = 81.84

Expression too large to display

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x)
[Out] 286*a^10*b/n*(x^n)^7*c^3-1430*a^9*b^3/n*(x^n)^7*c^2+1287*a^8*b^5/n*(x^n)^7*
c+429*a^10/n*(x^n)^6*b^2*c^2+429/2*a^6/n*(x^n)^8*b^8+143/2*a^10/n*(x^n)^8*c
^4-26*a^11*b^3/n*(x^n)^3-c^13/n*(x^n)^26*a+13/2*c^12/n*(x^n)^26*b^2+13/2*a^
12/n*(x^n)^2*b^2+13/2*c^12/n*(x^n)^24*a^2+143/2*c^10/n*(x^n)^24*b^4+143/2*c
^10/n*(x^n)^20*a^4+429/2*c^6/n*(x^n)^20*b^8-26*a^11/n*(x^n)^6*c^3+429/2*a^8
/n*(x^n)^6*b^6+13/2*a^12/n*(x^n)^4*c^2+143/2*a^10/n*(x^n)^4*b^4+1/14*c^14/n
*(x^n)^28+1/14/n*(x^n)^14*b^14+429/2*a^8/n*(x^n)^12*c^6+13/2*a^2/n*(x^n)^12
*b^12-143*a^9/n*(x^n)^10*c^5+143/2*a^4/n*(x^n)^10*b^10-a^13/n*(x^n)^2*c+715
*b*c^9/n*(x^n)^19*a^4-4290*b^3*c^8/n*(x^n)^19*a^3+5148*b^5*c^7/n*(x^n)^19*a
^2-1716*b^7*c^6/n*(x^n)^19*a-1716/7/n*(x^n)^14*a^7*c^7-143*a^5*b^9/n*(x^n)^
9+b^13*c/n*(x^n)^15+26*c^11*b^3/n*(x^n)^25-143*c^9/n*(x^n)^18*a^5+143/2*c^4
/n*(x^n)^18*b^10+1716/7*b^7*c^7/n*(x^n)^21-26*c^11/n*(x^n)^22*a^3+429/2*c^8
/n*(x^n)^22*b^6-a*b^13/n*(x^n)^13-b*a^13/n*x^n-5148*a^7/n*(x^n)^12*b^2*c^5+
15015*a^6/n*(x^n)^12*b^4*c^4-12012*a^5/n*(x^n)^12*b^6*c^3+6435/2*a^4/n*(x^n
)^12*b^8*c^2-286*a^3/n*(x^n)^12*b^10*c-1716/7*a^7*b^7/n*(x^n)^7+143*c^9*b^5
/n*(x^n)^23+429/2*c^8/n*(x^n)^16*a^6+13/2*c^2/n*(x^n)^16*b^12+b*c^13/n*(x^n
```

```
)^27+26*b^11*c^3/n*(x^n)^17-26*a^3*b^11/n*(x^n)^11+143*b^9*c^5/n*(x^n)^19-1
43*a^9*b^5/n*(x^n)^5+6435/2*a^8/n*(x^n)^10*b^2*c^4-8580*a^7/n*(x^n)^10*b^4*
c^3+6006*a^6/n*(x^n)^10*b^6*c^2-1287*a^5/n*(x^n)^10*b^8*c+6006/n*(x^n)^14*a
^6*b^2*c^6-18018/n*(x^n)^14*a^5*b^4*c^5+15015/n*(x^n)^14*a^4*b^6*c^4-4290/n
*(x^n)^14*a^3*b^8*c^3+429/n*(x^n)^14*a^2*b^10*c^2-13/n*(x^n)^14*a*b^12*c-14
30*c^9/n*(x^n)^20*a^3*b^2+1716*b*c^7/n*(x^n)^15*a^6-12012*b^3*c^6/n*(x^n)^1
5*a^5+18018*b^5*c^5/n*(x^n)^15*a^4-8580*b^7*c^4/n*(x^n)^15*a^3+1430*b^9*c^3
/n*(x^n)^15*a^2-78*b^11*c^2/n*(x^n)^15*a+1430*b^3*c^9/n*(x^n)^21*a^2-1287*b
^5*c^8/n*(x^n)^21*a+78*c^11*b/n*(x^n)^23*a^2-286*c^10*b^3/n*(x^n)^23*a+5148
*b^7*c^5/n*(x^n)^17*a^2-715*b^9*c^4/n*(x^n)^17*a+1287*a^8*b/n*(x^n)^11*c^5-
8580*a^7*b^3/n*(x^n)^11*c^4+12012*a^6*b^5/n*(x^n)^11*c^3-5148*a^5*b^7/n*(x^
n)^11*c^2+715*a^4*b^9/n*(x^n)^11*c+6435/2*c^8/n*(x^n)^20*a^2*b^4-1716*c^7/n
*(x^n)^20*a*b^6+6435/2*c^8/n*(x^n)^18*a^4*b^2-8580*c^7/n*(x^n)^18*a^3*b^4+6
006*c^6/n*(x^n)^18*a^2*b^6-1287*c^5/n*(x^n)^18*a*b^8-286*b*c^10/n*(x^n)^21*
a^3-78*a^11*b/n*(x^n)^5*c^2+286*a^10*b^3/n*(x^n)^5*c+13*a^12*b/n*(x^n)^3*c-
715*a^9*b/n*(x^n)^9*c^4+4290*a^8*b^3/n*(x^n)^9*c^3-5148*a^7*b^5/n*(x^n)^9*c
^2+1716*a^6*b^7/n*(x^n)^9*c-1430*a^9/n*(x^n)^8*b^2*c^3+6435/2*a^8/n*(x^n)^8
*b^4*c^2-1716*a^7/n*(x^n)^8*b^6*c-78*c^11/n*(x^n)^24*a*b^2+429*c^10/n*(x^n)
^22*a^2*b^2-715*c^9/n*(x^n)^22*a*b^4-715*a^9/n*(x^n)^6*b^4*c-78*a^11/n*(x^n
)^4*b^2*c-13*c^12*b/n*(x^n)^25*a-1287*b*c^8/n*(x^n)^17*a^5+8580*b^3*c^7/n*(
x^n)^17*a^4-12012*b^5*c^6/n*(x^n)^17*a^3-1716*a^7*b/n*(x^n)^13*c^6+12012*a^
6*b^3/n*(x^n)^13*c^5-18018*a^5*b^5/n*(x^n)^13*c^4+8580*a^4*b^7/n*(x^n)^13*c
^3-1430*a^3*b^9/n*(x^n)^13*c^2+78*a^2*b^11/n*(x^n)^13*c-5148*c^7/n*(x^n)^16
*a^5*b^2+15015*c^6/n*(x^n)^16*a^4*b^4-12012*c^5/n*(x^n)^16*a^3*b^6+6435/2*c
^4/n*(x^n)^16*a^2*b^8-286*c^3/n*(x^n)^16*a*b^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. $2(23) = 46$.

Time = 0.33 (sec) , antiderivative size = 1299, normalized size of antiderivative = 51.96

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="fricas
")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) - 14*a^13*b*x^n + 7*(13*b^2*c^12 -
 2*a*c^13)*x^(26*n) + 182*(2*b^3*c^11 - a*b*c^12)*x^(25*n) + 91*(11*b^4*c^1
0 - 12*a*b^2*c^11 + a^2*c^12)*x^(24*n) + 182*(11*b^5*c^9 - 22*a*b^3*c^10 +
6*a^2*b*c^11)*x^(23*n) + 91*(33*b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 -
 4*a^3*c^11)*x^(22*n) + 286*(12*b^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 1
4*a^3*b*c^10)*x^(21*n) + 1001*(3*b^8*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 -
20*a^3*b^2*c^9 + a^4*c^10)*x^(20*n) + 2002*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2
*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^(19*n) + 1001*(b^10*c^4 - 18*a*b
```

```
^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7 + 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^(
18*n) + 182*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*a^2*b^7*c^5 - 924*a^3*b^5*c^6
+ 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^(17*n) + 91*(b^12*c^2 - 44*a*b^10*c^3 +
 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 - 792*a^5*b^2*c^7 +
33*a^6*c^8)*x^(16*n) + 14*(b^13*c - 78*a*b^11*c^2 + 1430*a^2*b^9*c^3 - 8580
*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^(1
5*n) + (b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 - 60060*a^3*b^8*c^3 + 21021
0*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6 - 3432*a^7*c^7)*x^(1
4*n) - 14*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 1
8018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^(13*n) + 91*(a^2*b
^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4
 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^(12*n) - 182*(2*a^3*b^11 - 55*a^4*b^9*c
+ 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 99*a^8*b*c^5)*x^(11
*n) + 1001*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a^7*b^4*c^3 + 45
*a^8*b^2*c^4 - 2*a^9*c^5)*x^(10*n) - 2002*(a^5*b^9 - 12*a^6*b^7*c + 36*a^7*
b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^(9*n) + 1001*(3*a^6*b^8 - 24*a^7*
b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^(8*n) - 286*(12*a^7*b
^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^(7*n) + 91*(33*a^8*b^
6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)*x^(6*n) - 182*(11*a^9*b^5
 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^(5*n) + 91*(11*a^10*b^4 - 12*a^11*b^2*c
+ a^12*c^2)*x^(4*n) - 182*(2*a^11*b^3 - a^12*b*c)*x^(3*n) + 7*(13*a^12*b^2
- 2*a^13*c)*x^(2*n))/n
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx=\text{Timed out}$$

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**13,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2045 vs. $2(23) = 46$.

Time = 0.27 (sec) , antiderivative size = 2045, normalized size of antiderivative = 81.80

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx=\text{Too large to display}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="maxima
")
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n - a*c^1
3*x^(26*n)/n + 26*b^3*c^11*x^(25*n)/n - 13*a*b*c^12*x^(25*n)/n + 143/2*b^4*
c^10*x^(24*n)/n - 78*a*b^2*c^11*x^(24*n)/n + 13/2*a^2*c^12*x^(24*n)/n + 143
*b^5*c^9*x^(23*n)/n - 286*a*b^3*c^10*x^(23*n)/n + 78*a^2*b*c^11*x^(23*n)/n
+ 429/2*b^6*c^8*x^(22*n)/n - 715*a*b^4*c^9*x^(22*n)/n + 429*a^2*b^2*c^10*x^
(22*n)/n - 26*a^3*c^11*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n - 1287*a*b^5*
c^8*x^(21*n)/n + 1430*a^2*b^3*c^9*x^(21*n)/n - 286*a^3*b*c^10*x^(21*n)/n +
429/2*b^8*c^6*x^(20*n)/n - 1716*a*b^6*c^7*x^(20*n)/n + 6435/2*a^2*b^4*c^8*x
^(20*n)/n - 1430*a^3*b^2*c^9*x^(20*n)/n + 143/2*a^4*c^10*x^(20*n)/n + 143*b
^9*c^5*x^(19*n)/n - 1716*a*b^7*c^6*x^(19*n)/n + 5148*a^2*b^5*c^7*x^(19*n)/n
 - 4290*a^3*b^3*c^8*x^(19*n)/n + 715*a^4*b*c^9*x^(19*n)/n + 143/2*b^10*c^4*
x^(18*n)/n - 1287*a*b^8*c^5*x^(18*n)/n + 6006*a^2*b^6*c^6*x^(18*n)/n - 8580
*a^3*b^4*c^7*x^(18*n)/n + 6435/2*a^4*b^2*c^8*x^(18*n)/n - 143*a^5*c^9*x^(18
*n)/n + 26*b^11*c^3*x^(17*n)/n - 715*a*b^9*c^4*x^(17*n)/n + 5148*a^2*b^7*c^
5*x^(17*n)/n - 12012*a^3*b^5*c^6*x^(17*n)/n + 8580*a^4*b^3*c^7*x^(17*n)/n -
 1287*a^5*b*c^8*x^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n - 286*a*b^10*c^3*x^(1
6*n)/n + 6435/2*a^2*b^8*c^4*x^(16*n)/n - 12012*a^3*b^6*c^5*x^(16*n)/n + 150
15*a^4*b^4*c^6*x^(16*n)/n - 5148*a^5*b^2*c^7*x^(16*n)/n + 429/2*a^6*c^8*x^(
16*n)/n + b^13*c*x^(15*n)/n - 78*a*b^11*c^2*x^(15*n)/n + 1430*a^2*b^9*c^3*x
^(15*n)/n - 8580*a^3*b^7*c^4*x^(15*n)/n + 18018*a^4*b^5*c^5*x^(15*n)/n - 12
012*a^5*b^3*c^6*x^(15*n)/n + 1716*a^6*b*c^7*x^(15*n)/n + 1/14*b^14*x^(14*n)
/n - 13*a*b^12*c*x^(14*n)/n + 429*a^2*b^10*c^2*x^(14*n)/n - 4290*a^3*b^8*c^
3*x^(14*n)/n + 15015*a^4*b^6*c^4*x^(14*n)/n - 18018*a^5*b^4*c^5*x^(14*n)/n
+ 6006*a^6*b^2*c^6*x^(14*n)/n - 1716/7*a^7*c^7*x^(14*n)/n - a*b^13*x^(13*n)
/n + 78*a^2*b^11*c*x^(13*n)/n - 1430*a^3*b^9*c^2*x^(13*n)/n + 8580*a^4*b^7*
c^3*x^(13*n)/n - 18018*a^5*b^5*c^4*x^(13*n)/n + 12012*a^6*b^3*c^5*x^(13*n)/
n - 1716*a^7*b*c^6*x^(13*n)/n + 13/2*a^2*b^12*x^(12*n)/n - 286*a^3*b^10*c*x
^(12*n)/n + 6435/2*a^4*b^8*c^2*x^(12*n)/n - 12012*a^5*b^6*c^3*x^(12*n)/n +
15015*a^6*b^4*c^4*x^(12*n)/n - 5148*a^7*b^2*c^5*x^(12*n)/n + 429/2*a^8*c^6*
x^(12*n)/n - 26*a^3*b^11*x^(11*n)/n + 715*a^4*b^9*c*x^(11*n)/n - 5148*a^5*b
^7*c^2*x^(11*n)/n + 12012*a^6*b^5*c^3*x^(11*n)/n - 8580*a^7*b^3*c^4*x^(11*n
)/n + 1287*a^8*b*c^5*x^(11*n)/n + 143/2*a^4*b^10*x^(10*n)/n - 1287*a^5*b^8*
c*x^(10*n)/n + 6006*a^6*b^6*c^2*x^(10*n)/n - 8580*a^7*b^4*c^3*x^(10*n)/n +
6435/2*a^8*b^2*c^4*x^(10*n)/n - 143*a^9*c^5*x^(10*n)/n - 143*a^5*b^9*x^(9*n
)/n + 1716*a^6*b^7*c*x^(9*n)/n - 5148*a^7*b^5*c^2*x^(9*n)/n + 4290*a^8*b^3*
c^3*x^(9*n)/n - 715*a^9*b*c^4*x^(9*n)/n + 429/2*a^6*b^8*x^(8*n)/n - 1716*a^
7*b^6*c*x^(8*n)/n + 6435/2*a^8*b^4*c^2*x^(8*n)/n - 1430*a^9*b^2*c^3*x^(8*n)
/n + 143/2*a^10*c^4*x^(8*n)/n - 1716/7*a^7*b^7*x^(7*n)/n + 1287*a^8*b^5*c*x
^(7*n)/n - 1430*a^9*b^3*c^2*x^(7*n)/n + 286*a^10*b*c^3*x^(7*n)/n + 429/2*a^
8*b^6*x^(6*n)/n - 715*a^9*b^4*c*x^(6*n)/n + 429*a^10*b^2*c^2*x^(6*n)/n - 26
*a^11*c^3*x^(6*n)/n - 143*a^9*b^5*x^(5*n)/n + 286*a^10*b^3*c*x^(5*n)/n - 78
*a^11*b*c^2*x^(5*n)/n + 143/2*a^10*b^4*x^(4*n)/n - 78*a^11*b^2*c*x^(4*n)/n
+ 13/2*a^12*c^2*x^(4*n)/n - 26*a^11*b^3*x^(3*n)/n + 13*a^12*b*c*x^(3*n)/n +
 13/2*a^12*b^2*x^(2*n)/n - a^13*c*x^(2*n)/n - a^13*b*x^n/n
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1693 vs. $2(23) = 46$.

Time = 0.37 (sec) , antiderivative size = 1693, normalized size of antiderivative = 67.72

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx = \text{Too large to display}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) - 14*a*c^13
*x^(26*n) + 364*b^3*c^11*x^(25*n) - 182*a*b*c^12*x^(25*n) + 1001*b^4*c^10*x
^(24*n) - 1092*a*b^2*c^11*x^(24*n) + 91*a^2*c^12*x^(24*n) + 2002*b^5*c^9*x^
(23*n) - 4004*a*b^3*c^10*x^(23*n) + 1092*a^2*b*c^11*x^(23*n) + 3003*b^6*c^8
*x^(22*n) - 10010*a*b^4*c^9*x^(22*n) + 6006*a^2*b^2*c^10*x^(22*n) - 364*a^3
*c^11*x^(22*n) + 3432*b^7*c^7*x^(21*n) - 18018*a*b^5*c^8*x^(21*n) + 20020*a
^2*b^3*c^9*x^(21*n) - 4004*a^3*b*c^10*x^(21*n) + 3003*b^8*c^6*x^(20*n) - 24
024*a*b^6*c^7*x^(20*n) + 45045*a^2*b^4*c^8*x^(20*n) - 20020*a^3*b^2*c^9*x^(
20*n) + 1001*a^4*c^10*x^(20*n) + 2002*b^9*c^5*x^(19*n) - 24024*a*b^7*c^6*x^
(19*n) + 72072*a^2*b^5*c^7*x^(19*n) - 60060*a^3*b^3*c^8*x^(19*n) + 10010*a^
4*b*c^9*x^(19*n) + 1001*b^10*c^4*x^(18*n) - 18018*a*b^8*c^5*x^(18*n) + 8408
4*a^2*b^6*c^6*x^(18*n) - 120120*a^3*b^4*c^7*x^(18*n) + 45045*a^4*b^2*c^8*x^
(18*n) - 2002*a^5*c^9*x^(18*n) + 364*b^11*c^3*x^(17*n) - 10010*a*b^9*c^4*x^
(17*n) + 72072*a^2*b^7*c^5*x^(17*n) - 168168*a^3*b^5*c^6*x^(17*n) + 120120*
a^4*b^3*c^7*x^(17*n) - 18018*a^5*b*c^8*x^(17*n) + 91*b^12*c^2*x^(16*n) - 40
04*a*b^10*c^3*x^(16*n) + 45045*a^2*b^8*c^4*x^(16*n) - 168168*a^3*b^6*c^5*x^
(16*n) + 210210*a^4*b^4*c^6*x^(16*n) - 72072*a^5*b^2*c^7*x^(16*n) + 3003*a^
6*c^8*x^(16*n) + 14*b^13*c*x^(15*n) - 1092*a*b^11*c^2*x^(15*n) + 20020*a^2*
b^9*c^3*x^(15*n) - 120120*a^3*b^7*c^4*x^(15*n) + 252252*a^4*b^5*c^5*x^(15*n
) - 168168*a^5*b^3*c^6*x^(15*n) + 24024*a^6*b*c^7*x^(15*n) + b^14*x^(14*n)
- 182*a*b^12*c*x^(14*n) + 6006*a^2*b^10*c^2*x^(14*n) - 60060*a^3*b^8*c^3*x^
(14*n) + 210210*a^4*b^6*c^4*x^(14*n) - 252252*a^5*b^4*c^5*x^(14*n) + 84084*
a^6*b^2*c^6*x^(14*n) - 3432*a^7*c^7*x^(14*n) - 14*a*b^13*x^(13*n) + 1092*a^
2*b^11*c*x^(13*n) - 20020*a^3*b^9*c^2*x^(13*n) + 120120*a^4*b^7*c^3*x^(13*n
) - 252252*a^5*b^5*c^4*x^(13*n) + 168168*a^6*b^3*c^5*x^(13*n) - 24024*a^7*b
*c^6*x^(13*n) + 91*a^2*b^12*x^(12*n) - 4004*a^3*b^10*c*x^(12*n) + 45045*a^4
*b^8*c^2*x^(12*n) - 168168*a^5*b^6*c^3*x^(12*n) + 210210*a^6*b^4*c^4*x^(12*
n) - 72072*a^7*b^2*c^5*x^(12*n) + 3003*a^8*c^6*x^(12*n) - 364*a^3*b^11*x^(1
1*n) + 10010*a^4*b^9*c*x^(11*n) - 72072*a^5*b^7*c^2*x^(11*n) + 168168*a^6*b
^5*c^3*x^(11*n) - 120120*a^7*b^3*c^4*x^(11*n) + 18018*a^8*b*c^5*x^(11*n) +
1001*a^4*b^10*x^(10*n) - 18018*a^5*b^8*c*x^(10*n) + 84084*a^6*b^6*c^2*x^(10
*n) - 120120*a^7*b^4*c^3*x^(10*n) + 45045*a^8*b^2*c^4*x^(10*n) - 2002*a^9*c
^5*x^(10*n) - 2002*a^5*b^9*x^(9*n) + 24024*a^6*b^7*c*x^(9*n) - 72072*a^7*b^
5*c^2*x^(9*n) + 60060*a^8*b^3*c^3*x^(9*n) - 10010*a^9*b*c^4*x^(9*n) + 3003*
a^6*b^8*x^(8*n) - 24024*a^7*b^6*c*x^(8*n) + 45045*a^8*b^4*c^2*x^(8*n) - 200
```

```
20*a^9*b^2*c^3*x^(8*n) + 1001*a^10*c^4*x^(8*n) - 3432*a^7*b^7*x^(7*n) + 180
18*a^8*b^5*c*x^(7*n) - 20020*a^9*b^3*c^2*x^(7*n) + 4004*a^10*b*c^3*x^(7*n)
+ 3003*a^8*b^6*x^(6*n) - 10010*a^9*b^4*c*x^(6*n) + 6006*a^10*b^2*c^2*x^(6*n
) - 364*a^11*c^3*x^(6*n) - 2002*a^9*b^5*x^(5*n) + 4004*a^10*b^3*c*x^(5*n) -
 1092*a^11*b*c^2*x^(5*n) + 1001*a^10*b^4*x^(4*n) - 1092*a^11*b^2*c*x^(4*n)
+ 91*a^12*c^2*x^(4*n) - 364*a^11*b^3*x^(3*n) + 182*a^12*b*c*x^(3*n) + 91*a^
12*b^2*x^(2*n) - 14*a^13*c*x^(2*n) - 14*a^13*b*x^n)/n
```

Mupad [B] (verification not implemented)

Time = 11.07 (sec) , antiderivative size = 1401, normalized size of antiderivative = 56.04

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx = \text{Too large to display}$$

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n - a + c*x^(2*n))^13,x)
```

```
[Out] x^(n - 1)*((x^(11*n + 1)*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 - 286*a^3*b^10*
c + (6435*a^4*b^8*c^2)/2 - 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 - 5148*a^7
*b^2*c^5))/n + (x^(15*n + 1)*((429*a^6*c^8)/2 + (13*b^12*c^2)/2 - 286*a*b^1
0*c^3 + (6435*a^2*b^8*c^4)/2 - 12012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 - 5148
*a^5*b^2*c^7))/n - (x^(12*n + 1)*(a*b^13 - 78*a^2*b^11*c + 1716*a^7*b*c^6 +
 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^
5))/n + (x^(14*n + 1)*(b^13*c - 78*a*b^11*c^2 + 1716*a^6*b*c^7 + 1430*a^2*b
^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c^6))/n + (x^
(5*n + 1)*((429*a^8*b^6)/2 - 26*a^11*c^3 - 715*a^9*b^4*c + 429*a^10*b^2*c^2
))/n - (x^(21*n + 1)*(26*a^3*c^11 - (429*b^6*c^8)/2 + 715*a*b^4*c^9 - 429*a
^2*b^2*c^10))/n + (x^(9*n + 1)*((143*a^4*b^10)/2 - 143*a^9*c^5 - 1287*a^5*b
^8*c + 6006*a^6*b^6*c^2 - 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2))/n - (x^
(17*n + 1)*(143*a^5*c^9 - (143*b^10*c^4)/2 + 1287*a*b^8*c^5 - 6006*a^2*b^6*
c^6 + 8580*a^3*b^4*c^7 - (6435*a^4*b^2*c^8)/2))/n + (x^(13*n + 1)*(b^14/14
- (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 - 4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^
4 - 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6 - 13*a*b^12*c))/n + (x^(7*n + 1)*(
(429*a^6*b^8)/2 + (143*a^10*c^4)/2 - 1716*a^7*b^6*c + (6435*a^8*b^4*c^2)/2
- 1430*a^9*b^2*c^3))/n + (x^(19*n + 1)*((143*a^4*c^10)/2 + (429*b^8*c^6)/2
- 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 - 1430*a^3*b^2*c^9))/n + (c^14*x^(2
7*n + 1))/(14*n) - (a^12*x^(n + 1)*(a*c - (13*b^2)/2))/n + (a^10*x^(3*n + 1
)*((143*b^4)/2 + (13*a^2*c^2)/2 - 78*a*b^2*c))/n + (c^10*x^(23*n + 1)*((143
*b^4)/2 + (13*a^2*c^2)/2 - 78*a*b^2*c))/n + (b*c^13*x^(26*n + 1))/n - (c^12
*x^(25*n + 1)*(a*c - (13*b^2)/2))/n - (a^13*b*x)/n - (143*a^7*b*x^(6*n + 1)
*(12*b^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/(7*n) + (143*b*c^7*x^
(20*n + 1)*(12*b^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/(7*n) - (14
3*a^5*b*x^(8*n + 1)*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12
*a*b^6*c))/n + (143*b*c^5*x^(18*n + 1)*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 -
30*a^3*b^2*c^3 - 12*a*b^6*c))/n - (13*a^3*b*x^(10*n + 1)*(2*b^10 - 99*a^5*c
```

```
^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/n +
 (13*b*c^3*x^(16*n + 1)*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^
4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/n - (13*a^9*b*x^(4*n + 1)*(11*b^4 +
6*a^2*c^2 - 22*a*b^2*c))/n + (13*b*c^9*x^(22*n + 1)*(11*b^4 + 6*a^2*c^2 - 2
2*a*b^2*c))/n + (13*a^11*b*x^(2*n + 1)*(a*c - 2*b^2))/n - (13*b*c^11*x^(24*
n + 1)*(a*c - 2*b^2))/n)
```

3.101 $\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(bx+cx^2\right)^{14}$$

```
[Out] 1/14*(c*x^2+b*x)^14
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {643}

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(bx+cx^2\right)^{14}$$

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b*x + c*x^2)^14/14
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{1}{14}\left(bx+cx^2\right)^{14}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. $2(15) = 30$.

Time = 0.01 (sec) , antiderivative size = 172, normalized size of antiderivative = 11.47

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx = \frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (1
43*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c
^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/
2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result
gosper	$\frac{(cx+b)^{14}x^{14}}{14}$
default	$\frac{(c\,x^2+bx)^{14}}{14}$
norman	$26b^3c^{11}x^{25}+\frac{13}{2}x^{26}b^2c^{12}+b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}x^{14}b^{14}+b^{13}c\,x^{15}+\frac{13}{2}x^{16}b^{12}c^2+26b^{11}c^3x^{17}$
risch	$26b^3c^{11}x^{25}+\frac{13}{2}x^{26}b^2c^{12}+b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}x^{14}b^{14}+b^{13}c\,x^{15}+\frac{13}{2}x^{16}b^{12}c^2+26b^{11}c^3x^{17}$
parallelrisch	$26b^3c^{11}x^{25}+\frac{13}{2}x^{26}b^2c^{12}+b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}x^{14}b^{14}+b^{13}c\,x^{15}+\frac{13}{2}x^{16}b^{12}c^2+26b^{11}c^3x^{17}$

```
[In] int((2*c*x+b)*(c*x^2+b*x)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*(c*x+b)^14*x^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 10.27

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx = \frac{1}{14}\,c^{14}x^{28}+bc^{13}x^{27}+\frac{13}{2}\,b^2c^{12}x^{26}+26\,b^3c^{11}x^{25}+\frac{143}{2}\,b^4c^{10}x^{24}+143\,b^5c^9x^{23}+\frac{429}{2}\,b^6c^8x^{22}+\frac{1716}{7}\,b^7c^7x^{21}+\frac{429}{2}\,b^8c^6x^{20}+143\,b^9c^5x^{19}+\frac{143}{2}\,b^{10}c^4x^{18}+26\,b^{11}c^3x^{17}+\frac{13}{2}\,b^{12}c^2x^{16}+b^{13}cx^{15}+\frac{1}{14}\,b^{14}x^{14}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143/
2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*x^
21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^11*
c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(10) = 20.

Time = 0.05 (sec) , antiderivative size = 175, normalized size of antiderivative = 11.67

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx = \frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13b^{12}c^2x^{16}}{2}+26b^{11}c^3x^{17}+\frac{143b^{10}c^4x^{18}}{2}+143b^9c^5x^{19}+\frac{429b^8c^6x^{20}}{2}+\frac{1716b^7c^7x^{21}}{7}+\frac{429b^6c^8x^{22}}{2}+143b^5c^9x^{23}+\frac{143b^4c^{10}x^{24}}{2}+26b^3c^{11}x^{25}+\frac{13b^2c^{12}x^{26}}{2}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

```
[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**1
7 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 +
1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*
b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*
x**27 + c**14*x**28/14
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(cx^2+bx\right)^{14}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="maxima")
[Out] 1/14*(c*x^2 + b*x)^14
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(cx^2+bx\right)^{14}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="giac")
[Out] 1/14*(c*x^2 + b*x)^14
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 10.27

$$\begin{aligned}\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=&\ \frac{b^{14}\,x^{14}}{14}+b^{13}\,c\,x^{15}+\frac{13\,b^{12}\,c^2\,x^{16}}{2}+26\,b^{11}\,c^3\,x^{17}\\&+\frac{143\,b^{10}\,c^4\,x^{18}}{2}+143\,b^9\,c^5\,x^{19}+\frac{429\,b^8\,c^6\,x^{20}}{2}\\&+\frac{1716\,b^7\,c^7\,x^{21}}{7}+\frac{429\,b^6\,c^8\,x^{22}}{2}+143\,b^5\,c^9\,x^{23}+\frac{143\,b^4\,c^{10}\,x^{24}}{2}\\&+26\,b^3\,c^{11}\,x^{25}+\frac{13\,b^2\,c^{12}\,x^{26}}{2}+b\,c^{13}\,x^{27}+\frac{c^{14}\,x^{28}}{14}\end{aligned}$$

```
[In] int((b*x + c*x^2)^13*(b + 2*c*x),x)
[Out] (b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2*
x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (42
9*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*
c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2
```

3.102 $\int x(b+2cx^2)(bx^2+cx^4)^{13}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^{13}\,dx=\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

```
[Out] 1/28*x^28*(c*x^2+b)^14
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1598, 457, 75}

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^{13}\,dx=\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

```
[In] Int[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]
[Out] (x^28*(b + c*x^2)^14)/28
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx, x, x^2\right)\\ &= \frac{1}{28}x^{28}\left(b+cx^2\right)^{14}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 182 vs. $2(16) = 32$.

Time = 0.01 (sec) , antiderivative size = 182, normalized size of antiderivative = 11.38

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{13}\,dx = {} & \frac{b^{14}x^{28}}{28}+\frac{1}{2}b^{13}cx^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}\\ &+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}\\ &+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{143}{2}b^5c^9x^{46}+\frac{143}{4}b^4c^{10}x^{48}\\ &+13b^3c^{11}x^{50}+\frac{13}{4}b^2c^{12}x^{52}+\frac{1}{2}bc^{13}x^{54}+\frac{c^{14}x^{56}}{28}\end{aligned}$$

```
[In] Integrate[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]
```

```
[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
+ (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (85
8*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*
c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 +
(c^14*x^56)/28
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{x^{28}\left(c\,x^{2}+b\right)^{14}}{28}$
default	$\frac{\left(b^{2}-\left(2c\,x^{2}+b\right)^{2}\right)^{14}}{7516192768c^{14}}$
risch	$\frac{1}{2}b\,c^{13}x^{54}+\frac{1}{28}c^{14}x^{56}+\frac{143}{2}x^{46}b^{5}c^{9}+\frac{143}{4}x^{48}b^{4}c^{10}+13x^{50}b^{3}c^{11}+\frac{13}{4}x^{52}b^{2}c^{12}+\frac{429}{4}x^{44}b^{6}c^{8}+\frac{858}{7}$
parallelrisch	$\frac{1}{2}b\,c^{13}x^{54}+\frac{1}{28}c^{14}x^{56}+\frac{143}{2}x^{46}b^{5}c^{9}+\frac{143}{4}x^{48}b^{4}c^{10}+13x^{50}b^{3}c^{11}+\frac{13}{4}x^{52}b^{2}c^{12}+\frac{429}{4}x^{44}b^{6}c^{8}+\frac{858}{7}$

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*x^28*(c*x^2+b)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{13}\,dx &= \frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}\\ &\quad+\frac{143}{4}\,b^4c^{10}x^{48}+\frac{143}{2}\,b^5c^9x^{46}+\frac{429}{4}\,b^6c^8x^{44}+\frac{858}{7}\,b^7c^7x^{42}\\ &\quad+\frac{429}{4}\,b^8c^6x^{40}+\frac{143}{2}\,b^9c^5x^{38}+\frac{143}{4}\,b^{10}c^4x^{36}\\ &\quad+13\,b^{11}c^3x^{34}+\frac{13}{4}\,b^{12}c^2x^{32}+\frac{1}{2}\,b^{13}cx^{30}+\frac{1}{28}\,b^{14}x^{28}\end{aligned}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="fricas")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(12) = 24$.

Time = 0.05 (sec) , antiderivative size = 182, normalized size of antiderivative = 11.38

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^{13}\,dx = \frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4} + \frac{143b^5c^9x^{46}}{2} + \frac{143b^4c^{10}x^{48}}{4} + 13b^3c^{11}x^{50} + \frac{13b^2c^{12}x^{52}}{4} + \frac{bc^{13}x^{54}}{2} + \frac{c^{14}x^{56}}{28}$$

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**13,x)
```

```
[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x**34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**40/4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/2 + 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 + b*c**13*x**54/2 + c**14*x**56/28
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^{13}\,dx = \frac{1}{28}\,c^{14}x^{56} + \frac{1}{2}\,bc^{13}x^{54} + \frac{13}{4}\,b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}\,b^4c^{10}x^{48} + \frac{143}{2}\,b^5c^9x^{46} + \frac{429}{4}\,b^6c^8x^{44} + \frac{858}{7}\,b^7c^7x^{42} + \frac{429}{4}\,b^8c^6x^{40} + \frac{143}{2}\,b^9c^5x^{38} + \frac{143}{4}\,b^{10}c^4x^{36} + 13\,b^{11}c^3x^{34} + \frac{13}{4}\,b^{12}c^2x^{32} + \frac{1}{2}\,b^{13}cx^{30} + \frac{1}{28}\,b^{14}x^{28}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 + 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^{13}\,dx=\frac{1}{28}\left(cx^4+bx^2\right)^{14}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="giac")
[Out] 1/28*(c*x^4 + b*x^2)^14
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\begin{aligned}\int x(b+2cx^2)\left(bx^2+cx^4\right)^{13}\,dx=&\frac{b^{14}\,x^{28}}{28}+\frac{b^{13}\,c\,x^{30}}{2}+\frac{13\,b^{12}\,c^2\,x^{32}}{4}\\&+13\,b^{11}\,c^3\,x^{34}+\frac{143\,b^{10}\,c^4\,x^{36}}{4}+\frac{143\,b^9\,c^5\,x^{38}}{2}\\&+\frac{429\,b^8\,c^6\,x^{40}}{4}+\frac{858\,b^7\,c^7\,x^{42}}{7}+\frac{429\,b^6\,c^8\,x^{44}}{4}\\&+\frac{143\,b^5\,c^9\,x^{46}}{2}+\frac{143\,b^4\,c^{10}\,x^{48}}{4}+13\,b^3\,c^{11}\,x^{50}\\&+\frac{13\,b^2\,c^{12}\,x^{52}}{4}+\frac{b\,c^{13}\,x^{54}}{2}+\frac{c^{14}\,x^{56}}{28}\end{aligned}$$

```
[In] int(x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x)
[Out] (b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*b
^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x
^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4
 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^
2*c^12*x^52)/4
```

3.103 $\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^{13} dx$

Optimal result

Integrand size = 25, antiderivative size = 16

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^{13} dx = \frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

```
[Out] 1/42*x^42*(c*x^3+b)^14
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1598, 457, 75}

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^{13} dx = \frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

```
[In] Int[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx\\ &= \frac{1}{3}\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^3\right)\\ &= \frac{1}{42}x^{42}\left(b+cx^3\right)^{14}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 186 vs. $2(16) = 32$.

Time = 0.00 (sec) , antiderivative size = 186, normalized size of antiderivative = 11.62

$$\begin{aligned}\int x^2\left(b+2cx^3\right)\left(bx^3+cx^6\right)^{13}\,dx = {}&\frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}\\ &+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}\\ &+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{6}b^4c^{10}x^{72}\\ &+\frac{26}{3}b^3c^{11}x^{75}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}bc^{13}x^{81}+\frac{c^{14}x^{84}}{42}\end{aligned}$$

```
[In] Integrate[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]
```

```
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51
)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 +
 (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*
b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^
81)/3 + (c^14*x^84)/42
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{x^{42}\left(c\,x^{3}+b\right)^{14}}{42}$
default	$\frac{\left(b^{2}-\left(2c\,x^{3}+b\right)^{2}\right)^{14}}{11274289152c^{14}}$
risch	$\frac{1}{42}c^{14}x^{84}+\frac{13}{6}x^{78}b^{2}c^{12}+\frac{1}{3}b\,c^{13}x^{81}+\frac{143}{3}x^{69}b^{5}c^{9}+\frac{143}{6}x^{72}b^{4}c^{10}+\frac{26}{3}x^{75}b^{3}c^{11}+\frac{572}{7}x^{63}b^{7}c^{7}+\frac{143}{2}x$
parallelrisch	$\frac{1}{42}c^{14}x^{84}+\frac{13}{6}x^{78}b^{2}c^{12}+\frac{1}{3}b\,c^{13}x^{81}+\frac{143}{3}x^{69}b^{5}c^{9}+\frac{143}{6}x^{72}b^{4}c^{10}+\frac{26}{3}x^{75}b^{3}c^{11}+\frac{572}{7}x^{63}b^{7}c^{7}+\frac{143}{2}x$

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x,method=_RETURNVERBOSE)
[Out] 1/42*x^42*(c*x^3+b)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(14) = 28.

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^2\left(b+2cx^3\right)\left(bx^3+cx^6\right)^{13}\,dx=\frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^2c^{12}x^{78}+\frac{26}{3}\,b^3c^{11}x^{75}+\frac{143}{6}\,b^4c^{10}x^{72}+\frac{143}{3}\,b^5c^9x^{69}+\frac{143}{2}\,b^6c^8x^{66}+\frac{572}{7}\,b^7c^7x^{63}+\frac{143}{2}\,b^8c^6x^{60}+\frac{143}{3}\,b^9c^5x^{57}+\frac{143}{6}\,b^{10}c^4x^{54}+\frac{26}{3}\,b^{11}c^3x^{51}+\frac{13}{6}\,b^{12}c^2x^{48}+\frac{1}{3}\,b^{13}cx^{45}+\frac{1}{42}\,b^{14}x^{42}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="fricas")
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(12) = 24$.

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 11.56

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^{13}\,dx = \frac{b^{14}x^{42}}{42} + \frac{b^{13}cx^{45}}{3} + \frac{13b^{12}c^2x^{48}}{6} + \frac{26b^{11}c^3x^{51}}{3} + \frac{143b^{10}c^4x^{54}}{6} + \frac{143b^9c^5x^{57}}{3} + \frac{143b^8c^6x^{60}}{2} + \frac{572b^7c^7x^{63}}{7} + \frac{143b^6c^8x^{66}}{2} + \frac{143b^5c^9x^{69}}{3} + \frac{143b^4c^{10}x^{72}}{6} + \frac{26b^3c^{11}x^{75}}{3} + \frac{13b^2c^{12}x^{78}}{6} + \frac{bc^{13}x^{81}}{3} + \frac{c^{14}x^{84}}{42}$$

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**13,x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x*
*51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x**6
0/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**69/3
 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78/6 +
 b*c**13*x**81/3 + c**14*x**84/42
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^{13}\,dx = \frac{1}{42}\,c^{14}x^{84} + \frac{1}{3}\,bc^{13}x^{81} + \frac{13}{6}\,b^2c^{12}x^{78} + \frac{26}{3}\,b^3c^{11}x^{75} + \frac{143}{6}\,b^4c^{10}x^{72} + \frac{143}{3}\,b^5c^9x^{69} + \frac{143}{2}\,b^6c^8x^{66} + \frac{572}{7}\,b^7c^7x^{63} + \frac{143}{2}\,b^8c^6x^{60} + \frac{143}{3}\,b^9c^5x^{57} + \frac{143}{6}\,b^{10}c^4x^{54} + \frac{26}{3}\,b^{11}c^3x^{51} + \frac{13}{6}\,b^{12}c^2x^{48} + \frac{1}{3}\,b^{13}cx^{45} + \frac{1}{42}\,b^{14}x^{42}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^2 (b + 2cx^3) (bx^3 + cx^6)^{13} dx = \frac{1}{42} (cx^6 + bx^3)^{14}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="giac")
[Out] 1/42*(c*x^6 + b*x^3)^14
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^2 (b + 2cx^3) (bx^3 + cx^6)^{13} dx = \frac{b^{14} x^{42}}{42} + \frac{b^{13} c x^{45}}{3} + \frac{13 b^{12} c^2 x^{48}}{6} + \frac{26 b^{11} c^3 x^{51}}{3} + \frac{143 b^{10} c^4 x^{54}}{6} + \frac{143 b^9 c^5 x^{57}}{3} + \frac{143 b^8 c^6 x^{60}}{2} + \frac{572 b^7 c^7 x^{63}}{7} + \frac{143 b^6 c^8 x^{66}}{2} + \frac{143 b^5 c^9 x^{69}}{3} + \frac{143 b^4 c^{10} x^{72}}{6} + \frac{26 b^3 c^{11} x^{75}}{3} + \frac{13 b^2 c^{12} x^{78}}{6} + \frac{b c^{13} x^{81}}{3} + \frac{c^{14} x^{84}}{42}$$

```
[In] int(x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x)
[Out] (b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*b
^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c
^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^6
6)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3
+ (13*b^2*c^12*x^78)/6
```

3.104 $\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 21

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

```
[Out] 1/14*x^(14*n)*(b+c*x^n)^14/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1598, 457, 75}

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx$$

$$= \frac{\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx, x, x^n\right)}{n}$$

$$= \frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx = \frac{x^{14n}(b+cx^n)^{14}}{14n}$$

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(19) = 38.

Time = 0.02 (sec) , antiderivative size = 230, normalized size of antiderivative = 10.95

$$\frac{c^{14}x^{28n}}{14n}+\frac{b\,c^{13}x^{27n}}{n}+\frac{13c^{12}x^{26n}b^2}{2n}+\frac{26c^{11}b^3x^{25n}}{n}+\frac{143c^{10}x^{24n}b^4}{2n}+\frac{143c^9b^5x^{23n}}{n}+\frac{429c^8x^{22n}b^6}{2n}+\frac{1716b^7c^7x^{21n}}{7n}+429$$

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x)
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*c^12/n*(x^n)^26*b^2+26*c^11*b^3
/n*(x^n)^25+143/2*c^10/n*(x^n)^24*b^4+143*c^9*b^5/n*(x^n)^23+429/2*c^8/n*(x
^n)^22*b^6+1716/7*b^7*c^7/n*(x^n)^21+429/2*c^6/n*(x^n)^20*b^8+143*b^9*c^5/n
*(x^n)^19+143/2*c^4/n*(x^n)^18*b^10+26*b^11*c^3/n*(x^n)^17+13/2*c^2/n*(x^n)
^16*b^12+b^13*c/n*(x^n)^15+1/14/n*(x^n)^14*b^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(19) = 38.

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 9.00

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx = \frac{c^{14}x^{28\,n}+14\,bc^{13}x^{27\,n}+91\,b^2c^{12}x^{26\,n}+364\,b^3c^{11}x^{25\,n}+1001\,b^4c^{10}x^{24\,n}+2002\,b^5c^9x^{23\,n}+3003\,b^6c^8x^{22\,n}+}{}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="fricas")
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**13,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(19) = 38.

Time = 0.20 (sec) , antiderivative size = 229, normalized size of antiderivative = 10.90

$$\begin{aligned}\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx &= \frac{c^{14}x^{28\,n}}{14\,n}+\frac{bc^{13}x^{27\,n}}{n}+\frac{13\,b^2c^{12}x^{26\,n}}{2\,n}\\&\quad+\frac{26\,b^3c^{11}x^{25\,n}}{n}+\frac{143\,b^4c^{10}x^{24\,n}}{2\,n}+\frac{143\,b^5c^9x^{23\,n}}{n}\\&\quad+\frac{429\,b^6c^8x^{22\,n}}{2\,n}+\frac{1716\,b^7c^7x^{21\,n}}{7\,n}+\frac{429\,b^8c^6x^{20\,n}}{2\,n}\\&\quad+\frac{143\,b^9c^5x^{19\,n}}{n}+\frac{143\,b^{10}c^4x^{18\,n}}{2\,n}+\frac{26\,b^{11}c^3x^{17\,n}}{n}\\&\quad+\frac{13\,b^{12}c^2x^{16\,n}}{2\,n}+\frac{b^{13}cx^{15\,n}}{n}+\frac{b^{14}x^{14\,n}}{14\,n}\end{aligned}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="maxima")
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b^
3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n + 42
9/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20*n)
/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x^(17
*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14*n)/n
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(19) = 38$.

Time = 0.29 (sec) , antiderivative size = 189, normalized size of antiderivative = 9.00

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx = \frac{c^{14}x^{28\,n} + 14\,bc^{13}x^{27\,n} + 91\,b^2c^{12}x^{26\,n} + 364\,b^3c^{11}x^{25\,n} + 1001\,b^4c^{10}x^{24\,n} + 2002\,b^5c^9x^{23\,n} + 3003\,b^6c^8x^{22\,n} + 3}{}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 229, normalized size of antiderivative = 10.90

$$\begin{aligned}\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx = {} & \frac{b^{14}\,x^{14\,n}}{14\,n} + \frac{c^{14}\,x^{28\,n}}{14\,n} + \frac{13\,b^{12}\,c^2\,x^{16\,n}}{2\,n} \\ & + \frac{26\,b^{11}\,c^3\,x^{17\,n}}{n} + \frac{143\,b^{10}\,c^4\,x^{18\,n}}{2\,n} + \frac{143\,b^9\,c^5\,x^{19\,n}}{n} \\ & + \frac{429\,b^8\,c^6\,x^{20\,n}}{2\,n} + \frac{1716\,b^7\,c^7\,x^{21\,n}}{7\,n} + \frac{429\,b^6\,c^8\,x^{22\,n}}{2\,n} \\ & + \frac{143\,b^5\,c^9\,x^{23\,n}}{n} + \frac{143\,b^4\,c^{10}\,x^{24\,n}}{2\,n} + \frac{26\,b^3\,c^{11}\,x^{25\,n}}{n} \\ & + \frac{13\,b^2\,c^{12}\,x^{26\,n}}{2\,n} + \frac{b^{13}\,c\,x^{15\,n}}{n} + \frac{b\,c^{13}\,x^{27\,n}}{n}\end{aligned}$$

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x)
```

```
[Out] (b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(2
*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9*c
^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/(7*
n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4*c^1
0*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/(2*n)
 + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

3.105 $\int \frac{b+2cx}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 11

$$\int \frac{b+2cx}{a+bx+cx^2}\,dx=\log\left(a+bx+cx^2\right)$$

```
[Out] ln(c*x^2+b*x+a)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {642}

$$\int \frac{b+2cx}{a+bx+cx^2}\,dx=\log\left(a+bx+cx^2\right)$$

```
[In] Int[(b + 2*c*x)/(a + b*x + c*x^2),x]
```

```
[Out] Log[a + b*x + c*x^2]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\text{integral}=\log\left(a+bx+cx^2\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{b+2cx}{a+bx+cx^2}\,dx = \log(a+x(b+cx))$$

```
[In] Integrate[(b + 2*c*x)/(a + b*x + c*x^2),x]
[Out] Log[a + x*(b + c*x)]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\ln\left(c\,x^2+bx+a\right)$	12
default	$\ln\left(c\,x^2+bx+a\right)$	12
norman	$\ln\left(c\,x^2+bx+a\right)$	12
risch	$\ln\left(c\,x^2+bx+a\right)$	12
parallelrisch	$\ln\left(c\,x^2+bx+a\right)$	12

```
[In] int((2*c*x+b)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
[Out] ln(c*x^2+b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{a+bx+cx^2}\,dx = \log\left(cx^2+bx+a\right)$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="fricas")
[Out] log(c*x^2 + b*x + a)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{b+2cx}{a+bx+cx^2} \, dx = \log \left(a+bx+cx^2\right)$$

```
[In] integrate((2*c*x+b)/(c*x**2+b*x+a),x)
[Out] log(a + b*x + c*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{a+bx+cx^2} \, dx = \log \left(cx^2+bx+a\right)$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="maxima")
[Out] log(c*x^2 + b*x + a)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{b+2cx}{a+bx+cx^2} \, dx = \log \left(\left|cx^2+bx+a\right|\right)$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="giac")
[Out] log(abs(c*x^2 + b*x + a))
```

Mupad [B] (verification not implemented)

Time = 8.63 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{a+bx+cx^2} \, dx = \ln \left(c\,x^2+b\,x+a\right)$$

```
[In] int((b + 2*c*x)/(a + b*x + c*x^2),x)
[Out] log(a + b*x + c*x^2)
```

3.106 $\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 17

$$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

```
[Out] 1/2*ln(c*x^4+b*x^2+a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1261, 642}

$$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

```
[In] Int[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Log[a + b*x^2 + c*x^4]/2
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

```
[In] Integrate[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x]
[Out] Log[a + b*x^2 + c*x^4]/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2}$	16
norman	$\frac{\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2}$	16
risch	$\frac{\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2}$	16
parallelrisch	$\frac{\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2}$	16

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(c*x^4+b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\log\left(cx^4+bx^2+a\right)$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 1/2*log(c*x^4 + b*x^2 + a)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx = \frac{\log\left(a+bx^2+cx^4\right)}{2}$$

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a),x)
[Out] log(a + b*x**2 + c*x**4)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\log\left(cx^4+bx^2+a\right)$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 1/2*log(c*x^4 + b*x^2 + a)
```

Giac [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\log\left(\left|cx^4+bx^2+a\right|\right)$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/2*log(abs(c*x^4 + b*x^2 + a))
```

Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx=\frac{\ln\left(c\,x^4+b\,x^2+a\right)}{2}$$

```
[In] int((x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x)
[Out] log(a + b*x^2 + c*x^4)/2
```

3.107 $\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 17

$$\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

```
[Out] 1/3*ln(c*x^6+b*x^3+a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1482, 642}

$$\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

```
[In] Int[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] Log[a + b*x^3 + c*x^6]/3
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
```

```
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\log\left(a+bx^3+cx^6\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x]
[Out] Log[a + b*x^3 + c*x^6]/3
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{3}$	16
norman	$\frac{\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{3}$	16
risch	$\frac{\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{3}$	16
parallelrisch	$\frac{\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{3}$	16

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(c*x^6+b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\,\log\left(cx^6+bx^3+a\right)$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] 1/3*log(c*x^6 + b*x^3 + a)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx = \frac{\log\left(a+bx^3+cx^6\right)}{3}$$

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a),x)
[Out] log(a + b*x**3 + c*x**6)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\,\log\left(cx^6+bx^3+a\right)$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] 1/3*log(c*x^6 + b*x^3 + a)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\,\log\left(\left|cx^6+bx^3+a\right|\right)$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] 1/3*log(abs(c*x^6 + b*x^3 + a))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx=\frac{\ln\left(c\,x^6+b\,x^3+a\right)}{3}$$

```
[In] int((x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x)
[Out] log(a + b*x^3 + c*x^6)/3
```

3.108 $\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 19

$$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx = \frac{\log(a+bx^n+cx^{2n})}{n}$$

```
[Out] ln(a+b*x^n+c*x^(2*n))/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1482, 642}

$$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx = \frac{\log(a+bx^n+cx^{2n})}{n}$$

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a + b*x^n + c*x^(2*n)]/n
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
```

```
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^n\right)}{n}$$
$$= \frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx = -\frac{2\log\left(x^{-n}\right)}{n}+\frac{\log\left(c+ax^{-2n}+bx^{-n}\right)}{n}$$

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x]
[Out] (-2*Log[x^(-n)])/n + Log[c + a/x^(2*n) + b/x^n]/n
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

method	result	size
norman	$\frac{\ln\left(a+b\,e^{n\ln(x)}+c\,e^{2n\ln(x)}\right)}{n}$	24
risch	$\frac{\ln\left(x^{2n}+\frac{b\,x^{n}}{c}+\frac{a}{c}\right)}{n}$	25

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] 1/n*ln(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx = \frac{\log\left(cx^{2\,n}+bx^n+a\right)}{n}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] log(c*x^(2*n) + b*x^n + a)/n
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx = \frac{\log\left(\frac{cx^{2\,n}+bx^n+a}{c}\right)}{n}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] log((c*x^(2*n) + b*x^n + a)/c)/n
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx = \frac{\log\left(cx^{2\,n}+bx^n+a\right)}{n}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] log(c*x^(2*n) + b*x^n + a)/n
```

Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 121, normalized size of antiderivative = 6.37

$$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx = -\frac{2\,b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x^n}{\sqrt{4\,a\,c-b^2}}\right)-\ln\left(a+b\,x^n+c\,x^{2\,n}\right)\,\sqrt{4\,a\,c-b^2}}{n\,\sqrt{4\,a\,c-b^2}} - \frac{2\,b\,\mathrm{atanh}\left(\frac{b+2\,c\,x^n}{\sqrt{b^2-4\,a\,c}}\right)}{n\,\sqrt{b^2-4\,a\,c}}$$

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] - (2*b*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x^n)/(4*a*c - b^2)^(1/2)) - log(a
+ b*x^n + c*x^(2*n))*(4*a*c - b^2)^(1/2))/(n*(4*a*c - b^2)^(1/2)) - (2*b*at
anh((b + 2*c*x^n)/(b^2 - 4*a*c)^(1/2)))/(n*(b^2 - 4*a*c)^(1/2))
```

3.109 $\int \frac{b+2cx}{(a+bx+cx^2)^8} dx$

Optimal result

Integrand size = 19, antiderivative size = 16

$$\int \frac{b+2cx}{(a+bx+cx^2)^8} dx = -\frac{1}{7(a+bx+cx^2)^7}$$

```
[Out] -1/7/(c*x^2+b*x+a)^7
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {643}

$$\int \frac{b+2cx}{(a+bx+cx^2)^8} dx = -\frac{1}{7(a+bx+cx^2)^7}$$

```
[In] Int[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]
```

```
[Out] -1/7*1/(a + b*x + c*x^2)^7
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{7(a+bx+cx^2)^7}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx = -\frac{1}{7(a+x(b+cx))^7}$$

```
[In] Integrate[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]
[Out] -1/7*1/(a + x*(b + c*x))^7
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{7(cx^2+bx+a)^7}$	15
derivativedivides	$-\frac{1}{7(cx^2+bx+a)^7}$	15
default	$-\frac{1}{7(cx^2+bx+a)^7}$	15
norman	$-\frac{1}{7(cx^2+bx+a)^7}$	15
risch	$-\frac{1}{7(cx^2+bx+a)^7}$	15
parallelrisch	$-\frac{1}{7(cx^2+bx+a)^7}$	15

```
[In] int((2*c*x+b)/(c*x^2+b*x+a)^8,x,method=_RETURNVERBOSE)
[Out] -1/7/(c*x^2+b*x+a)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(14) = 28.

Time = 0.28 (sec) , antiderivative size = 350, normalized size of antiderivative = 21.88

$$\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx =$$

$$-\frac{}{7\left(c^7x^{14}+7\,bc^6x^{13}+7\left(3\,b^2c^5+ac^6\right)x^{12}+7\left(5\,b^3c^4+6\,abc^5\right)x^{11}+7\left(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5\right)x^{10}+7\left(\right.\right.}$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="fricas")
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 7*(3*b^2*c^5 + a*c^6)*x^12 + 7*(5*b^3*c^4 +
 6*a*b*c^5)*x^11 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^10 + 7*(3*b^5
```

```
*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^9 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*
b^2*c^3 + 5*a^3*c^4)*x^8 + 7*a^6*b*x + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2
+ 140*a^3*b*c^3)*x^7 + a^7 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a
^4*c^3)*x^6 + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^5 + 7*(5*a^3*b^
4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^4 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^3 + 7*(3*a^
5*b^2 + a^6*c)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(15) = 30$.

Time = 2.55 (sec) , antiderivative size = 359, normalized size of antiderivative = 22.44

$$\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx =$$

$$-\frac{}{7a^7 + 49a^6bx + 49bc^6x^{13} + 7c^7x^{14} + x^{12}\cdot(49ac^6 + 147b^2c^5) + x^{11}\cdot(294abc^5 + 245b^3c^4) + x^{10}\cdot(147a^2c^5}$$

```
[In] integrate((2*c*x+b)/(c*x**2+b*x+a)**8,x)
```

```
[Out] -1/(7*a**7 + 49*a**6*b*x + 49*b*c**6*x**13 + 7*c**7*x**14 + x**12*(49*a*c**
6 + 147*b**2*c**5) + x**11*(294*a*b*c**5 + 245*b**3*c**4) + x**10*(147*a**2
*c**5 + 735*a*b**2*c**4 + 245*b**4*c**3) + x**9*(735*a**2*b*c**4 + 980*a*b*
*3*c**3 + 147*b**5*c**2) + x**8*(245*a**3*c**4 + 1470*a**2*b**2*c**3 + 735*
a*b**4*c**2 + 49*b**6*c) + x**7*(980*a**3*b*c**3 + 1470*a**2*b**3*c**2 + 29
4*a*b**5*c + 7*b**7) + x**6*(245*a**4*c**3 + 1470*a**3*b**2*c**2 + 735*a**2
*b**4*c + 49*a*b**6) + x**5*(735*a**4*b*c**2 + 980*a**3*b**3*c + 147*a**2*b
**5) + x**4*(147*a**5*c**2 + 735*a**4*b**2*c + 245*a**3*b**4) + x**3*(294*a
**5*b*c + 245*a**4*b**3) + x**2*(49*a**6*c + 147*a**5*b**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx = -\frac{1}{7\,(cx^2+bx+a)^7}$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x + a)^7
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx = -\frac{1}{7\,(cx^2+bx+a)^7}$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="giac")
[Out] -1/7/(c*x^2 + b*x + a)^7
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 358, normalized size of antiderivative = 22.38

$$\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx =$$

$$-\frac{}{7\,\left(x^5\,\left(105\,a^4\,b\,c^2+140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^9\,\left(105\,a^2\,b\,c^4+140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^7\,\left(140\,a^3\,b\,c^3+2\right.\right.}$$

```
[In] int((b + 2*c*x)/(a + b*x + c*x^2)^8,x)
[Out] -1/(7*(x^5*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^9*(21*b^5*c^2 +
 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^7*(b^7 + 140*a^3*b*c^3 + 210*a^2*b^3*c^
2 + 42*a*b^5*c) + x^3*(35*a^4*b^3 + 42*a^5*b*c) + x^11*(35*b^3*c^4 + 42*a*b
*c^5) + x^4*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^10*(21*a^2*c^5 +
35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^6*(7*a*b^6 + 35*a^4*c^3 + 105*a^2*b^4
*c + 210*a^3*b^2*c^2) + x^8*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c^2 + 210*a^2
*b^2*c^3) + c^7*x^14 + x^2*(7*a^6*c + 21*a^5*b^2) + x^12*(7*a*c^6 + 21*b^2*
c^5) + 7*b*c^6*x^13 + 7*a^6*b*x))
```

3.110 $\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8} dx$

Optimal result

Integrand size = 24, antiderivative size = 18

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8} dx = -\frac{1}{14(a+bx^2+cx^4)^7}$$

```
[Out] -1/14/(c*x^4+b*x^2+a)^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1261, 643}

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8} dx = -\frac{1}{14(a+bx^2+cx^4)^7}$$

```
[In] Int[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/14*1/(a + b*x^2 + c*x^4)^7
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx, x, x^2\right)$$
$$= -\frac{1}{14\,(a+bx^2+cx^4)^7}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx = -\frac{1}{14\,(a+bx^2+cx^4)^7}$$

```
[In] Integrate[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]
[Out] -1/14*1/(a + b*x^2 + c*x^4)^7
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{14(c\,x^4+b\,x^2+a)^7}$	17
default	$-\frac{1}{14(c\,x^4+b\,x^2+a)^7}$	17
norman	$-\frac{1}{14(c\,x^4+b\,x^2+a)^7}$	17
risch	$-\frac{1}{14(c\,x^4+b\,x^2+a)^7}$	17
parallelrisch	$-\frac{1}{14(c\,x^4+b\,x^2+a)^7}$	17

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x,method=_RETURNVERBOSE)
[Out] -1/14/(c*x^4+b*x^2+a)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(16) = 32$.

Time = 0.30 (sec) , antiderivative size = 352, normalized size of antiderivative = 19.56

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx =$$

$$-\frac{}{14\,(c^7x^{28} + 7\,bc^6x^{26} + 7\,(3\,b^2c^5 + ac^6)x^{24} + 7\,(5\,b^3c^4 + 6\,abc^5)x^{22} + 7\,(5\,b^4c^3 + 15\,ab^2c^4 + 3\,a^2c^5)x^{20} + 7\,(}$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 + a*c^6)*x^24 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^16 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^14 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 + 7*(5*a^3*
b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^8 + a^7 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^6 +
 7*(3*a^5*b^2 + a^6*c)*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(17) = 34$.

Time = 4.19 (sec) , antiderivative size = 360, normalized size of antiderivative = 20.00

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx =$$

$$-\frac{}{14a^7 + 98a^6bx^2 + 98bc^6x^{26} + 14c^7x^{28} + x^{24}\cdot(98ac^6 + 294b^2c^5) + x^{22}\cdot(588abc^5 + 490b^3c^4) + x^{20}\cdot(294a}$$

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a)**8,x)
```

```
[Out] -1/(14*a**7 + 98*a**6*b*x**2 + 98*b*c**6*x**26 + 14*c**7*x**28 + x**24*(98*
a*c**6 + 294*b**2*c**5) + x**22*(588*a*b*c**5 + 490*b**3*c**4) + x**20*(294
*a**2*c**5 + 1470*a*b**2*c**4 + 490*b**4*c**3) + x**18*(1470*a**2*b*c**4 +
1960*a*b**3*c**3 + 294*b**5*c**2) + x**16*(490*a**3*c**4 + 2940*a**2*b**2*c
**3 + 1470*a*b**4*c**2 + 98*b**6*c) + x**14*(1960*a**3*b*c**3 + 2940*a**2*b
**3*c**2 + 588*a*b**5*c + 14*b**7) + x**12*(490*a**4*c**3 + 2940*a**3*b**2*
c**2 + 1470*a**2*b**4*c + 98*a*b**6) + x**10*(1470*a**4*b*c**2 + 1960*a**3*
b**3*c + 294*a**2*b**5) + x**8*(294*a**5*c**2 + 1470*a**4*b**2*c + 490*a**3
*b**4) + x**6*(588*a**5*b*c + 490*a**4*b**3) + x**4*(98*a**6*c + 294*a**5*b
**2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 352, normalized size of antiderivative = 19.56

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx =$$

$$-\frac{}{14\,(c^7x^{28} + 7\,bc^6x^{26} + 7\,(3\,b^2c^5 + ac^6)x^{24} + 7\,(5\,b^3c^4 + 6\,abc^5)x^{22} + 7\,(5\,b^4c^3 + 15\,ab^2c^4 + 3\,a^2c^5)x^{20} + 7}$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 + a*c^6)*x^24 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^16 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^14 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 + 7*(5*a^3*
b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^8 + a^7 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^6 +
 7*(3*a^5*b^2 + a^6*c)*x^4)
```

Giac [A] (verification not implemented)

none

Time = 1.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx = -\frac{1}{14\,(cx^4+bx^2+a)^7}$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2 + a)^7
```

Mupad [B] (verification not implemented)

Time = 15.83 (sec) , antiderivative size = 360, normalized size of antiderivative = 20.00

$$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx =$$

$$-\frac{}{14\,\left(x^{10}\,\left(105\,a^4\,b\,c^2+140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^{18}\,\left(105\,a^2\,b\,c^4+140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^{14}\,\left(140\,a^3\,b\,c^3\right.\right.}$$

```
[In] int((x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x)
```

```
[Out] -1/(14*(x^10*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^18*(21*b^5*c^
2 + 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^14*(b^7 + 140*a^3*b*c^3 + 210*a^2*b^
3*c^2 + 42*a*b^5*c) + x^6*(35*a^4*b^3 + 42*a^5*b*c) + x^22*(35*b^3*c^4 + 42
*a*b*c^5) + x^8*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^20*(21*a^2*c^
5 + 35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^12*(7*a*b^6 + 35*a^4*c^3 + 105*a^
2*b^4*c + 210*a^3*b^2*c^2) + x^16*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c^2 + 2
10*a^2*b^2*c^3) + c^7*x^28 + x^4*(7*a^6*c + 21*a^5*b^2) + x^24*(7*a*c^6 + 2
1*b^2*c^5) + 7*a^6*b*x^2 + 7*b*c^6*x^26))
```

3.111 $\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 18

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx = -\frac{1}{21(a+bx^3+cx^6)^7}$$

```
[Out] -1/21/(c*x^6+b*x^3+a)^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1482, 643}

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx = -\frac{1}{21(a+bx^3+cx^6)^7}$$

```
[In] Int[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/21*1/(a + b*x^3 + c*x^6)^7
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
```

```
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx, x, x^3\right)$$
$$= -\frac{1}{21\,(a+bx^3+cx^6)^7}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx = -\frac{1}{21\,(a+bx^3+cx^6)^7}$$

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]
[Out] -1/21*1/(a + b*x^3 + c*x^6)^7
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{21(c\,x^6+b\,x^3+a)^7}$	17
default	$-\frac{1}{21(c\,x^6+b\,x^3+a)^7}$	17
risch	$-\frac{1}{21(c\,x^6+b\,x^3+a)^7}$	17
parallelrisch	$-\frac{1}{21(c\,x^6+b\,x^3+a)^7}$	17

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x,method=_RETURNVERBOSE)
[Out] -1/21/(c*x^6+b*x^3+a)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 352, normalized size of antiderivative = 19.56

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx =$$

$$-\frac{}{21\,(c^7x^{42}+7\,bc^6x^{39}+7\,(3\,b^2c^5+ac^6)x^{36}+7\,(5\,b^3c^4+6\,abc^5)x^{33}+7\,(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5)x^{30}+7}$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 + a*c^6)*x^36 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^24 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^21 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 + 7*(5*a^3*b^4 + 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^9 + a^7
+ 7*(3*a^5*b^2 + a^6*c)*x^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(17) = 34$.

Time = 14.85 (sec) , antiderivative size = 360, normalized size of antiderivative = 20.00

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx =$$

$$-\frac{}{21a^7+147a^6bx^3+147bc^6x^{39}+21c^7x^{42}+x^{36}\cdot(147ac^6+441b^2c^5)+x^{33}\cdot(882abc^5+735b^3c^4)+x^{30}\cdot(}$$

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a)**8,x)
```

```
[Out] -1/(21*a**7 + 147*a**6*b*x**3 + 147*b*c**6*x**39 + 21*c**7*x**42 + x**36*(1
47*a*c**6 + 441*b**2*c**5) + x**33*(882*a*b*c**5 + 735*b**3*c**4) + x**30*(
441*a**2*c**5 + 2205*a*b**2*c**4 + 735*b**4*c**3) + x**27*(2205*a**2*b*c**4
 + 2940*a*b**3*c**3 + 441*b**5*c**2) + x**24*(735*a**3*c**4 + 4410*a**2*b**
2*c**3 + 2205*a*b**4*c**2 + 147*b**6*c) + x**21*(2940*a**3*b*c**3 + 4410*a*
*2*b**3*c**2 + 882*a*b**5*c + 21*b**7) + x**18*(735*a**4*c**3 + 4410*a**3*b
**2*c**2 + 2205*a**2*b**4*c + 147*a*b**6) + x**15*(2205*a**4*b*c**2 + 2940*
a**3*b**3*c + 441*a**2*b**5) + x**12*(441*a**5*c**2 + 2205*a**4*b**2*c + 73
5*a**3*b**4) + x**9*(882*a**5*b*c + 735*a**4*b**3) + x**6*(147*a**6*c + 441
*a**5*b**2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(16) = 32.

Time = 0.27 (sec) , antiderivative size = 352, normalized size of antiderivative = 19.56

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx =$$

$$-\frac{}{21\,(c^7x^{42}+7\,bc^6x^{39}+7\,(3\,b^2c^5+ac^6)x^{36}+7\,(5\,b^3c^4+6\,abc^5)x^{33}+7\,(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5)x^{30}+7\,(}$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 + a*c^6)*x^36 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^24 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^21 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 + 7*(5*a^3*b^4 + 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^9 + a^7
+ 7*(3*a^5*b^2 + a^6*c)*x^6)
```

Giac [A] (verification not implemented)

none

Time = 2.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx = -\frac{1}{21\,(cx^6+bx^3+a)^7}$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3 + a)^7
```

Mupad [B] (verification not implemented)

Time = 18.83 (sec) , antiderivative size = 360, normalized size of antiderivative = 20.00

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx =$$

$$-\frac{}{21\,\left(x^{15}\,\left(105\,a^4\,b\,c^2+140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^{27}\,\left(105\,a^2\,b\,c^4+140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^{21}\,\left(140\,a^3\,b\,c^3+\right.\right.}$$

```
[In] int((x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x)
```

```
[Out] -1/(21*(x^15*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^27*(21*b^5*c^
2 + 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^21*(b^7 + 140*a^3*b*c^3 + 210*a^2*b^
3*c^2 + 42*a*b^5*c) + x^9*(35*a^4*b^3 + 42*a^5*b*c) + x^33*(35*b^3*c^4 + 42
*a*b*c^5) + x^12*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^30*(21*a^2*c
^5 + 35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^18*(7*a*b^6 + 35*a^4*c^3 + 105*a
^2*b^4*c + 210*a^3*b^2*c^2) + x^24*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c^2 +
210*a^2*b^2*c^3) + c^7*x^42 + x^6*(7*a^6*c + 21*a^5*b^2) + x^36*(7*a*c^6 +
21*b^2*c^5) + 7*a^6*b*x^3 + 7*b*c^6*x^39))
```

3.112 $\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8} dx$

Optimal result

Integrand size = 30, antiderivative size = 23

$$\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8} dx = -\frac{1}{7n(a+bx^n+cx^{2n})^7}$$

```
[Out] -1/7/n/(a+b*x^n+c*x^(2*n))^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1482, 643}

$$\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8} dx = -\frac{1}{7n(a+bx^n+cx^{2n})^7}$$

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]
[Out] -1/7*1/(n*(a + b*x^n + c*x^(2*n))^7)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
```

```
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx, x, x^n\right)}{n}$$
$$= -\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx = -\frac{1}{7n\left(a+x^n\left(b+cx^n\right)\right)^7}$$

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]
[Out] -1/7*1/(n*(a + x^n*(b + c*x^n))^7)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$-\frac{1}{7n\left(a+b\,x^n+c\,x^{2n}\right)^7}$$

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x)
[Out] -1/7/n/(a+b*x^n+c*(x^n)^2)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(21) = 42.

Time = 0.35 (sec) , antiderivative size = 394, normalized size of antiderivative = 17.13

$$\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx =$$
$$-\frac{}{7\left(c^7nx^{14n}+7\,bc^6nx^{13n}+7\,a^6bnx^n+a^7n+7\left(3\,b^2c^5+ac^6\right)nx^{12n}+7\left(5\,b^3c^4+6\,abc^5\right)nx^{11n}+7\left(5\,b^4c^3\right.\right.}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n + a^7*n + 7*(3*b^
2*c^5 + a*c^6)*n*x^(12*n) + 7*(5*b^3*c^4 + 6*a*b*c^5)*n*x^(11*n) + 7*(5*b^4
*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*n*x^(10*n) + 7*(3*b^5*c^2 + 20*a*b^3*c^3 +
 15*a^2*b*c^4)*n*x^(9*n) + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3
*c^4)*n*x^(8*n) + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*c^3)*n*x^
(7*n) + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*n*x^(6*n) + 7
*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*n*x^(5*n) + 7*(5*a^3*b^4 + 15*a^
4*b^2*c + 3*a^5*c^2)*n*x^(4*n) + 7*(5*a^4*b^3 + 6*a^5*b*c)*n*x^(3*n) + 7*(3
*a^5*b^2 + a^6*c)*n*x^(2*n))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(21) = 42$.

Time = 0.54 (sec) , antiderivative size = 416, normalized size of antiderivative = 18.09

$$\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx =$$

$$-\frac{}{7\left(c^7nx^{14n}+7\,bc^6nx^{13n}+7\,a^6bnx^n+a^7n+7\,(3\,b^2c^5n+ac^6n)x^{12n}+7\,(5\,b^3c^4n+6\,abc^5n)x^{11n}+7\,(5\,b^4\right.}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n + a^7*n + 7*(3*b^
2*c^5*n + a*c^6*n)*x^(12*n) + 7*(5*b^3*c^4*n + 6*a*b*c^5*n)*x^(11*n) + 7*(5
*b^4*c^3*n + 15*a*b^2*c^4*n + 3*a^2*c^5*n)*x^(10*n) + 7*(3*b^5*c^2*n + 20*a
*b^3*c^3*n + 15*a^2*b*c^4*n)*x^(9*n) + 7*(b^6*c*n + 15*a*b^4*c^2*n + 30*a^2
*b^2*c^3*n + 5*a^3*c^4*n)*x^(8*n) + (b^7*n + 42*a*b^5*c*n + 210*a^2*b^3*c^2
*n + 140*a^3*b*c^3*n)*x^(7*n) + 7*(a*b^6*n + 15*a^2*b^4*c*n + 30*a^3*b^2*c^
2*n + 5*a^4*c^3*n)*x^(6*n) + 7*(3*a^2*b^5*n + 20*a^3*b^3*c*n + 15*a^4*b*c^2
*n)*x^(5*n) + 7*(5*a^3*b^4*n + 15*a^4*b^2*c*n + 3*a^5*c^2*n)*x^(4*n) + 7*(5
*a^4*b^3*n + 6*a^5*b*c*n)*x^(3*n) + 7*(3*a^5*b^2*n + a^6*c*n)*x^(2*n))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx = -\frac{1}{7\left(cx^{2n}+bx^n+a\right)^7 n}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="giac")
[Out] -1/7/((c*x^(2*n) + b*x^n + a)^7*n)
```

Mupad [B] (verification not implemented)

Time = 22.40 (sec) , antiderivative size = 496, normalized size of antiderivative = 21.57

$$\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx =$$

$$-\frac{}{7\,a^7\,n+7\,b^7\,n\,x^{7\,n}+7\,c^7\,n\,x^{14\,n}+49\,a^6\,b\,n\,x^n+49\,a\,b^6\,n\,x^{6\,n}+49\,a^6\,c\,n\,x^{2\,n}+49\,a\,c^6\,n\,x^{12\,n}+49\,b^6\,c\,n}$$

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x)
[Out] -1/(7*a^7*n + 7*b^7*n*x^(7*n) + 7*c^7*n*x^(14*n) + 49*a^6*b*n*x^n + 49*a*b^
6*n*x^(6*n) + 49*a^6*c*n*x^(2*n) + 49*a*c^6*n*x^(12*n) + 49*b^6*c*n*x^(8*n)
 + 49*b*c^6*n*x^(13*n) + 147*a^5*b^2*n*x^(2*n) + 245*a^4*b^3*n*x^(3*n) + 24
5*a^3*b^4*n*x^(4*n) + 147*a^2*b^5*n*x^(5*n) + 147*a^5*c^2*n*x^(4*n) + 245*a
^4*c^3*n*x^(6*n) + 245*a^3*c^4*n*x^(8*n) + 147*a^2*c^5*n*x^(10*n) + 147*b^5
*c^2*n*x^(9*n) + 245*b^4*c^3*n*x^(10*n) + 245*b^3*c^4*n*x^(11*n) + 147*b^2*
c^5*n*x^(12*n) + 735*a^4*b^2*c*n*x^(4*n) + 980*a^3*b^3*c*n*x^(5*n) + 735*a^
4*b*c^2*n*x^(5*n) + 735*a^2*b^4*c*n*x^(6*n) + 980*a^3*b*c^3*n*x^(7*n) + 735
*a*b^4*c^2*n*x^(8*n) + 980*a*b^3*c^3*n*x^(9*n) + 735*a^2*b*c^4*n*x^(9*n) +
735*a*b^2*c^4*n*x^(10*n) + 1470*a^3*b^2*c^2*n*x^(6*n) + 1470*a^2*b^3*c^2*n*
x^(7*n) + 1470*a^2*b^2*c^3*n*x^(8*n) + 294*a^5*b*c*n*x^(3*n) + 294*a*b^5*c*
n*x^(7*n) + 294*a*b*c^5*n*x^(11*n))
```

3.113 $\int \frac{b+2cx}{-a+bx+cx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 13

$$\int \frac{b+2cx}{-a+bx+cx^2}\,dx = \log\left(a-bx-cx^2\right)$$

```
[Out] ln(-c*x^2-b*x+a)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {642}

$$\int \frac{b+2cx}{-a+bx+cx^2}\,dx = \log\left(a-bx-cx^2\right)$$

```
[In] Int[(b + 2*c*x)/(-a + b*x + c*x^2),x]
```

```
[Out] Log[a - b*x - c*x^2]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\text{integral} = \log\left(a-bx-cx^2\right)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{b+2cx}{-a+bx+cx^2}\,dx=\log(-a+x(b+cx))$$

```
[In] Integrate[(b + 2*c*x)/(-a + b*x + c*x^2),x]
```

```
[Out] Log[-a + x*(b + c*x)]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\ln\left(c\,x^2+bx-a\right)$	14
default	$\ln\left(-c\,x^2-bx+a\right)$	14
norman	$\ln\left(-c\,x^2-bx+a\right)$	14
risch	$\ln\left(-c\,x^2-bx+a\right)$	14
parallelrisch	$\ln\left(c\,x^2+bx-a\right)$	14

```
[In] int((2*c*x+b)/(c*x^2+b*x-a),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*x^2+b*x-a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{-a+bx+cx^2}\,dx=\log\left(cx^2+bx-a\right)$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x - a)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{b+2cx}{-a+bx+cx^2}\,dx=\log\left(-a+bx+cx^2\right)$$

```
[In] integrate((2*c*x+b)/(c*x**2+b*x-a),x)
[Out] log(-a + b*x + c*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{-a+bx+cx^2}\,dx=\log\left(cx^2+bx-a\right)$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="maxima")
[Out] log(c*x^2 + b*x - a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{b+2cx}{-a+bx+cx^2}\,dx=\log\left(\left|cx^2+bx-a\right|\right)$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="giac")
[Out] log(abs(c*x^2 + b*x - a))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{-a+bx+cx^2}\,dx=\ln\left(c\,x^2+b\,x-a\right)$$

```
[In] int((b + 2*c*x)/(b*x - a + c*x^2),x)
[Out] log(b*x - a + c*x^2)
```

3.114 $\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 19

$$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4} dx = \frac{1}{2}\log\left(a-bx^2-cx^4\right)$$

```
[Out] 1/2*ln(-c*x^4-b*x^2+a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1261, 642}

$$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4} dx = \frac{1}{2}\log\left(a-bx^2-cx^4\right)$$

```
[In] Int[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4),x]
```

```
[Out] Log[a - b*x^2 - c*x^4]/2
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{-a+bx+cx^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\log\left(a-bx^2-cx^4\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx = \frac{1}{2}\log\left(-a+bx^2+cx^4\right)$$

```
[In] Integrate[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4),x]
[Out] Log[-a + b*x^2 + c*x^4]/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln\left(-c\,x^{4}-b\,x^{2}+a\right)}{2}$	18
norman	$\frac{\ln\left(-c\,x^{4}-b\,x^{2}+a\right)}{2}$	18
risch	$\frac{\ln\left(-c\,x^{4}-b\,x^{2}+a\right)}{2}$	18
parallelrisch	$\frac{\ln\left(c\,x^{4}+b\,x^{2}-a\right)}{2}$	18

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(-c*x^4-b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx = \frac{1}{2}\,\log\left(cx^4+bx^2-a\right)$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="fricas")
[Out] 1/2*log(c*x^4 + b*x^2 - a)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx = \frac{\log\left(-a+bx^2+cx^4\right)}{2}$$

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a),x)
[Out] log(-a + b*x**2 + c*x**4)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx = \frac{1}{2}\,\log\left(cx^4+bx^2-a\right)$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="maxima")
[Out] 1/2*log(c*x^4 + b*x^2 - a)
```

Giac [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx = \frac{1}{2}\,\log\left(\left|cx^4+bx^2-a\right|\right)$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="giac")
[Out] 1/2*log(abs(c*x^4 + b*x^2 - a))
```

Mupad [B] (verification not implemented)

Time = 8.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx=\frac{\ln\left(c\,x^4+b\,x^2-a\right)}{2}$$

```
[In] int((x*(b + 2*c*x^2))/(b*x^2 - a + c*x^4),x)
[Out] log(b*x^2 - a + c*x^4)/2
```

3.115 $\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6} dx$

Optimal result

Integrand size = 28, antiderivative size = 19

$$\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6} dx = \frac{1}{3}\log\left(a-bx^3-cx^6\right)$$

```
[Out] 1/3*ln(-c*x^6-b*x^3+a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1482, 642}

$$\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6} dx = \frac{1}{3}\log\left(a-bx^3-cx^6\right)$$

```
[In] Int[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6),x]
```

```
[Out] Log[a - b*x^3 - c*x^6]/3
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
```

```
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{-a+bx+cx^2}\,dx, x, x^3\right) \\ &= \frac{1}{3}\log\left(a-bx^3-cx^6\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(-a+bx^3+cx^6\right)$$

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6),x]
[Out] Log[-a + b*x^3 + c*x^6]/3
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln(-c\,x^6-b\,x^3+a)}{3}$	18
norman	$\frac{\ln(-c\,x^6-b\,x^3+a)}{3}$	18
risch	$\frac{\ln(-c\,x^6-b\,x^3+a)}{3}$	18
parallelrisch	$\frac{\ln(c\,x^6+b\,x^3-a)}{3}$	18

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(-c*x^6-b*x^3+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(cx^6+bx^3-a\right)$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="fricas")
[Out] 1/3*log(c*x^6 + b*x^3 - a)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx = \frac{\log\left(-a+bx^3+cx^6\right)}{3}$$

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a),x)
[Out] log(-a + b*x**3 + c*x**6)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(cx^6+bx^3-a\right)$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="maxima")
[Out] 1/3*log(c*x^6 + b*x^3 - a)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx = \frac{1}{3}\log\left(\left|cx^6+bx^3-a\right|\right)$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="giac")
[Out] 1/3*log(abs(c*x^6 + b*x^3 - a))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx=\frac{\ln\left(c\,x^6+b\,x^3-a\right)}{3}$$

```
[In] int((x^2*(b + 2*c*x^3))/(b*x^3 - a + c*x^6),x)
[Out] log(b*x^3 - a + c*x^6)/3
```

3.116 $\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 21

$$\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}} \, dx = \frac{\log(a-bx^n-cx^{2n})}{n}$$

```
[Out] ln(a-b*x^n-c*x^(2*n))/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1482, 642}

$$\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}} \, dx = \frac{\log(a-bx^n-cx^{2n})}{n}$$

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n)),x]
[Out] Log[a - b*x^n - c*x^(2*n)]/n
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
```

```
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{b+2cx}{-a+bx+cx^2}\,dx, x, x^n\right)}{n}$$
$$= \frac{\log\left(a - bx^n - cx^{2n}\right)}{n}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}}\,dx = -\frac{2\log\left(x^{-n}\right)}{n} + \frac{\log\left(c - ax^{-2n} + bx^{-n}\right)}{n}$$

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n)),x]
[Out] (-2*Log[x^(-n)])/n + Log[c - a/x^(2*n) + b/x^n]/n
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

method	result	size
norman	$\frac{\ln\left(-c\,{\mathrm e}^{2 n \ln\left(x \right)}-b\,{\mathrm e}^{n \ln\left(x \right)}+a \right)}{n}$	26
risch	$\frac{\ln\left(x^{2 n}+\frac{b\,x^{n}}{c}-\frac{a}{c}\right)}{n}$	26

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] 1/n*ln(-c*exp(n*ln(x))^2-b*exp(n*ln(x))+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}}\,dx = \frac{\log\left(cx^{2\,n} + bx^n - a\right)}{n}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] log(c*x^(2*n) + b*x^n - a)/n
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}}\,dx = \frac{\log\left(\frac{cx^{2\,n}+bx^n-a}{c}\right)}{n}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] log((c*x^(2*n) + b*x^n - a)/c)/n
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}}\,dx = \frac{\log\left(cx^{2\,n}+bx^n-a\right)}{n}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] log(c*x^(2*n) + b*x^n - a)/n
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 199, normalized size of antiderivative = 9.48

$$\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}}\,dx = \ln\left(\frac{2\,c\,x^n}{n}-\left(\frac{1}{n}+\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)\,(b+2\,c\,x^n)\right)\,\left(\frac{1}{n}+\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)+\ln\left(\frac{2\,c\,x^n}{n}-\left(\frac{1}{n}-\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)\,(b+2\,c\,x^n)\right)\,\left(\frac{1}{n}-\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)-\frac{2\,b\,\mathrm{atanh}\left(\frac{b+2\,c\,x^n}{\sqrt{b^2+4\,a\,c}}\right)}{n\,\sqrt{b^2+4\,a\,c}}$$

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n - a + c*x^(2*n)),x)
```

```
[Out] log((2*c*x^n)/n - (1/n + (b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n))*(b + 2*
c*x^n))*(1/n + (b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n)) + log((2*c*x^n)/n
 - (1/n - (b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n))*(b + 2*c*x^n))*(1/n -
(b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n)) - (2*b*atanh((b + 2*c*x^n)/(4*a*
c + b^2)^(1/2)))/(n*(4*a*c + b^2)^(1/2))
```

3.117 $\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx = \frac{1}{7\,(a-bx-cx^2)^7}$$

```
[Out] 1/7/(-c*x^2-b*x+a)^7
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {643}

$$\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx = \frac{1}{7\,(a-bx-cx^2)^7}$$

```
[In] Int[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]
```

```
[Out] 1/(7*(a - b*x - c*x^2)^7)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{7\,(a-bx-cx^2)^7}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx = \frac{1}{7(a-x(b+cx))^7}$$

```
[In] Integrate[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]
[Out] 1/(7*(a - x*(b + c*x))^7)
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{1}{7(-c\,x^2-bx+a)^7}$	17
derivativedivides	$-\frac{1}{7(c\,x^2+bx-a)^7}$	17
default	$\frac{1}{7(-c\,x^2-bx+a)^7}$	17
norman	$\frac{1}{7(-c\,x^2-bx+a)^7}$	17
risch	$\frac{1}{7(-c\,x^2-bx+a)^7}$	17
parallelrisch	$-\frac{1}{7(c\,x^2+bx-a)^7}$	17

```
[In] int((2*c*x+b)/(c*x^2+b*x-a)^8,x,method=_RETURNVERBOSE)
[Out] 1/7/(-c*x^2-b*x+a)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 354, normalized size of antiderivative = 19.67

$$\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx =$$

$$-\frac{}{7\left(c^7x^{14}+7\,bc^6x^{13}+7\left(3\,b^2c^5-ac^6\right)x^{12}+7\left(5\,b^3c^4-6\,abc^5\right)x^{11}+7\left(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5\right)x^{10}+7\,(3\right.}$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="fricas")
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 7*(3*b^2*c^5 - a*c^6)*x^12 + 7*(5*b^3*c^4 -
 6*a*b*c^5)*x^11 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^10 + 7*(3*b^5
```

```
*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^9 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^2*
b^2*c^3 - 5*a^3*c^4)*x^8 + 7*a^6*b*x + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2
- 140*a^3*b*c^3)*x^7 - a^7 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a
^4*c^3)*x^6 + 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^5 - 7*(5*a^3*b^
4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^4 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^3 - 7*(3*a^
5*b^2 - a^6*c)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(14) = 28$.

Time = 2.75 (sec) , antiderivative size = 359, normalized size of antiderivative = 19.94

$$\int \frac{b+2cx}{\left(-a+bx+cx^2\right)^8}\,dx =$$

$$-\frac{1}{-7a^7+49a^6bx+49bc^6x^{13}+7c^7x^{14}+x^{12}\left(-49ac^6+147b^2c^5\right)+x^{11}\left(-294abc^5+245b^3c^4\right)+x^{10}\cdot\left(147a\right.}$$

```
[In] integrate((2*c*x+b)/(c*x**2+b*x-a)**8,x)
```

```
[Out] -1/(-7*a**7 + 49*a**6*b*x + 49*b*c**6*x**13 + 7*c**7*x**14 + x**12*(-49*a*c
**6 + 147*b**2*c**5) + x**11*(-294*a*b*c**5 + 245*b**3*c**4) + x**10*(147*a
**2*c**5 - 735*a*b**2*c**4 + 245*b**4*c**3) + x**9*(735*a**2*b*c**4 - 980*a
*b**3*c**3 + 147*b**5*c**2) + x**8*(-245*a**3*c**4 + 1470*a**2*b**2*c**3 -
735*a*b**4*c**2 + 49*b**6*c) + x**7*(-980*a**3*b*c**3 + 1470*a**2*b**3*c**2
 - 294*a*b**5*c + 7*b**7) + x**6*(245*a**4*c**3 - 1470*a**3*b**2*c**2 + 735
*a**2*b**4*c - 49*a*b**6) + x**5*(735*a**4*b*c**2 - 980*a**3*b**3*c + 147*a
**2*b**5) + x**4*(-147*a**5*c**2 + 735*a**4*b**2*c - 245*a**3*b**4) + x**3*
(-294*a**5*b*c + 245*a**4*b**3) + x**2*(49*a**6*c - 147*a**5*b**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{b+2cx}{\left(-a+bx+cx^2\right)^8}\,dx = -\frac{1}{7\left(cx^2+bx-a\right)^7}$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x - a)^7
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx = -\frac{1}{7\,(cx^2+bx-a)^7}$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="giac")
[Out] -1/7/(c*x^2 + b*x - a)^7
```

Mupad [B] (verification not implemented)

Time = 10.65 (sec) , antiderivative size = 358, normalized size of antiderivative = 19.89

$$\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx =$$

$$-\frac{}{7\,\left(x^5\,\left(105\,a^4\,b\,c^2-140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^9\,\left(105\,a^2\,b\,c^4-140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^7\,\left(-140\,a^3\,b\,c^3+2\right.\right.}$$

```
[In] int((b + 2*c*x)/(b*x - a + c*x^2)^8,x)
[Out] -1/(7*(x^5*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^9*(21*b^5*c^2 -
 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^7*(b^7 - 140*a^3*b*c^3 + 210*a^2*b^3*c^
2 - 42*a*b^5*c) + x^3*(35*a^4*b^3 - 42*a^5*b*c) + x^11*(35*b^3*c^4 - 42*a*b
*c^5) - x^4*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^10*(21*a^2*c^5 +
35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^6*(7*a*b^6 - 35*a^4*c^3 - 105*a^2*b^4
*c + 210*a^3*b^2*c^2) + x^8*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c^2 + 210*a^2
*b^2*c^3) + c^7*x^14 + x^2*(7*a^6*c - 21*a^5*b^2) - x^12*(7*a*c^6 - 21*b^2*
c^5) + 7*b*c^6*x^13 + 7*a^6*b*x))
```

3.118 $\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8} dx$

Optimal result

Integrand size = 26, antiderivative size = 20

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8} dx = \frac{1}{14(a-bx^2-cx^4)^7}$$

```
[Out] 1/14/(-c*x^4-b*x^2+a)^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1261, 643}

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8} dx = \frac{1}{14(a-bx^2-cx^4)^7}$$

```
[In] Int[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]
```

```
[Out] 1/(14*(a - b*x^2 - c*x^4)^7)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx, x, x^2\right)\\ &= \frac{1}{14\,(a-bx^2-cx^4)^7}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx = -\frac{1}{14\,(-a+bx^2+cx^4)^7}$$

```
[In] Integrate[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]
[Out] -1/14*1/(-a + b*x^2 + c*x^4)^7
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{1}{14(-c\,x^4-b\,x^2+a)^7}$	19
default	$\frac{1}{14(-c\,x^4-b\,x^2+a)^7}$	19
norman	$\frac{1}{14(-c\,x^4-b\,x^2+a)^7}$	19
risch	$\frac{1}{14(-c\,x^4-b\,x^2+a)^7}$	19
parallelrisch	$-\frac{1}{14(c\,x^4+b\,x^2-a)^7}$	19

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x,method=_RETURNVERBOSE)
[Out] 1/14/(-c*x^4-b*x^2+a)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(18) = 36$.

Time = 0.30 (sec) , antiderivative size = 356, normalized size of antiderivative = 17.80

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx =$$

$$-\frac{}{14\,(c^7x^{28}+7\,bc^6x^{26}+7\,(3\,b^2c^5-ac^6)x^{24}+7\,(5\,b^3c^4-6\,abc^5)x^{22}+7\,(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5)x^{20}+7}$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 - a*c^6)*x^24 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^16 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^14 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 - 7*(5*a^3*
b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^8 - a^7 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^6 -
 7*(3*a^5*b^2 - a^6*c)*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(15) = 30$.

Time = 4.36 (sec) , antiderivative size = 360, normalized size of antiderivative = 18.00

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx =$$

$$-\frac{}{-14a^7+98a^6bx^2+98bc^6x^{26}+14c^7x^{28}+x^{24}\,(-98ac^6+294b^2c^5)+x^{22}\,(-588abc^5+490b^3c^4)+x^{20}\cdot(}$$

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a)**8,x)
```

```
[Out] -1/(-14*a**7 + 98*a**6*b*x**2 + 98*b*c**6*x**26 + 14*c**7*x**28 + x**24*(-9
8*a*c**6 + 294*b**2*c**5) + x**22*(-588*a*b*c**5 + 490*b**3*c**4) + x**20*(
294*a**2*c**5 - 1470*a*b**2*c**4 + 490*b**4*c**3) + x**18*(1470*a**2*b*c**4
 - 1960*a*b**3*c**3 + 294*b**5*c**2) + x**16*(-490*a**3*c**4 + 2940*a**2*b*
*2*c**3 - 1470*a*b**4*c**2 + 98*b**6*c) + x**14*(-1960*a**3*b*c**3 + 2940*a
**2*b**3*c**2 - 588*a*b**5*c + 14*b**7) + x**12*(490*a**4*c**3 - 2940*a**3*
b**2*c**2 + 1470*a**2*b**4*c - 98*a*b**6) + x**10*(1470*a**4*b*c**2 - 1960*
a**3*b**3*c + 294*a**2*b**5) + x**8*(-294*a**5*c**2 + 1470*a**4*b**2*c - 49
0*a**3*b**4) + x**6*(-588*a**5*b*c + 490*a**4*b**3) + x**4*(98*a**6*c - 294
*a**5*b**2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(18) = 36.

Time = 0.29 (sec) , antiderivative size = 356, normalized size of antiderivative = 17.80

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx =$$

$$-\frac{}{14\,(c^7x^{28}+7\,bc^6x^{26}+7\,(3\,b^2c^5-ac^6)x^{24}+7\,(5\,b^3c^4-6\,abc^5)x^{22}+7\,(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5)x^{20}+7\,(}$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 - a*c^6)*x^24 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^16 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^14 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 - 7*(5*a^3*
b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^8 - a^7 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^6 -
 7*(3*a^5*b^2 - a^6*c)*x^4)
```

Giac [A] (verification not implemented)

none

Time = 1.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx = -\frac{1}{14\,(cx^4+bx^2-a)^7}$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2 - a)^7
```

Mupad [B] (verification not implemented)

Time = 15.10 (sec) , antiderivative size = 360, normalized size of antiderivative = 18.00

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx =$$

$$-\frac{}{14\,\left(x^{10}\,\left(105\,a^4\,b\,c^2-140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^{18}\,\left(105\,a^2\,b\,c^4-140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^{14}\,\left(-140\,a^3\,b\,c^3\right.}$$

```
[In] int((x*(b + 2*c*x^2))/(b*x^2 - a + c*x^4)^8,x)
```

```
[Out] -1/(14*(x^10*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^18*(21*b^5*c^
2 - 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^14*(b^7 - 140*a^3*b*c^3 + 210*a^2*b^
3*c^2 - 42*a*b^5*c) + x^6*(35*a^4*b^3 - 42*a^5*b*c) + x^22*(35*b^3*c^4 - 42
*a*b*c^5) - x^8*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^20*(21*a^2*c^
5 + 35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^12*(7*a*b^6 - 35*a^4*c^3 - 105*a^
2*b^4*c + 210*a^3*b^2*c^2) + x^16*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c^2 + 2
10*a^2*b^2*c^3) + c^7*x^28 + x^4*(7*a^6*c - 21*a^5*b^2) - x^24*(7*a*c^6 - 2
1*b^2*c^5) + 7*a^6*b*x^2 + 7*b*c^6*x^26))
```

3.119 $\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 20

$$\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx = \frac{1}{21\,(a-bx^3-cx^6)^7}$$

```
[Out] 1/21/(-c*x^6-b*x^3+a)^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1482, 643}

$$\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx = \frac{1}{21\,(a-bx^3-cx^6)^7}$$

```
[In] Int[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]
```

```
[Out] 1/(21*(a - b*x^3 - c*x^6)^7)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
```

```
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx, x, x^3\right)\\ &= \frac{1}{21\,(a-bx^3-cx^6)^7}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx = -\frac{1}{21\,(-a+bx^3+cx^6)^7}$$

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]
[Out] -1/21*1/(-a + b*x^3 + c*x^6)^7
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{1}{21(-c\,x^6-b\,x^3+a)^7}$	19
default	$\frac{1}{21(-c\,x^6-b\,x^3+a)^7}$	19
risch	$\frac{1}{21(-c\,x^6-b\,x^3+a)^7}$	19
parallelrisch	$-\frac{1}{21(c\,x^6+b\,x^3-a)^7}$	19

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x,method=_RETURNVERBOSE)
[Out] 1/21/(-c*x^6-b*x^3+a)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(18) = 36.

Time = 0.30 (sec) , antiderivative size = 356, normalized size of antiderivative = 17.80

$$\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx =$$

$$-\frac{}{21\left(c^7x^{42}+7\,bc^6x^{39}+7\left(3\,b^2c^5-ac^6\right)x^{36}+7\left(5\,b^3c^4-6\,abc^5\right)x^{33}+7\left(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5\right)x^{30}+7\left(\right.\right.}$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 - a*c^6)*x^36 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^24 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^21 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 - 7*(5*a^3*b^4 - 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^9 - a^7
- 7*(3*a^5*b^2 - a^6*c)*x^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(15) = 30.

Time = 14.85 (sec) , antiderivative size = 360, normalized size of antiderivative = 18.00

$$\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx =$$

$$-\frac{}{-21a^7+147a^6bx^3+147bc^6x^{39}+21c^7x^{42}+x^{36}\left(-147ac^6+441b^2c^5\right)+x^{33}\left(-882abc^5+735b^3c^4\right)+x^{30}}$$

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a)**8,x)
```

```
[Out] -1/(-21*a**7 + 147*a**6*b*x**3 + 147*b*c**6*x**39 + 21*c**7*x**42 + x**36*(
-147*a*c**6 + 441*b**2*c**5) + x**33*(-882*a*b*c**5 + 735*b**3*c**4) + x**3
0*(441*a**2*c**5 - 2205*a*b**2*c**4 + 735*b**4*c**3) + x**27*(2205*a**2*b*c
**4 - 2940*a*b**3*c**3 + 441*b**5*c**2) + x**24*(-735*a**3*c**4 + 4410*a**2
*b**2*c**3 - 2205*a*b**4*c**2 + 147*b**6*c) + x**21*(-2940*a**3*b*c**3 + 44
10*a**2*b**3*c**2 - 882*a*b**5*c + 21*b**7) + x**18*(735*a**4*c**3 - 4410*a
**3*b**2*c**2 + 2205*a**2*b**4*c - 147*a*b**6) + x**15*(2205*a**4*b*c**2 -
2940*a**3*b**3*c + 441*a**2*b**5) + x**12*(-441*a**5*c**2 + 2205*a**4*b**2*
c - 735*a**3*b**4) + x**9*(-882*a**5*b*c + 735*a**4*b**3) + x**6*(147*a**6*
c - 441*a**5*b**2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(18) = 36.

Time = 0.28 (sec) , antiderivative size = 356, normalized size of antiderivative = 17.80

$$\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx =$$

$$-\frac{}{21\left(c^7x^{42}+7\,bc^6x^{39}+7\left(3\,b^2c^5-ac^6\right)x^{36}+7\left(5\,b^3c^4-6\,abc^5\right)x^{33}+7\left(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5\right)x^{30}+7\right.}$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 - a*c^6)*x^36 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^24 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^21 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 - 7*(5*a^3*b^4 - 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^9 - a^7
- 7*(3*a^5*b^2 - a^6*c)*x^6)
```

Giac [A] (verification not implemented)

none

Time = 2.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx = -\frac{1}{21\,(cx^6+bx^3-a)^7}$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3 - a)^7
```

Mupad [B] (verification not implemented)

Time = 17.24 (sec) , antiderivative size = 360, normalized size of antiderivative = 18.00

$$\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx =$$

$$-\frac{}{21\,\left(x^{15}\,\left(105\,a^4\,b\,c^2-140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^{27}\,\left(105\,a^2\,b\,c^4-140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^{21}\,\left(-140\,a^3\,b\,c\right.\right.}$$

```
[In] int((x^2*(b + 2*c*x^3))/(b*x^3 - a + c*x^6)^8,x)
```

```
[Out] -1/(21*(x^15*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^27*(21*b^5*c^
2 - 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^21*(b^7 - 140*a^3*b*c^3 + 210*a^2*b^
3*c^2 - 42*a*b^5*c) + x^9*(35*a^4*b^3 - 42*a^5*b*c) + x^33*(35*b^3*c^4 - 42
*a*b*c^5) - x^12*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^30*(21*a^2*c
^5 + 35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^18*(7*a*b^6 - 35*a^4*c^3 - 105*a
^2*b^4*c + 210*a^3*b^2*c^2) + x^24*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c^2 +
210*a^2*b^2*c^3) + c^7*x^42 + x^6*(7*a^6*c - 21*a^5*b^2) - x^36*(7*a*c^6 -
21*b^2*c^5) + 7*a^6*b*x^3 + 7*b*c^6*x^39))
```

3.120 $\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8} dx$

Optimal result

Integrand size = 32, antiderivative size = 25

$$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8} dx = \frac{1}{7n(a-bx^n-cx^{2n})^7}$$

```
[Out] 1/7/n/(a-b*x^n-c*x^(2*n))^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1482, 643}

$$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8} dx = \frac{1}{7n(a-bx^n-cx^{2n})^7}$$

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] 1/(7*n*(a - b*x^n - c*x^(2*n))^7)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
```

```
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx, x, x^n\right)}{n}$$
$$= \frac{1}{7n\,(a-bx^n-cx^{2n})^7}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx = \frac{1}{7n\,(a-x^n\,(b+cx^n))^7}$$

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]
[Out] 1/(7*n*(a - x^n*(b + c*x^n))^7)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\frac{1}{7n\,(a-b\,x^n-c\,x^{2n})^7}$$

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x)
[Out] 1/7/n/(-c*(x^n)^2-b*x^n+a)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(23) = 46.

Time = 0.31 (sec) , antiderivative size = 397, normalized size of antiderivative = 15.88

$$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx =$$
$$-\frac{}{7\,(c^7nx^{14\,n}+7\,bc^6nx^{13\,n}+7\,a^6bnx^n-a^7n+7\,(3\,b^2c^5-ac^6)nx^{12\,n}+7\,(5\,b^3c^4-6\,abc^5)nx^{11\,n}+7\,(5\,b^4c^3}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="fricas"
)
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n - a^7*n + 7*(3*b^
2*c^5 - a*c^6)*n*x^(12*n) + 7*(5*b^3*c^4 - 6*a*b*c^5)*n*x^(11*n) + 7*(5*b^4
*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*n*x^(10*n) + 7*(3*b^5*c^2 - 20*a*b^3*c^3 +
 15*a^2*b*c^4)*n*x^(9*n) + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 5*a^3
*c^4)*n*x^(8*n) + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3*b*c^3)*n*x^
(7*n) - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*n*x^(6*n) + 7
*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*n*x^(5*n) - 7*(5*a^3*b^4 - 15*a^
4*b^2*c + 3*a^5*c^2)*n*x^(4*n) + 7*(5*a^4*b^3 - 6*a^5*b*c)*n*x^(3*n) - 7*(3
*a^5*b^2 - a^6*c)*n*x^(2*n))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx=\text{Timed out}$$

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(23) = 46.

Time = 0.56 (sec) , antiderivative size = 419, normalized size of antiderivative = 16.76

$$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx=$$

$$-\frac{}{7\,(c^7nx^{14\,n}+7\,bc^6nx^{13\,n}+7\,a^6bnx^n-a^7n+7\,(3\,b^2c^5n-ac^6n)x^{12\,n}+7\,(5\,b^3c^4n-6\,abc^5n)x^{11\,n}+7\,(5\,b}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="maxima"
)
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n - a^7*n + 7*(3*b^
2*c^5*n - a*c^6*n)*x^(12*n) + 7*(5*b^3*c^4*n - 6*a*b*c^5*n)*x^(11*n) + 7*(5
*b^4*c^3*n - 15*a*b^2*c^4*n + 3*a^2*c^5*n)*x^(10*n) + 7*(3*b^5*c^2*n - 20*a
*b^3*c^3*n + 15*a^2*b*c^4*n)*x^(9*n) + 7*(b^6*c*n - 15*a*b^4*c^2*n + 30*a^2
*b^2*c^3*n - 5*a^3*c^4*n)*x^(8*n) + (b^7*n - 42*a*b^5*c*n + 210*a^2*b^3*c^2
*n - 140*a^3*b*c^3*n)*x^(7*n) - 7*(a*b^6*n - 15*a^2*b^4*c*n + 30*a^3*b^2*c^
2*n - 5*a^4*c^3*n)*x^(6*n) + 7*(3*a^2*b^5*n - 20*a^3*b^3*c*n + 15*a^4*b*c^2
*n)*x^(5*n) - 7*(5*a^3*b^4*n - 15*a^4*b^2*c*n + 3*a^5*c^2*n)*x^(4*n) + 7*(5
*a^4*b^3*n - 6*a^5*b*c*n)*x^(3*n) - 7*(3*a^5*b^2*n - a^6*c*n)*x^(2*n))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx = -\frac{1}{7\,(cx^{2\,n}+bx^n-a)^7n}$$

[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="giac")

[Out] -1/7/((c*x^(2*n) + b*x^n - a)^7*n)

Mupad [B] (verification not implemented)

Time = 22.03 (sec) , antiderivative size = 496, normalized size of antiderivative = 19.84

$$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx =$$

$$-\frac{}{7\,b^7\,n\,x^{7\,n}-7\,a^7\,n+7\,c^7\,n\,x^{14\,n}+49\,a^6\,b\,n\,x^n-49\,a\,b^6\,n\,x^{6\,n}+49\,a^6\,c\,n\,x^{2\,n}-49\,a\,c^6\,n\,x^{12\,n}+49\,b^6\,c\,n\,x}$$

[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n - a + c*x^(2*n))^8,x)

[Out] -1/(7*b^7*n*x^(7*n) - 7*a^7*n + 7*c^7*n*x^(14*n) + 49*a^6*b*n*x^n - 49*a*b^6*n*x^(6*n) + 49*a^6*c*n*x^(2*n) - 49*a*c^6*n*x^(12*n) + 49*b^6*c*n*x^(8*n) + 49*b*c^6*n*x^(13*n) - 147*a^5*b^2*n*x^(2*n) + 245*a^4*b^3*n*x^(3*n) - 245*a^3*b^4*n*x^(4*n) + 147*a^2*b^5*n*x^(5*n) - 147*a^5*c^2*n*x^(4*n) + 245*a^4*c^3*n*x^(6*n) - 245*a^3*c^4*n*x^(8*n) + 147*a^2*c^5*n*x^(10*n) + 147*b^5*c^2*n*x^(9*n) + 245*b^4*c^3*n*x^(10*n) + 245*b^3*c^4*n*x^(11*n) + 147*b^2*c^5*n*x^(12*n) + 735*a^4*b^2*c*n*x^(4*n) - 980*a^3*b^3*c*n*x^(5*n) + 735*a^4*b*c^2*n*x^(5*n) + 735*a^2*b^4*c*n*x^(6*n) - 980*a^3*b*c^3*n*x^(7*n) - 735*a*b^4*c^2*n*x^(8*n) - 980*a*b^3*c^3*n*x^(9*n) + 735*a^2*b*c^4*n*x^(9*n) - 735*a*b^2*c^4*n*x^(10*n) - 1470*a^3*b^2*c^2*n*x^(6*n) + 1470*a^2*b^3*c^2*n*x^(7*n) + 1470*a^2*b^2*c^3*n*x^(8*n) - 294*a^5*b*c*n*x^(3*n) - 294*a*b^5*c*n*x^(7*n) - 294*a*b*c^5*n*x^(11*n))

3.121 $\int \frac{b+2cx}{bx+cx^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 10

$$\int \frac{b+2cx}{bx+cx^2} dx = \log(bx+cx^2)$$

```
[Out] ln(c*x^2+b*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {642}

$$\int \frac{b+2cx}{bx+cx^2} dx = \log(bx+cx^2)$$

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] Log[b*x + c*x^2]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\text{integral} = \log(bx+cx^2)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{b+2cx}{bx+cx^2}\,dx=\log(x)+\log(b+cx)$$

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2),x]
[Out] Log[x] + Log[b + c*x]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
default	$\ln\left(x(cx+b)\right)$	9
norman	$\ln\left(x\right)+\ln\left(cx+b\right)$	10
parallelrisch	$\ln\left(x\right)+\ln\left(cx+b\right)$	10
derivativedivides	$\ln\left(c\,x^2+bx\right)$	11
risch	$\ln\left(c\,x^2+bx\right)$	11

```
[In] int((2*c*x+b)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
[Out] ln(x*(c*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{bx+cx^2}\,dx=\log\left(cx^2+bx\right)$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="fricas")
[Out] log(c*x^2 + b*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{bx+cx^2}\,dx=\log\left(bx+cx^2\right)$$

```
[In] integrate((2*c*x+b)/(c*x**2+b*x),x)
[Out] log(b*x + c*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{bx+cx^2}\,dx=\log\left(cx^2+bx\right)$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="maxima")
[Out] log(c*x^2 + b*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{b+2cx}{bx+cx^2}\,dx=\log\left(\left|cx^2+bx\right|\right)$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="giac")
[Out] log(abs(c*x^2 + b*x))
```

Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{bx+cx^2}\,dx=\ln\left(x\left(b+c\,x\right)\right)$$

```
[In] int((b + 2*c*x)/(b*x + c*x^2),x)
[Out] log(x*(b + c*x))
```

3.122 $\int \frac{x(b+2cx^2)}{bx^2+cx^4} dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4} dx = \frac{1}{2}\log\left(bx^2+cx^4\right)$$

```
[Out] 1/2*ln(c*x^4+b*x^2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1598, 457, 78}

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4} dx = \frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

```
[In] Int[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx, x, x^2\right) \\
&= \log(x)+\frac{1}{2}\log\left(b+cx^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx = \log(x)+\frac{1}{2}\log\left(b+cx^2\right)$$

```
[In] Integrate[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x]
[Out] Log[x] + Log[b + c*x^2]/2
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
default	$\ln(x)+\frac{\ln(c x^2+b)}{2}$	14
norman	$\ln(x)+\frac{\ln(c x^2+b)}{2}$	14
risch	$\ln(x)+\frac{\ln(c x^2+b)}{2}$	14
parallelrisch	$\ln(x)+\frac{\ln(c x^2+b)}{2}$	14

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] ln(x)+1/2*ln(c*x^2+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx = \frac{1}{2}\log\left(cx^2+b\right)+\log(x)$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="fricas")
[Out] 1/2*log(c*x^2 + b) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx = \log(x)+\frac{\log\left(\frac{b}{c}+x^2\right)}{2}$$

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2),x)
[Out] log(x) + log(b/c + x**2)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx = \frac{1}{2}\log\left(cx^2+b\right)+\frac{1}{2}\log\left(x^2\right)$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="maxima")
[Out] 1/2*log(c*x^2 + b) + 1/2*log(x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx=\frac{1}{2}\,\log\left(\left|cx^4+bx^2\right|\right)$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="giac")
[Out] 1/2*log(abs(c*x^4 + b*x^2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx=\frac{\ln\left(c\,x^2+b\right)}{2}+\ln\left(x\right)$$

```
[In] int((x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x)
[Out] log(b + c*x^2)/2 + log(x)
```

3.123 $\int \frac{x^2(b+2cx^3)}{bx^3+cx^6} dx$

Optimal result

Integrand size = 25, antiderivative size = 16

$$\int \frac{x^2(b+2cx^3)}{bx^3+cx^6} dx = \frac{1}{3}\log\left(bx^3+cx^6\right)$$

```
[Out] 1/3*ln(c*x^6+b*x^3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1598, 457, 78}

$$\int \frac{x^2(b+2cx^3)}{bx^3+cx^6} dx = \frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

```
[In] Int[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx\\ &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx, x, x^3\right)\\ &= \log(x)+\frac{1}{3}\log\left(b+cx^3\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx = \log(x)+\frac{1}{3}\log\left(b+cx^3\right)$$

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]
[Out] Log[x] + Log[b + c*x^3]/3
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
default	$\ln(x)+\frac{\ln(cx^3+b)}{3}$	14
norman	$\ln(x)+\frac{\ln(cx^3+b)}{3}$	14
risch	$\ln(x)+\frac{\ln(cx^3+b)}{3}$	14
parallelrisch	$\ln(x)+\frac{\ln(cx^3+b)}{3}$	14

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x,method=_RETURNVERBOSE)
[Out] ln(x)+1/3*ln(c*x^3+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx=\frac{1}{3}\log\left(cx^3+b\right)+\log(x)$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="fricas")
[Out] 1/3*log(c*x^3 + b) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx=\log(x)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3),x)
[Out] log(x) + log(b/c + x**3)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx=\frac{1}{3}\log\left(cx^3+b\right)+\frac{1}{3}\log\left(x^3\right)$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="maxima")
[Out] 1/3*log(c*x^3 + b) + 1/3*log(x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx = \frac{1}{3}\,\log\left(\left|cx^6+bx^3\right|\right)$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="giac")
[Out] 1/3*log(abs(c*x^6 + b*x^3))
```

Mupad [B] (verification not implemented)

Time = 8.65 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx = \frac{\ln\left(c\,x^3+b\right)}{3}+\ln\left(x\right)$$

```
[In] int((x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x)
[Out] log(b + c*x^3)/3 + log(x)
```

3.124 $\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}} dx$

Optimal result

Integrand size = 29, antiderivative size = 15

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}} dx = \log(x) + \frac{\log(b+cx^n)}{n}$$

```
[Out] ln(x)+ln(b+c*x^n)/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1598, 457, 78}

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}} dx = \frac{\log(b+cx^n)}{n} + \log(x)$$

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{b+2cx^n}{x\,(b+cx^n)}\,dx \\ &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^n\right)}{n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx, x, x^n\right)}{n} \\ &= \log(x)+\frac{\log\left(b+cx^n\right)}{n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx = \frac{\log\left(x^n\right)+\log\left(n(b+cx^n)\right)}{n}$$

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x]
[Out] (Log[x^n] + Log[n*(b + c*x^n)])/n
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
norman	$\ln\left(x\right)+\frac{\ln\left(c\,{\mathrm e}^{n\ln\left(x\right)}+b\right)}{n}$	18
risch	$\ln\left(x\right)+\frac{\ln\left(x^{n}+\frac{b}{c}\right)}{n}$	18

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] ln(x)+1/n*ln(c*exp(n*ln(x))+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx = \frac{n\log(x)+\log(cx^n+b)}{n}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] (n*log(x) + log(c*x^n + b))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(12) = 24.

Time = 7.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx = \begin{cases} \log(x) & \text{for}\: c=0 \wedge n=0 \\ -\frac{\log(x^{-n})}{n} & \text{for}\: c=0 \\ \frac{(b+2c)\log(x)}{b+c} & \text{for}\: n=0 \\ \frac{\log(x^n)}{n}+\frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n)),x)
[Out] Piecewise((log(x), Eq(c, 0) & Eq(n, 0)), (-log(x**(-n))/n, Eq(c, 0)), ((b +
 2*c)*log(x)/(b + c), Eq(n, 0)), (log(x**n)/n + log(b/c + x**n)/n, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(15) = 30.

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.13

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx = b\left(\frac{\log(x)}{b}-\frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right)+\frac{2\log\left(\frac{cx^n+b}{c}\right)}{n}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx = \frac{\log\left(|cx^n+b|\right)}{n}+\log\left(|x|\right)$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] log(abs(c*x^n + b))/n + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx = \frac{2\left(\ln\left(b+c\,x^n\right)-\mathrm{atanh}\left(\frac{2\,c\,x^n}{b}+1\right)\right)}{n}$$

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x)
[Out] (2*(log(b + c*x^n) - atanh((2*c*x^n)/b + 1)))/n
```

3.125 $\int \frac{b+2cx}{(bx+cx^2)^8} dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{b+2cx}{(bx+cx^2)^8} dx = -\frac{1}{7(bx+cx^2)^7}$$

```
[Out] -1/7/(c*x^2+b*x)^7
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{643\}$

$$\int \frac{b+2cx}{(bx+cx^2)^8} dx = -\frac{1}{7(bx+cx^2)^7}$$

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/7*1/(b*x + c*x^2)^7
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{7(bx+cx^2)^7}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx=-\frac{1}{7x^7(b+cx)^7}$$

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]
[Out] -1/7*1/(x^7*(b + c*x)^7)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result
gosper	$-\frac{1}{7x^7(cx+b)^7}$
norman	$-\frac{1}{7x^7(cx+b)^7}$
risch	$-\frac{1}{7x^7(cx+b)^7}$
parallelrisch	$-\frac{1}{7x^7(cx+b)^7}$
derivativedivides	$-\frac{1}{7(c\,x^2+bx)^7}$
default	$-\frac{1}{7b^7x^7}-\frac{132c^6}{b^{13}x}+\frac{66c^5}{b^{12}x^2}-\frac{30c^4}{b^{11}x^3}+\frac{12c^3}{b^{10}x^4}-\frac{4c^2}{b^9x^5}+\frac{c}{b^8x^6}+\frac{132c^7}{b^{13}(cx+b)}+\frac{66c^7}{b^{12}(cx+b)^2}+\frac{30c^7}{b^{11}(cx+b)^3}+$

```
[In] int((2*c*x+b)/(c*x^2+b*x)^8,x,method=_RETURNVERBOSE)
[Out] -1/7/x^7/(c*x+b)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(13) = 26$.

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.40

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7\left(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7\right)}$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="fricas")
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(14) = 28$.

Time = 0.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.80

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7b^7x^7+49b^6cx^8+147b^5c^2x^9+245b^4c^3x^{10}+245b^3c^4x^{11}+147b^2c^5x^{12}+49bc^6x^{13}+7c^7x^{14}}$$

```
[In] integrate((2*c*x+b)/(c*x**2+b*x)**8,x)
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10
 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**
14)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7\,(cx^2+bx)^7}$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="maxima")
[Out] -1/7/(c*x^2 + b*x)^7
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7\,(cx^2+bx)^7}$$

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="giac")
[Out] -1/7/(c*x^2 + b*x)^7
```

Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{(bx+cx^2)^8}\,dx = -\frac{1}{7\,x^7\,(b+c\,x)^7}$$

```
[In] int((b + 2*c*x)/(b*x + c*x^2)^8,x)
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

3.126 $\int \frac{x\left(b+2cx^2\right)}{\left(bx^2+cx^4\right)^8} dx$

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8} dx = -\frac{1}{14x^{14}(b+cx^2)^7}$$

```
[Out] -1/14/x^14/(c*x^2+b)^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1598, 457, 75}

$$\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8} dx = -\frac{1}{14x^{14}(b+cx^2)^7}$$

```
[In] Int[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx \\ &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^2\right) \\ &= -\frac{1}{14x^{14}\left(b+cx^2\right)^7} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x(b+2cx^2)}{\left(bx^2+cx^4\right)^8}\,dx = -\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

```
[In] Integrate[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
norman	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
risch	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
parallelrisch	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
default	$-\frac{1}{14b^7x^{14}}-\frac{66c^6}{b^{13}x^2}+\frac{33c^5}{b^{12}x^4}-\frac{15c^4}{b^{11}x^6}+\frac{6c^3}{b^{10}x^8}-\frac{2c^2}{b^9x^{10}}+\frac{c}{2b^8x^{12}}-\frac{c^8\left(-\frac{12b^3}{c\left(c\,x^2+b\right)^4}-\frac{b^5}{c\left(c\,x^2+b\right)^6}-\frac{66b}{c\left(c\,x^2+b\right)^2}-\cdots\right.}{2\cdots}$

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x,method=_RETURNVERBOSE)
[Out] -1/14/x^14/(c*x^2+b)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8}\,dx = -\frac{1}{14\,(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14})}$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="fricas")
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

Time = 0.66 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8}\,dx = -\frac{1}{14b^7x^{14}+98b^6cx^{16}+294b^5c^2x^{18}+490b^4c^3x^{20}+490b^3c^4x^{22}+294b^2c^5x^{24}+98bc^6x^{26}+14c^7x^{28}}$$

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2)**8,x)
[Out] -1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*x
**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c**
7*x**28)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8}\,dx = -\frac{1}{14\,(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14})}$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8}\,dx = -\frac{1}{14\,(cx^4+bx^2)^7}$$

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2)^7
```

Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8}\,dx = -\frac{1}{14\,x^{14}\,(c\,x^2+b)^7}$$

```
[In] int((x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x)
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

3.127 $\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8} dx$

Optimal result

Integrand size = 25, antiderivative size = 16

$$\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8} dx = -\frac{1}{21x^{21}(b+cx^3)^7}$$

```
[Out] -1/21/x^21/(c*x^3+b)^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1598, 457, 75}

$$\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8} dx = -\frac{1}{21x^{21}(b+cx^3)^7}$$

```
[In] Int[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx \\ &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^3\right) \\ &= -\frac{1}{21x^{21}\left(b+cx^3\right)^7} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^2(b+2cx^3)}{\left(bx^3+cx^6\right)^8}\,dx = -\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{1}{21x^{21}\left(c\,x^{3}+b\right)^{7}}$
risch	$-\frac{1}{21x^{21}\left(c\,x^{3}+b\right)^{7}}$
parallelrisch	$-\frac{1}{21x^{21}\left(c\,x^{3}+b\right)^{7}}$
default	$-\frac{1}{21b^{7}x^{21}}-\frac{44c^{6}}{b^{13}x^{3}}+\frac{22c^{5}}{b^{12}x^{6}}-\frac{10c^{4}}{b^{11}x^{9}}+\frac{4c^{3}}{b^{10}x^{12}}-\frac{4c^{2}}{3b^{9}x^{15}}+\frac{c}{3b^{8}x^{18}}-\frac{c^{8}\left(-\frac{12b^{3}}{c\left(c\,x^{3}+b\right)^{4}}-\frac{b^{5}}{c\left(c\,x^{3}+b\right)^{6}}-\frac{66b}{c\left(c\,x^{3}+b\right)^{2}}-\right.}{}$

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/x^21/(c*x^3+b)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(14) = 28.

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8} dx = -\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="fricas")
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(15) = 30.

Time = 0.93 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8} dx = -\frac{1}{21b^7x^{21}+147b^6cx^{24}+441b^5c^2x^{27}+735b^4c^3x^{30}+735b^3c^4x^{33}+441b^2c^5x^{36}+147bc^6x^{39}+21c^7x^{42}}$$

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3)**8,x)
[Out] -1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3*
x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21*c
**7*x**42)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(14) = 28.

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8} dx = -\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="maxima")
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8}\,dx = -\frac{1}{21\,(cx^6+bx^3)^7}$$

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="giac")
[Out] -1/21/(c*x^6 + b*x^3)^7
```

Mupad [B] (verification not implemented)

Time = 11.46 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8}\,dx = -\frac{1}{21\,x^{21}\,(c\,x^3+b)^7}$$

```
[In] int((x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x)
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

3.128 $\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8} dx$

Optimal result

Integrand size = 29, antiderivative size = 21

$$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8} dx = -\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

```
[Out] -1/7/n/(x^(7*n))/(b+c*x^n)^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1598, 457, 75}

$$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8} dx = -\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^n\right)}{n}\\ &= -\frac{x^{-7n}}{7n\,(b+cx^n)^7}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx = -\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(21) = 42.

Time = 69.56 (sec) , antiderivative size = 203, normalized size of antiderivative = 9.67

method	result
risch	$-\frac{132c^6x^{-n}}{b^{13}n}+\frac{66c^5x^{-2n}}{b^{12}n}-\frac{30c^4x^{-3n}}{b^{11}n}+\frac{12c^3x^{-4n}}{b^{10}n}-\frac{4c^2x^{-5n}}{b^9n}+\frac{c\,x^{-6n}}{b^8n}-\frac{x^{-7n}}{7b^7n}+\frac{c^7(924x^{6n}c^6+6006b\,c^5x^{5n}+16380b^2}{}$

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x,method=_RETURNVERBOSE)
[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c
^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^
```

```
7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(
x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(21) = 42.

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.00

$$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx = -\frac{1}{7\,(c^7nx^{14\,n}+7\,bc^6nx^{13\,n}+21\,b^2c^5nx^{12\,n}+35\,b^3c^4nx^{11\,n}+35\,b^4c^3nx^{10\,n}+21\,b^5c^2nx^{9\,n}+7\,b^6cnx^{8\,n}+b^7n\ldots}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="fricas")
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 21*b^2*c^5*n*x^(12*n) + 35*b^3*
c^4*n*x^(11*n) + 35*b^4*c^3*n*x^(10*n) + 21*b^5*c^2*n*x^(9*n) + 7*b^6*c*n*x
^(8*n) + b^7*n*x^(7*n))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n))**8,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(21) = 42.

Time = 0.23 (sec) , antiderivative size = 612, normalized size of antiderivative = 29.14

$$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx = -\frac{1}{105}\,b\left(\frac{360360\,c^{13}x^{13\,n}+2342340\,bc^{12}x^{12\,n}+6426420\,b^2c^{11}x^{11\,n}+9579570\,b^3c^{10}x^{10\,n}+8270262\,b^4c^9x^{9\,n}\ldots}{b^{14}c^7nx^{14\,n}+7\,b^{15}c^6nx^{13\,n}+21\,b^{16}c^5nx\ldots}\right. + \frac{1}{105}\,c\left(\frac{360360\,c^{12}x^{12\,n}+2342340\,bc^{11}x^{11\,n}+6426420\,b^2c^{10}x^{10\,n}+9579570\,b^3c^9x^{9\,n}+8270262\,b^4c^8x^{8\,n}+\ldots}{b^{13}c^7nx^{13\,n}+7\,b^{14}c^6nx^{12\,n}+21\,b^{15}c^5nx^{11\,n}+3\ldots}\right.$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="maxima")
```

```
[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^1
1*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*
b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8
*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^
(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*
n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*
n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*
c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c*((36036
0*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 957
9570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) +
934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 100
1*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c
^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^
4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x
^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n +
 b)/c)/(b^14*n))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx = -\frac{1}{7\,(cx^{2\,n}+bx^n)^7n}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="giac")
```

```
[Out] -1/7/((c*x^(2*n) + b*x^n)^7*n)
```

Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 107, normalized size of antiderivative = 5.10

$$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx = -\frac{1}{7\,b^7\,n\,x^{7\,n}+7\,c^7\,n\,x^{14\,n}+49\,b^6\,c\,n\,x^{8\,n}+49\,b\,c^6\,n\,x^{13\,n}+147\,b^5\,c^2\,n\,x^{9\,n}+245\,b^4\,c^3\,n\,x^{10\,n}+245\,b^3\,c^4\,n\,x$$

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x)
```

```
[Out] -1/(7*b^7*n*x^(7*n) + 7*c^7*n*x^(14*n) + 49*b^6*c*n*x^(8*n) + 49*b*c^6*n*x^
(13*n) + 147*b^5*c^2*n*x^(9*n) + 245*b^4*c^3*n*x^(10*n) + 245*b^3*c^4*n*x^(
11*n) + 147*b^2*c^5*n*x^(12*n))
```

3.129 $\int (b+2cx) \left(a+bx+cx^2\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int (b+2cx) \left(a+bx+cx^2\right)^p \, dx = \frac{\left(a+bx+cx^2\right)^{1+p}}{1+p}$$

```
[Out] (c*x^2+b*x+a)^(p+1)/(p+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{643\}$

$$\int (b+2cx) \left(a+bx+cx^2\right)^p \, dx = \frac{\left(a+bx+cx^2\right)^{p+1}}{p+1}$$

```
[In] Int[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]
```

```
[Out] (a + b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\left(a+bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx=\frac{(a+x(b+cx))^{1+p}}{1+p}$$

```
[In] Integrate[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]
[Out] (a + x*(b + c*x))^(1 + p)/(1 + p)
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{\left(c\,x^2+bx+a\right)^{1+p}}{1+p}$	21
derivativedivides	$\frac{\left(c\,x^2+bx+a\right)^{1+p}}{1+p}$	21
default	$\frac{\left(c\,x^2+bx+a\right)^{1+p}}{1+p}$	21
risch	$\frac{\left(c\,x^2+bx+a\right)\left(c\,x^2+bx+a\right)^p}{1+p}$	29
parallelrisch	$\frac{x^2\left(c\,x^2+bx+a\right)^p ac+ab\left(c\,x^2+bx+a\right)^p x+a^2\left(c\,x^2+bx+a\right)^p}{a(1+p)}$	61
norman	$\frac{a\,{\mathrm e}^{p\ln\left(c\,x^2+bx+a\right)}}{1+p}+\frac{bx\,{\mathrm e}^{p\ln\left(c\,x^2+bx+a\right)}}{1+p}+\frac{c\,x^2{\mathrm e}^{p\ln\left(c\,x^2+bx+a\right)}}{1+p}$	69

```
[In] int((2*c*x+b)*(c*x^2+b*x+a)^p,x,method=_RETURNVERBOSE)
[Out] (c*x^2+b*x+a)^(1+p)/(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx=\frac{(cx^2+bx+a)(cx^2+bx+a)^p}{p+1}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
[Out] (c*x^2 + b*x + a)*(c*x^2 + b*x + a)^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(15) = 30$.

Time = 29.58 (sec) , antiderivative size = 104, normalized size of antiderivative = 5.20

$$\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx = \begin{cases} \frac{a(a+bx+cx^2)^p}{p+1} + \frac{bx(a+bx+cx^2)^p}{p+1} + \frac{cx^2(a+bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log\left(\frac{b}{2c} + x - \frac{\sqrt{-4ac+b^2}}{2c}\right) + \log\left(\frac{b}{2c} + x + \frac{\sqrt{-4ac+b^2}}{2c}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((2*c*x+b)*(c*x**2+b*x+a)**p,x)
```

```
[Out] Piecewise((a*(a + b*x + c*x**2)**p/(p + 1) + b*x*(a + b*x + c*x**2)**p/(p +
 1) + c*x**2*(a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(b/(2*c) + x -
sqrt(-4*a*c + b**2)/(2*c)) + log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c)),
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx = \frac{(cx^2+bx+a)^{p+1}}{p+1}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] (c*x^2 + b*x + a)^(p + 1)/(p + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx = \frac{(cx^2+bx+a)^{p+1}}{p+1}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] (c*x^2 + b*x + a)^(p + 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int (b+2cx) \left(a+bx+cx^2\right)^p \, dx = \left(\frac{a}{p+1}+\frac{b\,x}{p+1}+\frac{c\,x^2}{p+1}\right)\,\left(c\,x^2+b\,x+a\right)^p$$

```
[In] int((b + 2*c*x)*(a + b*x + c*x^2)^p,x)
[Out] (a/(p + 1) + (b*x)/(p + 1) + (c*x^2)/(p + 1))*(a + b*x + c*x^2)^p
```

3.130 $\int x(b+2cx^2)(a+bx^2+cx^4)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 25

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^p \, dx = \frac{(a+bx^2+cx^4)^{1+p}}{2(1+p)}$$

```
[Out] 1/2*(c*x^4+b*x^2+a)^(p+1)/(p+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1261, 643}

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^p \, dx = \frac{(a+bx^2+cx^4)^{p+1}}{2(p+1)}$$

```
[In] Int[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)$$
$$= \frac{\left(a+bx^2+cx^4\right)^{1+p}}{2(1+p)}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^p\,dx = \frac{\left(a+bx^2+cx^4\right)^{1+p}}{2(1+p)}$$

```
[In] Integrate[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{\left(c\,x^4+b\,x^2+a\right)^{1+p}}{2+2p}$	24
risch	$\frac{\left(c\,x^4+b\,x^2+a\right)\left(c\,x^4+b\,x^2+a\right)^p}{2+2p}$	34
parallelrisch	$\frac{x^4\left(c\,x^4+b\,x^2+a\right)^p c^2+x^2\left(c\,x^4+b\,x^2+a\right)^p bc+\left(c\,x^4+b\,x^2+a\right)^p ac}{2c(1+p)}$	70
norman	$\frac{a\,\mathrm{e}^{p\ln\left(c\,x^4+b\,x^2+a\right)}}{2+2p}+\frac{b\,x^2\mathrm{e}^{p\ln\left(c\,x^4+b\,x^2+a\right)}}{2+2p}+\frac{c\,x^4\mathrm{e}^{p\ln\left(c\,x^4+b\,x^2+a\right)}}{2+2p}$	80

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x,method=_RETURNVERBOSE)
[Out] 1/2*(c*x^4+b*x^2+a)^(1+p)/(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2+a)(cx^4+bx^2+a)^p}{2\,(p+1)}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
[Out] 1/2*(c*x^4 + b*x^2 + a)*(c*x^4 + b*x^2 + a)^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(19) = 38.

Time = 104.63 (sec) , antiderivative size = 201, normalized size of antiderivative = 8.04

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^p\,dx$$
$$=\begin{cases}\frac{a(a+bx^2+cx^4)^p}{2p+2}+\frac{bx^2(a+bx^2+cx^4)^p}{2p+2}+\frac{cx^4(a+bx^2+cx^4)^p}{2p+2}\\ \frac{\log\left(x-\frac{\sqrt{2}\sqrt{-\frac{b}{c}-\frac{\sqrt{-4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x+\frac{\sqrt{2}\sqrt{-\frac{b}{c}-\frac{\sqrt{-4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x-\frac{\sqrt{2}\sqrt{-\frac{b}{c}+\frac{\sqrt{-4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x+\frac{\sqrt{2}\sqrt{-\frac{b}{c}+\frac{\sqrt{-4ac+b^2}}{c}}}{2}\right)}{2}\end{cases}$$

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**p,x)
[Out] Piecewise((a*(a + b*x**2 + c*x**4)**p/(2*p + 2) + b*x**2*(a + b*x**2 + c*x*
*4)**p/(2*p + 2) + c*x**4*(a + b*x**2 + c*x**4)**p/(2*p + 2), Ne(p, -1)), (
log(x - sqrt(2)*sqrt(-b/c - sqrt(-4*a*c + b**2)/c)/2)/2 + log(x + sqrt(2)*s
qrt(-b/c - sqrt(-4*a*c + b**2)/c)/2)/2 + log(x - sqrt(2)*sqrt(-b/c + sqrt(-
4*a*c + b**2)/c)/2)/2 + log(x + sqrt(2)*sqrt(-b/c + sqrt(-4*a*c + b**2)/c)/
2)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2+a)(cx^4+bx^2+a)^p}{2\,(p+1)}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
[Out] 1/2*(c*x^4 + b*x^2 + a)*(c*x^4 + b*x^2 + a)^p/(p + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2+a)^{p+1}}{2\,(p+1)}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
[Out] 1/2*(c*x^4 + b*x^2 + a)^(p + 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^p\,dx=\left(c\,x^4+b\,x^2+a\right)^p\left(\frac{a}{2\,p+2}+\frac{b\,x^2}{2\,p+2}+\frac{c\,x^4}{2\,p+2}\right)$$

```
[In] int(x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x)
[Out] (a + b*x^2 + c*x^4)^p*(a/(2*p + 2) + (b*x^2)/(2*p + 2) + (c*x^4)/(2*p + 2))
```

3.131 $\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^p\,dx$

Optimal result

Integrand size = 26, antiderivative size = 25

$$\int x^2\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\frac{\left(a+bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

```
[Out] 1/3*(c*x^6+b*x^3+a)^(p+1)/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {1482, 643}

$$\int x^2\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^p\,dx=\frac{\left(a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

```
[In] Int[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
```

```
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx, x, x^3\right)\\ &= \frac{\left(a+bx^3+cx^6\right)^{1+p}}{3(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^2\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^p\,dx = \frac{\left(a+bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

```
[In] Integrate[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{\left(c\,x^6+b\,x^3+a\right)^{1+p}}{3+3p}$	24
risch	$\frac{\left(c\,x^6+b\,x^3+a\right)\left(c\,x^6+b\,x^3+a\right)^p}{3+3p}$	34
parallelrisch	$\frac{x^6\left(c\,x^6+b\,x^3+a\right)^p c^2+x^3\left(c\,x^6+b\,x^3+a\right)^p bc+\left(c\,x^6+b\,x^3+a\right)^p ac}{3c(1+p)}$	70
norman	$\frac{a\,\mathrm{e}^{p\ln\left(c\,x^6+b\,x^3+a\right)}}{3+3p}+\frac{b\,x^3\mathrm{e}^{p\ln\left(c\,x^6+b\,x^3+a\right)}}{3+3p}+\frac{c\,x^6\mathrm{e}^{p\ln\left(c\,x^6+b\,x^3+a\right)}}{3+3p}$	80

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x,method=_RETURNVERBOSE)
[Out] 1/3*(c*x^6+b*x^3+a)^(1+p)/(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int x^2 \left(b+2 c x^3\right) \left(a+b x^3+c x^6\right)^p \, dx = \frac{(c x^6+b x^3+a)(c x^6+b x^3+a)^p}{3\,(p+1)}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
[Out] 1/3*(c*x^6 + b*x^3 + a)*(c*x^6 + b*x^3 + a)^p/(p + 1)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(b+2 c x^3\right) \left(a+b x^3+c x^6\right)^p \, dx = \text{Timed out}$$

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**p,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int x^2 \left(b+2 c x^3\right) \left(a+b x^3+c x^6\right)^p \, dx = \frac{(c x^6+b x^3+a)(c x^6+b x^3+a)^p}{3\,(p+1)}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3 + a)*(c*x^6 + b*x^3 + a)^p/(p + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int x^2 \left(b+2 c x^3\right) \left(a+b x^3+c x^6\right)^p \, dx = \frac{(c x^6+b x^3+a)^{p+1}}{3\,(p+1)}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
[Out] 1/3*(c*x^6 + b*x^3 + a)^(p + 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int x^2 \left(b+2cx^3\right) \left(a+bx^3+cx^6\right)^p \, dx = \left(c\,x^6+b\,x^3+a\right)^p \left(\frac{a}{3\,p+3}+\frac{b\,x^3}{3\,p+3}+\frac{c\,x^6}{3\,p+3}\right)$$

```
[In] int(x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x)
[Out] (a + b*x^3 + c*x^6)^p*(a/(3*p + 3) + (b*x^3)/(3*p + 3) + (c*x^6)/(3*p + 3))
```

3.132 $\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx$

Optimal result

Integrand size = 30, antiderivative size = 27

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\frac{\left(a+bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}$$

```
[Out] (a+b*x^n+c*x^(2*n))^(p+1)/n/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1482, 643}

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\frac{\left(a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
```

```
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx, x, x^n\right)}{n}\\ &= \frac{\left(a+bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx = \frac{(a+x^n(b+cx^n))^{1+p}}{n(1+p)}$$

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
[Out] (a + x^n*(b + c*x^n))^(1 + p)/(n*(1 + p))
```

Maple [A] (verified)

Time = 36.71 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

method	result	size
risch	$\frac{\left(a+b\,x^{n}+c\,x^{2 n}\right)\left(a+b\,x^{n}+c\,x^{2 n}\right)^{p}}{n(1+p)}$	40

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x,method=_RETURNVERBOSE)
[Out] (a+b*x^n+c*(x^n)^2)/n/(1+p)*(a+b*x^n+c*(x^n)^2)^p
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx = \frac{(cx^{2\,n}+bx^n+a)(cx^{2\,n}+bx^n+a)^p}{np+n}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
[Out] (c*x^(2*n) + b*x^n + a)*(c*x^(2*n) + b*x^n + a)^p/(n*p + n)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^{2n}+bx^n+a)(cx^{2n}+bx^n+a)^p}{n(p+1)}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] (c*x^(2*n) + b*x^n + a)*(c*x^(2*n) + b*x^n + a)^p/(n*(p + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^{2n}+bx^n+a)^{p+1}}{n(p+1)}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] (c*x^(2*n) + b*x^n + a)^(p + 1)/(n*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx=\left(a+b\,x^n+c\,x^{2\,n}\right)^p\left(\frac{a}{n\,(p+1)}+\frac{b\,x^n}{n\,(p+1)}+\frac{c\,x^{2\,n}}{n\,(p+1)}\right)$$

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x)
[Out] (a + b*x^n + c*x^(2*n))^p*(a/(n*(p + 1)) + (b*x^n)/(n*(p + 1)) + (c*x^(2*n)
)/(n*(p + 1)))
```

3.133 $\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx=\frac{\left(-a+bx+cx^2\right)^{1+p}}{1+p}$$

```
[Out] (c*x^2+b*x-a)^(p+1)/(p+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {643}

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx=\frac{\left(-a+bx+cx^2\right)^{p+1}}{p+1}$$

```
[In] Int[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]
```

```
[Out] (-a + b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{\left(-a+bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx=\frac{(-a+x(b+cx))^{1+p}}{1+p}$$

```
[In] Integrate[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]
[Out] (-a + x*(b + c*x))^(1 + p)/(1 + p)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{\left(c\,x^2+bx-a\right)^{1+p}}{1+p}$	23
derivativedivides	$\frac{\left(c\,x^2+bx-a\right)^{1+p}}{1+p}$	23
default	$\frac{\left(c\,x^2+bx-a\right)^{1+p}}{1+p}$	23
risch	$-\frac{\left(-c\,x^2-bx+a\right)\left(c\,x^2+bx-a\right)^p}{1+p}$	34
parallelrisch	$\frac{x^2\left(c\,x^2+bx-a\right)^p bc+x\left(c\,x^2+bx-a\right)^p b^2-ab\left(c\,x^2+bx-a\right)^p}{b(1+p)}$	68
norman	$\frac{bx\,\mathrm{e}^{p\ln\left(c\,x^2+bx-a\right)}}{1+p}+\frac{c\,x^2\mathrm{e}^{p\ln\left(c\,x^2+bx-a\right)}}{1+p}-\frac{a\,\mathrm{e}^{p\ln\left(c\,x^2+bx-a\right)}}{1+p}$	76

```
[In] int((2*c*x+b)*(c*x^2+b*x-a)^p,x,method=_RETURNVERBOSE)
[Out] (c*x^2+b*x-a)^(1+p)/(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx=\frac{(cx^2+bx-a)(cx^2+bx-a)^p}{p+1}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="fricas")
[Out] (c*x^2 + b*x - a)*(c*x^2 + b*x - a)^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(15) = 30$.

Time = 29.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.73

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx = \begin{cases} -\frac{a(-a+bx+cx^2)^p}{p+1} + \frac{bx(-a+bx+cx^2)^p}{p+1} + \frac{cx^2(-a+bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log\left(\frac{b}{2c} + x - \frac{\sqrt{4ac+b^2}}{2c}\right) + \log\left(\frac{b}{2c} + x + \frac{\sqrt{4ac+b^2}}{2c}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((2*c*x+b)*(c*x**2+b*x-a)**p,x)
```

```
[Out] Piecewise((-a*(-a + b*x + c*x**2)**p/(p + 1) + b*x*(-a + b*x + c*x**2)**p/(
p + 1) + c*x**2*(-a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(b/(2*c) +
x - sqrt(4*a*c + b**2)/(2*c)) + log(b/(2*c) + x + sqrt(4*a*c + b**2)/(2*c))
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx = \frac{(cx^2+bx-a)^{p+1}}{p+1}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="maxima")
```

```
[Out] (c*x^2 + b*x - a)^(p + 1)/(p + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx = \frac{(cx^2+bx-a)^{p+1}}{p+1}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="giac")
```

```
[Out] (c*x^2 + b*x - a)^(p + 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx=\left(\frac{b\,x}{p+1}-\frac{a}{p+1}+\frac{c\,x^2}{p+1}\right)\,\left(c\,x^2+b\,x-a\right)^p$$

```
[In] int((b + 2*c*x)*(b*x - a + c*x^2)^p,x)
[Out] ((b*x)/(p + 1) - a/(p + 1) + (c*x^2)/(p + 1))*(b*x - a + c*x^2)^p
```

3.134 $\int x(b+2cx^2)(-a+bx^2+cx^4)^p\,dx$

Optimal result

Integrand size = 26, antiderivative size = 27

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^p\,dx=\frac{\left(-a+bx^2+cx^4\right)^{1+p}}{2(1+p)}$$

```
[Out] 1/2*(c*x^4+b*x^2-a)^(p+1)/(p+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1261, 643}

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^p\,dx=\frac{\left(-a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

```
[In] Int[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx, x, x^2\right)\\ &= \frac{\left(-a+bx^2+cx^4\right)^{1+p}}{2(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^p\,dx = \frac{\left(-a+bx^2+cx^4\right)^{1+p}}{2(1+p)}$$

```
[In] Integrate[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]
[Out] (-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{\left(c\,x^4+b\,x^2-a\right)^{1+p}}{2+2p}$	26
risch	$-\frac{\left(-c\,x^4-b\,x^2+a\right)\left(c\,x^4+b\,x^2-a\right)^p}{2(1+p)}$	38
parallelrisch	$\frac{x^4\left(c\,x^4+b\,x^2-a\right)^p c^2+x^2\left(c\,x^4+b\,x^2-a\right)^p bc-\left(c\,x^4+b\,x^2-a\right)^p ac}{2c(1+p)}$	77
norman	$-\frac{a\,\mathrm{e}^{p\ln\left(c\,x^4+b\,x^2-a\right)}}{2(1+p)}+\frac{b\,x^2\mathrm{e}^{p\ln\left(c\,x^4+b\,x^2-a\right)}}{2+2p}+\frac{c\,x^4\mathrm{e}^{p\ln\left(c\,x^4+b\,x^2-a\right)}}{2+2p}$	86

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x,method=_RETURNVERBOSE)
[Out] 1/2*(c*x^4+b*x^2-a)^(1+p)/(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2-a)(cx^4+bx^2-a)^p}{2\,(p+1)}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="fricas")
[Out] 1/2*(c*x^4 + b*x^2 - a)*(c*x^4 + b*x^2 - a)^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(19) = 38$.

Time = 107.59 (sec) , antiderivative size = 201, normalized size of antiderivative = 7.44

$$\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^p\,dx = \begin{cases} -\frac{a(-a+bx^2+cx^4)^p}{2p+2}+\frac{bx^2(-a+bx^2+cx^4)^p}{2p+2}+\frac{cx^4(-a+bx^2+cx^4)^p}{2p+2} & \text{for } \\ \frac{\log\left(x-\frac{\sqrt{2}\sqrt{-\frac{b}{c}-\frac{\sqrt{4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x+\frac{\sqrt{2}\sqrt{-\frac{b}{c}-\frac{\sqrt{4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x-\frac{\sqrt{2}\sqrt{-\frac{b}{c}+\frac{\sqrt{4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x+\frac{\sqrt{2}\sqrt{-\frac{b}{c}+\frac{\sqrt{4ac+b^2}}{c}}}{2}\right)}{2} & \text{oth} \end{cases}$$

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**p,x)
[Out] Piecewise((-a*(-a + b*x**2 + c*x**4)**p/(2*p + 2) + b*x**2*(-a + b*x**2 + c
*x**4)**p/(2*p + 2) + c*x**4*(-a + b*x**2 + c*x**4)**p/(2*p + 2), Ne(p, -1)
), (log(x - sqrt(2)*sqrt(-b/c - sqrt(4*a*c + b**2)/c)/2)/2 + log(x + sqrt(2
)*sqrt(-b/c - sqrt(4*a*c + b**2)/c)/2)/2 + log(x - sqrt(2)*sqrt(-b/c + sqrt
(4*a*c + b**2)/c)/2)/2 + log(x + sqrt(2)*sqrt(-b/c + sqrt(4*a*c + b**2)/c)/
2)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2-a)(cx^4+bx^2-a)^p}{2\,(p+1)}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="maxima")
[Out] 1/2*(c*x^4 + b*x^2 - a)*(c*x^4 + b*x^2 - a)^p/(p + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2-a)^{p+1}}{2\,(p+1)}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="giac")
[Out] 1/2*(c*x^4 + b*x^2 - a)^(p + 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 8.61 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.93

$$\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^p\,dx=\left(c\,x^4+b\,x^2-a\right)^p\left(\frac{b\,x^2}{2\,p+2}-\frac{a}{2\,p+2}+\frac{c\,x^4}{2\,p+2}\right)$$

```
[In] int(x*(b + 2*c*x^2)*(b*x^2 - a + c*x^4)^p,x)
[Out] (b*x^2 - a + c*x^4)^p*((b*x^2)/(2*p + 2) - a/(2*p + 2) + (c*x^4)/(2*p + 2))
```

3.135 $\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^p\,dx$

Optimal result

Integrand size = 28, antiderivative size = 27

$$\int x^2\left(b+2cx^3\right)\left(-a+bx^3+cx^6\right)^p\,dx=\frac{\left(-a+bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

```
[Out] 1/3*(c*x^6+b*x^3-a)^(p+1)/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1482, 643}

$$\int x^2\left(b+2cx^3\right)\left(-a+bx^3+cx^6\right)^p\,dx=\frac{\left(-a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

```
[In] Int[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]
[Out] (-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
```

```
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx,x,x^3\right)$$
$$= \frac{\left(-a+bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^2\left(b+2cx^3\right)\left(-a+bx^3+cx^6\right)^p\,dx = \frac{\left(-a+bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

```
[In] Integrate[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]
[Out] (-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{\left(c\,x^{6}+b\,x^{3}-a\right)^{1+p}}{3+3p}$	26
risch	$-\frac{\left(-c\,x^{6}-b\,x^{3}+a\right)\left(c\,x^{6}+b\,x^{3}-a\right)^{p}}{3(1+p)}$	38
parallelrisch	$\frac{x^{6}\left(c\,x^{6}+b\,x^{3}-a\right)^{p}c^{2}+x^{3}\left(c\,x^{6}+b\,x^{3}-a\right)^{p}bc-\left(c\,x^{6}+b\,x^{3}-a\right)^{p}ac}{3c(1+p)}$	77
norman	$-\frac{a\,{e}^{p\ln\left(c\,x^{6}+b\,x^{3}-a\right)}}{3(1+p)}+\frac{b\,x^{3}{e}^{p\ln\left(c\,x^{6}+b\,x^{3}-a\right)}}{3+3p}+\frac{c\,x^{6}{e}^{p\ln\left(c\,x^{6}+b\,x^{3}-a\right)}}{3+3p}$	86

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x,method=_RETURNVERBOSE)
[Out] 1/3*(c*x^6+b*x^3-a)^(1+p)/(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^p\,dx=\frac{(cx^6+bx^3-a)(cx^6+bx^3-a)^p}{3\,(p+1)}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="fricas")
[Out] 1/3*(c*x^6 + b*x^3 - a)*(c*x^6 + b*x^3 - a)^p/(p + 1)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**p,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^p\,dx=\frac{(cx^6+bx^3-a)(cx^6+bx^3-a)^p}{3\,(p+1)}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3 - a)*(c*x^6 + b*x^3 - a)^p/(p + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^p\,dx=\frac{(cx^6+bx^3-a)^{p+1}}{3\,(p+1)}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="giac")
[Out] 1/3*(c*x^6 + b*x^3 - a)^(p + 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.93

$$\int x^2\left(b+2cx^3\right)\left(-a+bx^3+cx^6\right)^p\,dx=\left(c\,x^6+b\,x^3-a\right)^p\,\left(\frac{b\,x^3}{3\,p+3}-\frac{a}{3\,p+3}+\frac{c\,x^6}{3\,p+3}\right)$$

```
[In] int(x^2*(b + 2*c*x^3)*(b*x^3 - a + c*x^6)^p,x)
[Out] (b*x^3 - a + c*x^6)^p*((b*x^3)/(3*p + 3) - a/(3*p + 3) + (c*x^6)/(3*p + 3))
```

3.136 $\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx$

Optimal result

Integrand size = 32, antiderivative size = 29

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx=\frac{\left(-a+bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}$$

```
[Out] (-a+b*x^n+c*x^(2*n))^(p+1)/n/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1482, 643}

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx=\frac{\left(-a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^p,x]
[Out] (-a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
```

```
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx,x,x^n\right)}{n}\\&=\frac{\left(-a+bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx=\frac{(-a+x^n(b+cx^n))^{1+p}}{n(1+p)}$$

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^p,x]
[Out] (-a + x^n*(b + c*x^n))^(1 + p)/(n*(1 + p))
```

Maple [A] (verified)

Time = 35.81 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

method	result	size
risch	$-\frac{\left(a-b\,x^{n}-c\,x^{2n}\right)\left(-a+b\,x^{n}+c\,x^{2n}\right)^{p}}{n(1+p)}$	45

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x,method=_RETURNVERBOSE)
[Out] -(-c*(x^n)^2-b*x^n+a)/n/(1+p)*(c*(x^n)^2+b*x^n-a)^p
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^{2\,n}+bx^n-a)(cx^{2\,n}+bx^n-a)^p}{np+n}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas"
)
[Out] (c*x^(2*n) + b*x^n - a)*(c*x^(2*n) + b*x^n - a)^p/(n*p + n)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^{2n}+bx^n-a)(cx^{2n}+bx^n-a)^p}{n(p+1)}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima"
)
[Out] (c*x^(2*n) + b*x^n - a)*(c*x^(2*n) + b*x^n - a)^p/(n*(p + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^{2n}+bx^n-a)^{p+1}}{n(p+1)}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] (c*x^(2*n) + b*x^n - a)^(p + 1)/(n*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.03

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx=\left(\frac{b\,x^n}{n\,(p+1)}-\frac{a}{n\,(p+1)}+\frac{c\,x^{2\,n}}{n\,(p+1)}\right)\,{\left(b\,x^n-a+c\,x^{2\,n}\right)}^p$$

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n - a + c*x^(2*n))^p,x)
[Out] ((b*x^n)/(n*(p + 1)) - a/(n*(p + 1)) + (c*x^(2*n))/(n*(p + 1)))*(b*x^n - a
+ c*x^(2*n))^p
```

3.137 $\int (b+2cx)\left(bx+cx^2\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 19

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{\left(bx+cx^2\right)^{1+p}}{1+p}$$

```
[Out] (c*x^2+b*x)^(p+1)/(p+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {643}

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] (b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{\left(bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{(x(b+cx))^{1+p}}{1+p}$$

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]
[Out] (x*(b + c*x))^(1 + p)/(1 + p)
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{\left(c\,x^{2}+b x\right)^{1+p}}{1+p}$	20
default	$\frac{\left(c\,x^{2}+b x\right)^{1+p}}{1+p}$	20
risch	$\frac{x(cx+b)(x(cx+b))^{p}}{1+p}$	22
gosper	$\frac{x(cx+b)\left(c\,x^{2}+b x\right)^{p}}{1+p}$	24
parallelrisch	$\frac{x^{2}(x(cx+b))^{p}bc+x(x(cx+b))^{p}b^{2}}{b(1+p)}$	40
norman	$\frac{bx\,{\mathrm e}^{p\ln\left(c\,x^{2}+b x\right)}}{1+p}+\frac{c\,x^{2}{\mathrm e}^{p\ln\left(c\,x^{2}+b x\right)}}{1+p}$	46

```
[In] int((2*c*x+b)*(c*x^2+b*x)^p,x,method=_RETURNVERBOSE)
[Out] (c*x^2+b*x)^(1+p)/(1+p)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{(cx^2+bx)(cx^2+bx)^p}{p+1}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="fricas")
[Out] (c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(14) = 28$.

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\begin{cases}\frac{bx(bx+cx^2)^p}{p+1}+\frac{cx^2(bx+cx^2)^p}{p+1} & \text{for } p\neq -1\\ \log(x)+\log\left(\frac{b}{c}+x\right) & \text{otherwise}\end{cases}$$

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**p,x)
[Out] Piecewise((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)**p/(p + 1)
, Ne(p, -1)), (log(x) + log(b/c + x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{(cx^2+bx)^{p+1}}{p+1}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="maxima")
[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{(cx^2+bx)^{p+1}}{p+1}$$

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="giac")
[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{x\,\left(c\,x^2+b\,x\right)^p\,\left(b+c\,x\right)}{p+1}$$

```
[In] int((b*x + c*x^2)^p*(b + 2*c*x),x)
[Out] (x*(b*x + c*x^2)^p*(b + c*x))/(p + 1)
```

3.138 $\int x(b+2cx^2)\left(bx^2+cx^4\right)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 24

$$\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^p\,dx=\frac{\left(bx^2+cx^4\right)^{1+p}}{2(1+p)}$$

```
[Out] 1/2*(c*x^4+b*x^2)^(p+1)/(p+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1602}

$$\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^p\,dx=\frac{\left(bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

```
[In] Int[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] (b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{\left(bx^2+cx^4\right)^{1+p}}{2(1+p)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.04

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^p\,dx$$
$$=\frac{x^2(x^2(b+cx^2))^p\left(1+\frac{cx^2}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^2}{b}\right)+2c(1+p)x^2\,\text{Hype}\right.}{2(1+p)(2+p)}$$

```
[In] Integrate[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]
[Out] (x^2*(x^2*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -((
c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^2)/
b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

method	result	size
gosper	$\frac{x^2(c\,x^2+b)\left(c\,x^4+b\,x^2\right)^p}{2+2p}$	31
risch	$\frac{x^2(c\,x^2+b)\left(x^2(c\,x^2+b)\right)^p}{2+2p}$	31
parallelrisch	$\frac{x^4\left(x^2(c\,x^2+b)\right)^p bc+x^2\left(x^2(c\,x^2+b)\right)^p b^2}{2b(1+p)}$	51
norman	$\frac{b\,x^2\mathrm{e}^{p\ln\left(c\,x^4+b\,x^2\right)}}{2+2p}+\frac{c\,x^4\mathrm{e}^{p\ln\left(c\,x^4+b\,x^2\right)}}{2+2p}$	54

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*(c*x^2+b)/(1+p)*(c*x^4+b*x^2)^p
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2)(cx^4+bx^2)^p}{2\,(p+1)}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="fricas")
[Out] 1/2*(c*x^4 + b*x^2)*(c*x^4 + b*x^2)^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(17) = 34$.

Time = 8.59 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.12

$$\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^p\,dx=\begin{cases}\frac{bx^2(bx^2+cx^4)^p}{2p+2}+\frac{cx^4(bx^2+cx^4)^p}{2p+2} & \text{for } p\neq -1\\ \log\left(x\right)+\frac{\log\left(x-\sqrt{-\frac{b}{c}}\right)}{2}+\frac{\log\left(x+\sqrt{-\frac{b}{c}}\right)}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**p,x)
```

```
[Out] Piecewise((b*x**2*(b*x**2 + c*x**4)**p/(2*p + 2) + c*x**4*(b*x**2 + c*x**4)
**p/(2*p + 2), Ne(p, -1)), (log(x) + log(x - sqrt(-b/c))/2 + log(x + sqrt(-
b/c))/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2)e^{\left(p\log\left(cx^2+b\right)+2\,p\log\left(x\right)\right)}}{2\,(p+1)}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^p\,dx=\frac{(cx^4+bx^2)^{p+1}}{2\,(p+1)}$$

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*(c*x^4 + b*x^2)^(p + 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 8.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int x(b+2cx^2)\left(bx^2+cx^4\right)^p\,dx=\frac{x^2\,(c\,x^2+b)\,\left(c\,x^4+b\,x^2\right)^p}{2\,(p+1)}$$

```
[In] int(x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x)
[Out] (x^2*(b + c*x^2)*(b*x^2 + c*x^4)^p)/(2*(p + 1))
```

3.139 $\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 24

$$\int x^2\left(b+2cx^3\right)\left(bx^3+cx^6\right)^p\,dx=\frac{\left(bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

```
[Out] 1/3*(c*x^6+b*x^3)^(p+1)/(p+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1602}

$$\int x^2\left(b+2cx^3\right)\left(bx^3+cx^6\right)^p\,dx=\frac{\left(bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

```
[In] Int[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]
```

```
[Out] (b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{\left(bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.04

$$\int x^2(b+2cx^3)(bx^3+cx^6)^p\,dx = \frac{x^3(x^3(b+cx^3))^p\left(1+\frac{cx^3}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^3}{b}\right)+2c(1+p)x^3\,\text{Hype}\right.}{3(1+p)(2+p)}$$

```
[In] Integrate[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^3*(x^3*(b + c*x^3))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -((
c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^3)/
b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

method	result	size
gosper	$\frac{(c\,x^3+b)x^3\left(c\,x^6+b\,x^3\right)^p}{3+3p}$	31
risch	$\frac{x^3\left(c\,x^3+b\right)\left(x^3\left(c\,x^3+b\right)\right)^p}{3+3p}$	31
parallelrisch	$\frac{x^6\left(x^3\left(c\,x^3+b\right)\right)^p bc+x^3\left(x^3\left(c\,x^3+b\right)\right)^p b^2}{3b(1+p)}$	51
norman	$\frac{b\,x^3 e^{p\ln\left(c\,x^6+b\,x^3\right)}}{3+3p}+\frac{c\,x^6 e^{p\ln\left(c\,x^6+b\,x^3\right)}}{3+3p}$	54

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c*x^3+b)*x^3/(1+p)*(c*x^6+b*x^3)^p
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int x^2(b+2cx^3)(bx^3+cx^6)^p\,dx = \frac{(cx^6+bx^3)(cx^6+bx^3)^p}{3\,(p+1)}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="fricas")
```

```
[Out] 1/3*(c*x^6 + b*x^3)*(c*x^6 + b*x^3)^p/(p + 1)
```

Sympy [F(-1)]

Timed out.

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**p,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^p\,dx=\frac{(cx^6+bx^3)e^{(p\log(cx^3+b)+3\,p\log(x))}}{3\,(p+1)}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^p\,dx=\frac{(cx^6+bx^3)^{p+1}}{3\,(p+1)}$$

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="giac")
[Out] 1/3*(c*x^6 + b*x^3)^(p + 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^p\,dx=\frac{x^3\,(c\,x^3+b)\,\left(c\,x^6+b\,x^3\right)^p}{3\,(p+1)}$$

```
[In] int(x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x)
[Out] (x^3*(b + c*x^3)*(b*x^3 + c*x^6)^p)/(3*(p + 1))
```

3.140 $\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx$

Optimal result

Integrand size = 29, antiderivative size = 26

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx=\frac{\left(bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}$$

```
[Out] (b*x^n+c*x^(2*n))^(p+1)/n/(p+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2059, 643}

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx=\frac{\left(bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
```

```
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (b+2cx)\left(bx+cx^2\right)^p\,dx,x,x^n\right)}{n}\\ &= \frac{\left(bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.27

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx$$

$$= \frac{x^{-np}(x^n(b+cx^n))^p\left(1+\frac{cx^n}{b}\right)^{-p}\left(b(2+p)x^{n(1+p)}\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^n}{b}\right)+2c(1+p)x\right.}{n(1+p)(2+p)}$$

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((x^n*(b + c*x^n))^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p,
2 + p, -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 +
p, 3 + p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*x^(n*p)*(1 + (c*x^n)/b)^p)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 29.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 4.08

method	result	size
risch	$\frac{x^n(b+c\,x^n)(x^n)^p(b+c\,x^n)^p\mathrm{e}^{-\frac{i\,\mathrm{csgn}(ix^n(b+c\,x^n))\pi p(-\mathrm{csgn}(ix^n(b+c\,x^n))+\mathrm{csgn}(ix^n))(-\mathrm{csgn}(ix^n(b+c\,x^n))+\mathrm{csgn}(i(b+c\,x^n)))}{2}}}{n(1+p)}$	106

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x,method=_RETURNVERBOSE)
```

```
[Out] x^n*(b+c*x^n)/n/(1+p)*(x^n)^p*(b+c*x^n)^p*exp(-1/2*I*csgn(I*x^n*(b+c*x^n))*
Pi*p*(-csgn(I*x^n*(b+c*x^n))+csgn(I*x^n))*(-csgn(I*x^n*(b+c*x^n))+csgn(I*(b
+c*x^n))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^{2\,n}+bx^n)(cx^{2\,n}+bx^n)^p}{np+n}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
[Out] (c*x^(2*n) + b*x^n)*(c*x^(2*n) + b*x^n)^p/(n*p + n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(19) = 38.

Time = 11.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.85

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx$$
$$=\begin{cases}\frac{(b+2c)\log(x)}{b+c} & \text{for } n=0\wedge p=-1\\ (b+c)^p\,(b+2c)\log(x) & \text{for } n=0\\ \frac{\log(x^n)}{n}+\frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{for } p=-1\\ \frac{bxx^{n-1}(bx^n+cx^{2n})^p}{np+n}+\frac{cxx^nx^{n-1}(bx^n+cx^{2n})^p}{np+n} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**p,x)
[Out] Piecewise(((b + 2*c)*log(x)/(b + c), Eq(n, 0) & Eq(p, -1)), ((b + c)**p*(b
+ 2*c)*log(x), Eq(n, 0)), (log(x**n)/n + log(b/c + x**n)/n, Eq(p, -1)), (b*
x*x**(n - 1)*(b*x**n + c*x**(2*n))**p/(n*p + n) + c*x*x**n*x**(n - 1)*(b*x*
*n + c*x**(2*n))**p/(n*p + n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx=\frac{(cx^{2\,n}+bx^n)e^{(p\log(cx^n+b)+p\log(x^n))}}{n(p+1)}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] (c*x^(2*n) + b*x^n)*e^(p*log(c*x^n + b) + p*log(x^n))/(n*(p + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx = \frac{(cx^{2\,n}+bx^n)^{p+1}}{n(p+1)}$$

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] (c*x^(2*n) + b*x^n)^(p + 1)/(n*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx = \frac{x^n\,(b+c\,x^n)\,\left(b\,x^n+c\,x^{2\,n}\right)^p}{n\,(p+1)}$$

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x)
[Out] (x^n*(b + c*x^n)*(b*x^n + c*x^(2*n))^p)/(n*(p + 1))
```

3.141 $\int \frac{(fx)^m(d+ex^n)}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 196

$$\int \frac{(fx)^m\,(d+ex^n)}{a+bx^n+cx^{2n}}\,dx = \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)f(1+m)} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)f(1+m)}$$

```
[Out] (f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/
2)))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b-(-4*a*c+b^2)^(1/2))+(f*
x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2))
)*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1574, 371}

$$\int \frac{(fx)^m\,(d+ex^n)}{a+bx^n+cx^{2n}}\,dx = \frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)} + \frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

```
[In] Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (
1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^
2 - 4*a*c])*f*(1 + m)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 +
m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])/((b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1574

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^m}{b-\sqrt{b^2-4ac}+2cx^n}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^m}{b+\sqrt{b^2-4ac}+2cx^n}\right) dx \\
&= \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{(fx)^m}{b+\sqrt{b^2-4ac}+2cx^n}\,dx \\
&\quad +\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{(fx)^m}{b-\sqrt{b^2-4ac}+2cx^n}\,dx \\
&= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)f(1+m)} \\
&\quad +\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)f(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.81

$$\int \frac{(fx)^m (d+ex^n)}{a+bx^n+cx^{2n}}\,dx$$
$$= \frac{x(fx)^m \left(\left(bd+\sqrt{b^2-4ac}d-2ae\right) \text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+\left(-bd+\sqrt{b^2-4ac}\right.\right.}{2a\sqrt{b^2-4ac}(1+m)}$$

```
[In] Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x]
[Out] (x*(f*x)^m*((b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, (1 + m
)/n, (1 + m + n)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (-(b*d) + Sqrt[b^
2 - 4*a*c]*d + 2*a*e)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]))/(2*a*Sqrt[b^2 - 4*a*c]*(1 + m))
```

Maple [F]

$$\int \frac{(fx)^m (d+e\,x^n)}{a+b\,x^n+c\,x^{2n}}dx$$

```
[In] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
[Out] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{(fx)^m (d+ex^n)}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)(fx)^m}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]

$$\int \frac{(fx)^m (d+ex^n)}{a+bx^n+cx^{2n}}\,dx = \int \frac{(fx)^m (d+ex^n)}{a+bx^n+cx^{2n}}\,dx$$

```
[In] integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral((f*x)**m*(d + e*x**n)/(a + b*x**n + c*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{(fx)^m (d+ex^n)}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)(fx)^m}{cx^{2\,n}+bx^n+a}\,dx$$

[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)

Giac [F]

$$\int \frac{(fx)^m (d+ex^n)}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)(fx)^m}{cx^{2\,n}+bx^n+a}\,dx$$

[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^n)}{a+bx^n+cx^{2n}}\,dx = \int \frac{(f\,x)^m\,(d+e\,x^n)}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

[In] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x)

[Out] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)), x)

3.142 $\int \frac{(fx)^m(d+ex^n)}{(a+bx^n+cx^{2n})^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 374

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^2}\,dx = \frac{(fx)^{1+m}(b^2d-2acd-abe+c(bd-2ae)x^n)}{a(b^2-4ac)fn(a+bx^n+cx^{2n})}$$
$$-\frac{c\left((bd-2ae)(1+m-n)-\frac{4acd(1+m-2n)-b^2d(1+m-n)+2aben}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+}{n}\right.}{a(b^2-4ac)\left(b-\sqrt{b^2-4ac}\right)f(1+m)n}$$
$$-\frac{c\left((bd-2ae)(1+m-n)+\frac{4acd(1+m-2n)-b^2d(1+m-n)+2aben}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+}{n}\right.}{a(b^2-4ac)\left(b+\sqrt{b^2-4ac}\right)f(1+m)n}$$

```
[Out] (f*x)^(1+m)*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^n)/a/(-4*a*c+b^2)/f/n/(a+
b*x^n+c*x^(2*n))-c*(f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/
(b-(-4*a*c+b^2)^(1/2)))*((-2*a*e+b*d)*(1+m-n)+(-4*a*c*d*(1+m-2*n)+b^2*d*(1+
m-n)-2*a*b*e*n)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/f/(1+m)/n/(b-(-4*a*c+b^2
)^(1/2))-c*(f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a
*c+b^2)^(1/2)))*((-2*a*e+b*d)*(1+m-n)+(4*a*c*d*(1+m-2*n)-b^2*d*(1+m-n)+2*a*
b*e*n)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/f/(1+m)/n/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used

$= \{1572,\, 1574,\, 371\}$

$$\int \frac{(fx)^m\,(d+ex^n)}{(a+bx^n+cx^{2n})^2}\,dx = -\frac{c(fx)^{m+1}\left((m-n+1)(bd-2ae)-\frac{2aben+4acd(m-2n+1)+b^2(-d)(m-n+1)}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n}{n}\right.}{af(m+1)n\,(b^2-4ac)\left(b-\sqrt{b^2-4ac}\right)} - \frac{c(fx)^{m+1}\left(\frac{2aben+4acd(m-2n+1)+b^2(-d)(m-n+1)}{\sqrt{b^2-4ac}}+(m-n+1)(bd-2ae)\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n}{n}\right.}{af(m+1)n\,(b^2-4ac)\left(\sqrt{b^2-4ac}+b\right)} + \frac{(fx)^{m+1}\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{afn\,(b^2-4ac)\,(a+bx^n+cx^{2n})}$$

```
[In] Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(a*(b^2 - 4
*a*c)*f*n*(a + b*x^n + c*x^(2*n))) - (c*((b*d - 2*a*e)*(1 + m - n) - (4*a*c
*d*(1 + m - 2*n) - b^2*d*(1 + m - n) + 2*a*b*e*n)/Sqrt[b^2 - 4*a*c])*(f*x)^
(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m)*n) - (c
*((b*d - 2*a*e)*(1 + m - n) + (4*a*c*d*(1 + m - 2*n) - b^2*d*(1 + m - n) +
2*a*b*e*n)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n,
 (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b +
Sqrt[b^2 - 4*a*c])*f*(1 + m)*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1572

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m +
 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + (m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*c*
x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && ILtQ[p + 1, 0]
```

Rule 1574

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
```

```
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\text{integral} = \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)} - \frac{\int \frac{(fx)^m\left(-abe(1+m)-2acd(1+m-2n)+b^2d(1+m-n)+c(bd-2ae)(1+m-n)x^n\right)}{a+bx^n+cx^{2n}}\,dx}{a\left(b^2-4ac\right)n}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)} - \frac{\int \left(\frac{\left(c(bd-2ae)(1+m-n)+\frac{c\left(b^2d-4acd+b^2dm-4acdm-b^2dn+8acdn-2aben\right)}{\sqrt{b^2-4ac}}\right)(fx)^m}{b-\sqrt{b^2-4ac}+2cx^n} + \frac{\left(c(bd-2ae)(1+m-n)-\frac{c\left(b^2d-4acd+b^2d\cdots\right.}{\cdots}\right.}{b+\sqrt{b^2-4a\cdots}}\cdots\right.}{a\left(b^2-4ac\right)n}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)} - \frac{\left(c\left((bd-2ae)(1+m-n)-\frac{4acd(1+m-2n)-b^2d(1+m-n)+2aben}{\sqrt{b^2-4ac}}\right)\right)\int \frac{(fx)^m}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{a\left(b^2-4ac\right)n} - \frac{\left(c\left((bd-2ae)(1+m-n)+\frac{4acd(1+m-2n)-b^2d(1+m-n)+2aben}{\sqrt{b^2-4ac}}\right)\right)\int \frac{(fx)^m}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{a\left(b^2-4ac\right)n}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)} - \frac{c\left((bd-2ae)(1+m-n)-\frac{4acd(1+m-2n)-b^2d(1+m-n)+2aben}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx\cdots}{b-\sqrt{b^2\cdots}}\right.}{a\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)f(1+m)n} - \frac{c\left((bd-2ae)(1+m-n)+\frac{4acd(1+m-2n)-b^2d(1+m-n)+2aben}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx\cdots}{b+\sqrt{b^2\cdots}}\right.}{a\left(b^2-4ac\right)\left(b+\sqrt{b^2-4ac}\right)f(1+m)n}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5363 vs. 2(374) = 748.

Time = 7.23 (sec) , antiderivative size = 5363, normalized size of antiderivative = 14.34

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^2} dx = \text{Result too large to show}$$

```
[In] Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{(fx)^m (d+e\,x^n)}{(a+b\,x^n+c\,x^{2n})^2}dx$$

```
[In] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
[Out] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

Fricas [F]

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^2} dx = \int \frac{(ex^n+d)(fx)^m}{(cx^{2\,n}+bx^n+a)^2} dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
[Out] integral((e*x^n + d)*(f*x)^m/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 +
 2*(b*c*x^n + a*c)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^2} dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^2} dx = \int \frac{(ex^n+d)(fx)^m}{(cx^{2n}+bx^n+a)^2} dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] ((b^2*d*f^m - (2*c*d*f^m + b*e*f^m)*a)*x*x^m + (b*c*d*f^m - 2*a*c*e*f^m)*x*
e^(m*log(x) + n*log(x)))/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)
*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n) - integrate(((b^2*d*f^m*(m - n + 1)
 - (2*c*d*f^m*(m - 2*n + 1) + b*e*f^m*(m + 1))*a)*x^m + (b*c*d*f^m*(m - n +
 1) - 2*a*c*e*f^m*(m - n + 1))*e^(m*log(x) + n*log(x)))/(a^2*b^2*n - 4*a^3*
c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

Giac [F]

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^2} dx = \int \frac{(ex^n+d)(fx)^m}{(cx^{2n}+bx^n+a)^2} dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^2} dx = \int \frac{(f\,x)^m\,(d+e\,x^n)}{(a+b\,x^n+c\,x^{2\,n})^2} dx$$

```
[In] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x)
```

```
[Out] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2, x)
```

3.143 $\int \frac{(fx)^m(d+ex^n)}{(a+bx^n+cx^{2n})^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 816

$$\int \frac{(fx)^m\,(d+ex^n)}{(a+bx^n+cx^{2n})^3}\,dx = \frac{(fx)^{1+m}\,(b^2d-2acd-abe+c(bd-2ae)x^n)}{2a\,(b^2-4ac)\,fn\,(a+bx^n+cx^{2n})^2}$$
$$+\frac{(fx)^{1+m}\,((b^2-2ac)\,(abe(1+m)+2acd(1+m-4n)-b^2d(1+m-2n))+abc(bd-2ae)(1+m-3n)\ldots}{2a^2\,(b^2-4ac)^2\,fn^2\,(a+bx\ldots}$$
$$-\frac{c\Big((ab^2e(1+m)+2abcd(2+2m-7n)-4a^2ce(1+m-3n)-b^3d(1+m-2n))\,(1+m-n)+\frac{ab^3e(1+\ldots}{}\ldots}{\ldots}$$
$$-\frac{c\Big((ab^2e(1+m)+2abcd(2+2m-7n)-4a^2ce(1+m-3n)-b^3d(1+m-2n))\,(1+m-n)-\frac{ab^3e(1+\ldots}{}\ldots}{\ldots}$$

```
[Out] 1/2*(f*x)^(1+m)*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^n)/a/(-4*a*c+b^2)/f/n
/(a+b*x^n+c*x^(2*n))^2+1/2*(f*x)^(1+m)*((-2*a*c+b^2)*(a*b*e*(1+m)+2*a*c*d*(
1+m-4*n)-b^2*d*(1+m-2*n))+a*b*c*(-2*a*e+b*d)*(1+m-3*n)+c*(a*b^2*e*(1+m)+2*a
*b*c*d*(2+2*m-7*n)-4*a^2*c*e*(1+m-3*n)-b^3*d*(1+m-2*n))*x^n)/a^2/(-4*a*c+b^
2)^2/f/n^2/(a+b*x^n+c*x^(2*n))-1/2*c*(f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1
+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*((a*b^2*e*(1+m)+2*a*b*c*d*(2+2*m-
7*n)-4*a^2*c*e*(1+m-3*n)-b^3*d*(1+m-2*n))*(1+m-n)+(a*b^3*e*(1+m)*(1+m-n)-4*
a^2*b*c*e*(1+m^2+m*(2-n)-n-3*n^2)-b^4*d*(1+m^2+m*(2-3*n)-3*n+2*n^2)+6*a*b^2
*c*d*(1+m^2+m*(2-4*n)-4*n+3*n^2)-8*a^2*c^2*d*(1+m^2+m*(2-6*n)-6*n+8*n^2))/(
-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2/f/(1+m)/n^2/(b-(-4*a*c+b^2)^(1/2))-1/
2*c*(f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)
^(1/2)))*((a*b^2*e*(1+m)+2*a*b*c*d*(2+2*m-7*n)-4*a^2*c*e*(1+m-3*n)-b^3*d*(1
+m-2*n))*(1+m-n)+(-a*b^3*e*(1+m)*(1+m-n)+4*a^2*b*c*e*(1+m^2+m*(2-n)-n-3*n^2
)+b^4*d*(1+m^2+m*(2-3*n)-3*n+2*n^2)-6*a*b^2*c*d*(1+m^2+m*(2-4*n)-4*n+3*n^2)
+8*a^2*c^2*d*(1+m^2+m*(2-6*n)-6*n+8*n^2))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b
^2)^2/f/(1+m)/n^2/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 3.53 (sec) , antiderivative size = 816, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1572, 1574, 371}

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^3} dx =$$

$$-\frac{c\Big((-d(m-2n+1)b^3+ae(m+1)b^2+2acd(2m-7n+2)b-4a^2ce(m-3n+1))(m-n+1)+\frac{-d(\cdots}{\cdots}}{\cdots}$$

$$-\frac{c\Big((-d(m-2n+1)b^3+ae(m+1)b^2+2acd(2m-7n+2)b-4a^2ce(m-3n+1))(m-n+1)-\frac{-d(\cdots}{\cdots}}{\cdots}$$

$$+\frac{(c(-d(m-2n+1)b^3+ae(m+1)b^2+2acd(2m-7n+2)b-4a^2ce(m-3n+1))x^n+(b^2-2ac)(-\cdots}{2a^2(b^2-4ac)^2 fn^2(bx^{\cdots}}$$

$$+\frac{(c(bd-2ae)x^n+b^2d-2acd-abe)(fx)^{m+1}}{2a(b^2-4ac)fn(bx^n+cx^{2n}+a)^2}$$

```
[In] Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(2*a*(b^2 -
 4*a*c)*f*n*(a + b*x^n + c*x^(2*n))^2) + ((f*x)^(1 + m)*((b^2 - 2*a*c)*(a*b
*e*(1 + m) + 2*a*c*d*(1 + m - 4*n) - b^2*d*(1 + m - 2*n)) + a*b*c*(b*d - 2*
a*e)*(1 + m - 3*n) + c*(a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) - 4*a^2
*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*f*n^
2*(a + b*x^n + c*x^(2*n))) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*
n) - 4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) + (a*b^3*e*
(1 + m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n - 3*n^2) - b^4*d
*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*d*(1 + m^2 + m*(2 - 4*n)
 - 4*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))/Sqrt[b
^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b - Sqrt[b^2 -
4*a*c])*f*(1 + m)*n^2) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) -
 4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) - (a*b^3*e*(1 +
 m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n - 3*n^2) - b^4*d*(1
+ m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*d*(1 + m^2 + m*(2 - 4*n) - 4
*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))/Sqrt[b^2 -
 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b + Sqrt[b^2 - 4*a*
c])*f*(1 + m)*n^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1572

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m +
 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + (m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*c*
x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && ILtQ[p + 1, 0]
```

Rule 1574

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\text{integral} = \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} - \frac{\int \frac{(fx)^m\left(-abe(1+m)-2acd(1+m-4n)+b^2d(1+m-2n)+c(bd-2ae)(1+m-3n)x^n\right)}{\left(a+bx^n+cx^{2n}\right)^2}\,dx}{2a\left(b^2-4ac\right)n}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} + \frac{(fx)^{1+m}\left(\left(b^2-2ac\right)\left(abe(1+m)+2acd(1+m-4n)-b^2d(1+m-2n)\right)+abc(bd-2ae)(1+m\right.}{2a^2\left(b^2-4ac\right)^2fn^2} + \frac{\int \frac{(fx)^m\left(\left(abe(1+m)+2acd(1+m-4n)-b^2d(1+m-2n)\right)\left(2ac(1+m-2n)-b^2(1+m-n)\right)-abc(bd-2ae)(1+m)(1+m-3n)-c\left(ab^2e(1+\right.\right.}{a+bx^n+cx^{2n}}}{2a^2\left(b^2-4ac\right)^2n^2}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} + \frac{(fx)^{1+m}\left(\left(b^2-2ac\right)\left(abe(1+m)+2acd(1+m-4n)-b^2d(1+m-2n)\right)+abc(bd-2ae)(1+m\right.}{2a^2\left(b^2-4ac\right)^2fn^2} + \frac{\int \left(\frac{\left(-c\left(ab^2e(1+m)+2abcd(2+2m-7n)-4a^2ce(1+m-3n)-b^3d(1+m-2n)\right)(1+m-n)+\frac{c\left(b^4d-6ab^2cd+8a^2c^2d-ab^3e+4a^2bce+2b^4dn\right.}{}\right.}{}\right.}{}$$

$$= \frac{(fx)^{1+m}(b^2d - 2acd - abe + c(bd - 2ae)x^n)}{2a(b^2 - 4ac)fn(a + bx^n + cx^{2n})^2}$$
$$+ \frac{(fx)^{1+m}((b^2 - 2ac)(abe(1+m) + 2acd(1+m-4n) - b^2d(1+m-2n)) + abc(bd - 2ae)(1+ \ldots}{2a^2(b^2 - 4ac)^2 f\ldots}$$
$$- \frac{\left(c\left((ab^2e(1+m) + 2abcd(2+2m-7n) - 4a^2ce(1+m-3n) - b^3d(1+m-2n))(1+m-n \ldots\right.\right.}{\ldots}$$
$$- \frac{\left(c\left((ab^2e(1+m) + 2abcd(2+2m-7n) - 4a^2ce(1+m-3n) - b^3d(1+m-2n))(1+m-n \ldots\right.\right.}{\ldots}$$

$$= \frac{(fx)^{1+m}(b^2d - 2acd - abe + c(bd - 2ae)x^n)}{2a(b^2 - 4ac)fn(a + bx^n + cx^{2n})^2}$$
$$+ \frac{(fx)^{1+m}((b^2 - 2ac)(abe(1+m) + 2acd(1+m-4n) - b^2d(1+m-2n)) + abc(bd - 2ae)(1+ \ldots}{2a^2(b^2 - 4ac)^2 f\ldots}$$
$$- \frac{c\left((ab^2e(1+m) + 2abcd(2+2m-7n) - 4a^2ce(1+m-3n) - b^3d(1+m-2n))(1+m-n) \ldots\right.}{\ldots}$$
$$- \frac{c\left((ab^2e(1+m) + 2abcd(2+2m-7n) - 4a^2ce(1+m-3n) - b^3d(1+m-2n))(1+m-n) \ldots\right.}{\ldots}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 20515 vs. 2(816) = 1632.

Time = 8.44 (sec) , antiderivative size = 20515, normalized size of antiderivative = 25.14

$$\int \frac{(fx)^m(d + ex^n)}{(a + bx^n + cx^{2n})^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{(fx)^m(d + e\,x^n)}{(a + b\,x^n + c\,x^{2n})^3}dx$$

```
[In] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
[Out] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

Fricas [F]

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^3} \, dx = \int \frac{(ex^n+d)(fx)^m}{(cx^{2n}+bx^n+a)^3} \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(f*x)^m/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b^2*x^(2*n) +
 3*a^2*b*x^n + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a
*b*c*x^n + a^2*c)*x^(2*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^3} \, dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^3} \, dx = \int \frac{(ex^n+d)(fx)^m}{(cx^{2n}+bx^n+a)^3} \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((a*b^4*d*f^m*(m - 3*n + 1) + 2*(b*c*e*f^m*(2*m - 5*n + 2) + 2*c^2*d*f
^m*(m - 6*n + 1))*a^3 - (b^2*c*d*f^m*(5*m - 21*n + 5) + b^3*e*f^m*(m - n +
1))*a^2)*x*x^m + (b^3*c^2*d*f^m*(m - 2*n + 1) + 4*a^2*c^3*e*f^m*(m - 3*n +
1) - (2*b*c^3*d*f^m*(2*m - 7*n + 2) + b^2*c^2*e*f^m*(m + 1))*a)*x*e^(m*log(
x) + 3*n*log(x)) + (2*b^4*c*d*f^m*(m - 2*n + 1) + 2*(b*c^2*e*f^m*(4*m - 9*n
 + 4) + 2*c^3*d*f^m*(m - 4*n + 1))*a^2 - (b^2*c^2*d*f^m*(9*m - 29*n + 9) +
2*b^3*c*e*f^m*(m + 1))*a)*x*e^(m*log(x) + 2*n*log(x)) + (b^5*d*f^m*(m - 2*n
 + 1) + 4*a^3*c^2*e*f^m*(m - 5*n + 1) + (b^2*c*e*f^m*(3*m - 4*n + 3) + 2*b*
c^2*d*f^m*n)*a^2 - (4*b^3*c*d*f^m*(m - 3*n + 1) + b^4*e*f^m*(m + 1))*a)*x*e
^(m*log(x) + n*log(x)))/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*a^6*c^2*n^2 + (
a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n) + 2*(a^2*b^5*
c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2*b^6*n^2 - 6*a^
3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a^4*b^3*c*n^2 +
16*a^5*b*c^2*n^2)*x^n) + integrate(1/2*(((m^2 - m*(3*n - 2) + 2*n^2 - 3*n +
 1)*b^4*d*f^m + 2*(2*(m^2 - 2*m*(3*n - 1) + 8*n^2 - 6*n + 1)*c^2*d*f^m + (2
```

```
*m^2 - m*(5*n - 4) - 5*n + 2)*b*c*e*f^m)*a^2 - ((5*m^2 - m*(21*n - 10) + 16
*n^2 - 21*n + 5)*b^2*c*d*f^m + (m^2 - m*(n - 2) - n + 1)*b^3*e*f^m)*a)*x^m
+ ((m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^3*c*d*f^m + 4*(m^2 - 2*m*(2*n -
1) + 3*n^2 - 4*n + 1)*a^2*c^2*e*f^m - (2*(2*m^2 - m*(9*n - 4) + 7*n^2 - 9*n
 + 2)*b*c^2*d*f^m + (m^2 - m*(n - 2) - n + 1)*b^2*c*e*f^m)*a)*e^(m*log(x) +
 n*log(x)))/(a^3*b^4*n^2 - 8*a^4*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^
2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c^3*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*
c*n^2 + 16*a^4*b*c^2*n^2)*x^n), x)
```

Giac [F]

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(ex^n+d)(fx)^m}{(cx^{2\,n}+bx^n+a)^3}\,dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^3}\,dx = \int \frac{(f\,x)^m\,(d+e\,x^n)}{(a+b\,x^n+c\,x^{2\,n})^3}\,dx$$

```
[In] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x)
[Out] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3, x)
```

3.144 $\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx$

Optimal result

Integrand size = 59, antiderivative size = 47

$$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx=-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

```
[Out] -3*ln(c^(2/3)-c^(1/3)*d^(1/3)*x^(1/3)+d^(2/3)*x^(2/3))/c^(1/3)/d^(2/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {1608, 1482, 642}

$$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx=-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

```
[In] Int[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3)*x +
c^(1/3)*d*x^(4/3)),x]
```

```
[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^(1/3)*d^(2
/3))
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{\left(c\sqrt[3]{d}-c^{2/3}d^{2/3}\sqrt[3]{x}+\sqrt[3]{c}dx^{2/3}\right)x^{2/3}}\,dx\\
&= 3\text{Subst}\left(\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{c\sqrt[3]{d}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^2}\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx=-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

```
[In] Integrate[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3
)*x + c^(1/3)*d*x^(4/3)),x]
```

```
[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^(1/3)*d^(2
/3))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{3\ln\left(c^{\frac{2}{3}} d^{\frac{2}{3}} x^{\frac{1}{3}}-c^{\frac{1}{3}} d\, x^{\frac{2}{3}}-c\, d^{\frac{1}{3}}\right)}{d^{\frac{2}{3}} c^{\frac{1}{3}}}$	36
default	$-\frac{3\ln\left(c^{\frac{2}{3}} d^{\frac{2}{3}} x^{\frac{1}{3}}-c^{\frac{1}{3}} d\, x^{\frac{2}{3}}-c\, d^{\frac{1}{3}}\right)}{d^{\frac{2}{3}} c^{\frac{1}{3}}}$	36

```
[In] int((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+c^(1/3
)*d*x^(4/3)),x,method=_RETURNVERBOSE)
[Out] -3/d^(2/3)/c^(1/3)*ln(c^(2/3)*d^(2/3)*x^(1/3)-c^(1/3)*d*x^(2/3)-c*d^(1/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx=-\frac{3\log\left(dx^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}}+c^{\frac{2}{3}}d^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}d^{\frac{2}{3}}}$$

```
[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+
c^(1/3)*d*x^(4/3)),x, algorithm="fricas")
[Out] -3*log(d*x^(2/3) - c^(1/3)*d^(2/3)*x^(1/3) + c^(2/3)*d^(1/3))/(c^(1/3)*d^(2
/3))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(44) = 88$.

Time = 2.61 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.53

$$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx=-\frac{3\log\left(-\frac{\sqrt[3]{c}}{2\sqrt[3]{d}}+\sqrt[3]{x}-\frac{\sqrt{3}\sqrt{-c^{\frac{4}{3}}d^{\frac{4}{3}}}}{2\sqrt[3]{c}d}\right)}{\sqrt[3]{c}d^{\frac{2}{3}}}-\frac{3\log\left(-\frac{\sqrt[3]{c}}{2\sqrt[3]{d}}+\sqrt[3]{x}+\frac{\sqrt{3}\sqrt{-c^{\frac{4}{3}}d^{\frac{4}{3}}}}{2\sqrt[3]{c}d}\right)}{\sqrt[3]{c}d^{\frac{2}{3}}}$$

```
[In] integrate((c**(1/3)-2*d**(1/3)*x**(1/3))/(c*d**(1/3)*x**(2/3)-c**(2/3)*d**(
2/3)*x+c**(1/3)*d*x**(4/3)),x)
[Out] -3*log(-c**(1/3)/(2*d**(1/3)) + x**(1/3) - sqrt(3)*sqrt(-c**(4/3)*d**(4/3))
/(2*c**(1/3)*d))/(c**(1/3)*d**(2/3)) - 3*log(-c**(1/3)/(2*d**(1/3)) + x**(1
/3) + sqrt(3)*sqrt(-c**(4/3)*d**(4/3))/(2*c**(1/3)*d))/(c**(1/3)*d**(2/3))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx=-\frac{3\,\log\left(c^{\frac{1}{3}}dx^{\frac{2}{3}}-c^{\frac{2}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}}+cd^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}d^{\frac{2}{3}}}$$

```
[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+
c^(1/3)*d*x^(4/3)),x, algorithm="maxima")
```

```
[Out] -3*log(c^(1/3)*d*x^(2/3) - c^(2/3)*d^(2/3)*x^(1/3) + c*d^(1/3))/(c^(1/3)*d^
(2/3))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+
c^(1/3)*d*x^(4/3)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[%%%{%%{[%%%{1,[1]%%%},0]:[1,0,0,%%%{-1,[1]%%%}]%%},[1]%%%
},0]:[
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx=-\frac{3\,\ln\left(x^{2/3}+\frac{c^{2/3}}{d^{2/3}}-\frac{c^{1/3}\,x^{1/3}}{d^{1/3}}\right)}{c^{1/3}\,d^{2/3}}$$

```
[In] int((c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3)*x +
c^(1/3)*d*x^(4/3)),x)
```

```
[Out] -(3*log(x^(2/3) + c^(2/3)/d^(2/3) - (c^(1/3)*x^(1/3))/d^(1/3)))/(c^(1/3)*d^
(2/3))
```

3.145 $\int \frac{(fx)^m(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 245

$$\begin{aligned}\int \frac{(fx)^m(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx &= \frac{2c(fx)^{1+m}(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\text{AppellF1}\left(\frac{1+m}{n},1,-q,\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)f(1+m)} \\ &\quad - \frac{2c(fx)^{1+m}(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\text{AppellF1}\left(\frac{1+m}{n},1,-q,\frac{1+m+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)f(1+m)}\end{aligned}$$

```
[Out] 2*c*(f*x)^(1+m)*(d+e*x^n)^q*AppellF1((1+m)/n,1,-q,(1+m+n)/n,-2*c*x^n/(b-(-4
*a*c+b^2)^(1/2)),-e*x^n/d)/f/(1+m)/((1+e*x^n/d)^q)/(b-(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2)-2*c*(f*x)^(1+m)*(d+e*x^n)^q*AppellF1((1+m)/n,1,-q,(1+m+n)
/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/f/(1+m)/((1+e*x^n/d)^q)/(-4*a*
c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used

= {1570, 525, 524}

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$
$$= \frac{2c(fx)^{m+1}(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{m+1}{n},1,-q,\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}$$
$$-\frac{2c(fx)^{m+1}(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{m+1}{n},1,-q,\frac{m+n+1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[In] Int[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*c*(f*x)^(1 + m)*(d + e*x^n)^q*AppellF1[(1 + m)/n, 1, -q, (1 + m + n)/n,
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b -
Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^n)/d)^q) - (2*c*(f*x)^(1 + m)*(d + e
*x^n)^q*AppellF1[(1 + m)/n, 1, -q, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*f*(1 +
 m)*(1 + (e*x^n)/d)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1570

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
2*(c/r), Int[(f*x)^m*((d + e*x^n)^q/(b - r + 2*c*x^n)), x], x] - Dist[2*(c/
r), Int[(f*x)^m*((d + e*x^n)^q/(b + r + 2*c*x^n)), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(2c)\int \frac{(fx)^m(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(fx)^m(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}\\
&= \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{(fx)^m\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}\\
&\quad - \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{(fx)^m\left(1+\frac{ex^n}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}\\
&= \frac{2c(fx)^{1+m}(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{1+m}{n};1,-q;\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)f(1+m)}\\
&\quad - \frac{2c(fx)^{1+m}(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{1+m}{n};1,-q;\frac{1+m+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)f(1+m)}
\end{aligned}$$

Mathematica [F]

$$\int \frac{(fx)^m(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(fx)^m(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

```
[In] Integrate[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
[Out] Integrate[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

Maple [F]

$$\int \frac{(fx)^m(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2n}}dx$$

```
[In] int((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
[Out] int((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q (fx)^m}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((f*x)**m*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q (fx)^m}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q (fx)^m}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(f\,x)^m\,(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int(((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x)
[Out] int(((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x)
```

3.146 $\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 210

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = -\frac{2cx^3(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{3}{n},1,-q,\frac{3+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)} - \frac{2cx^3(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{3}{n},1,-q,\frac{3+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(b^2-4ac+b\sqrt{b^2-4ac}\right)}$$

```
[Out] -2/3*c*x^3*(d+e*x^n)^q*AppellF1(3/n,1,-q,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(
1/2)),-e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2/3*c*x^3*
(d+e*x^n)^q*AppellF1(3/n,1,-q,(3+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^
n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1570, 525, 524}

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = -\frac{2cx^3(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\operatorname{AppellF1}\left(\frac{3}{n},1,-q,\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)} - \frac{2cx^3(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\operatorname{AppellF1}\left(\frac{3}{n},1,-q,\frac{n+3}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[In] Int[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x^3*(d + e*x^n)^q*AppellF1[3/n, 1, -q, (3 + n)/n, (-2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c]), -((e*x^n)/d)])/(3*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 +
 (e*x^n)/d)^q) - (2*c*x^3*(d + e*x^n)^q*AppellF1[3/n, 1, -q, (3 + n)/n, (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(3*(b^2 - 4*a*c + b*Sqrt[b^
2 - 4*a*c])*(1 + (e*x^n)/d)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1570

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
2*(c/r), Int[(f*x)^m*((d + e*x^n)^q/(b - r + 2*c*x^n)), x], x] - Dist[2*(c/
r), Int[(f*x)^m*((d + e*x^n)^q/(b + r + 2*c*x^n)), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(2c)\int \frac{x^2(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{x^2(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} \\
&= \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{x^2\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} \\
&\quad - \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{x^2\left(1+\frac{ex^n}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}
\end{aligned}
$$

$$= -\frac{2cx^3(d+ex^n)^q \left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(\frac{3}{n};1,-q;\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)} - \frac{2cx^3(d+ex^n)^q \left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(\frac{3}{n};1,-q;\frac{3+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(b^2-4ac+b\sqrt{b^2-4ac}\right)}$$

Mathematica [F]

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

```
[In] Integrate[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
[Out] Integrate[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

Maple [F]

$$\int \frac{x^2(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2n}}dx$$

```
[In] int(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
[Out] int(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q x^2}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(x**2*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q x^2}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q x^2}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{x^2\,(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int((x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x)
[Out] int((x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x)
```

3.147 $\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 206

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = -\frac{cx^2(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{2}{n},1,-q,\frac{2+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{cx^2(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{2}{n},1,-q,\frac{2+n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

```
[Out] -c*x^2*(d+e*x^n)^q*AppellF1(2/n,1,-q,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)
),-e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-c*x^2*(d+e*x^n
)^q*AppellF1(2/n,1,-q,(2+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/((1
+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1570, 525, 524}

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = -\frac{cx^2(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\operatorname{AppellF1}\left(\frac{2}{n},1,-q,\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2} - \frac{cx^2(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\operatorname{AppellF1}\left(\frac{2}{n},1,-q,\frac{n+2}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[In] Int[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((c*x^2*(d + e*x^n)^q*AppellF1[2/n, 1, -q, (2 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + (e
*x^n)/d)^q)) - (c*x^2*(d + e*x^n)^q*AppellF1[2/n, 1, -q, (2 + n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*
a*c])*(1 + (e*x^n)/d)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1570

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
2*(c/r), Int[(f*x)^m*((d + e*x^n)^q/(b - r + 2*c*x^n)), x], x] - Dist[2*(c/
r), Int[(f*x)^m*((d + e*x^n)^q/(b + r + 2*c*x^n)), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(2c)\int \frac{x(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{x(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} \\
&= \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{x\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} \\
&\quad - \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{x\left(1+\frac{ex^n}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}
\end{aligned}$$

$$= -\frac{cx^2(d+ex^n)^q \left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(\frac{2}{n};1,-q;\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{cx^2(d+ex^n)^q \left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(\frac{2}{n};1,-q;\frac{2+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

Mathematica [F]

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

```
[In] Integrate[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
[Out] Integrate[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

Maple [F]

$$\int \frac{x(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2n}}dx$$

```
[In] int(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
[Out] int(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q x}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(x*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q x}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q x}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{x\,(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int((x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x)
[Out] int((x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x)
```

3.148 $\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}} dx$

Optimal result

Integrand size = 26, antiderivative size = 194

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}} dx = -\frac{2cx(d+ex^n)^q \left(1+\frac{ex^n}{d}\right)^{-q} \text{AppellF1}\left(\frac{1}{n},1,-q,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{2cx(d+ex^n)^q \left(1+\frac{ex^n}{d}\right)^{-q} \text{AppellF1}\left(\frac{1}{n},1,-q,1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

```
[Out] -2*c*x*(d+e*x^n)^q*AppellF1(1/n,1,-q,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),
-e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2*c*x*(d+e*x^n)^
q*AppellF1(1/n,1,-q,1+1/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/((1+e*x
^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1442, 441, 440}

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}} dx = -\frac{2cx(d+ex^n)^q \left(\frac{ex^n}{d}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{n},1,-q,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2} - \frac{2cx(d+ex^n)^q \left(\frac{ex^n}{d}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{n},1,-q,1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[In] Int[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x*(d + e*x^n)^q*AppellF1[n^(-1), 1, -q, 1 + n^(-1), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 +
 (e*x^n)/d)^q) - (2*c*x*(d + e*x^n)^q*AppellF1[n^(-1), 1, -q, 1 + n^(-1), (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^
2 - 4*a*c])*(1 + (e*x^n)/d)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1442

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[2*(c/r), Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(2c)\int \frac{(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}\\
&= \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}\\
&\quad - \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}\\
&= -\frac{2cx(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}\\
&\quad - \frac{2cx(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}
\end{aligned}
$$

Mathematica [F]

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx=\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

```
[In] Integrate[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x]
[Out] Integrate[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x]
```

Maple [F]

$$\int \frac{(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2n}}dx$$

```
[In] int((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
[Out] int((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx=\int \frac{(ex^n+d)^q}{cx^{2\,n}+bx^n+a}\,dx$$

```
[In] integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx=\text{Timed out}$$

```
[In] integrate((d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

Giac [F]

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(ex^n+d)^q}{cx^{2n}+bx^n+a}\,dx$$

```
[In] integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \int \frac{(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

```
[In] int((d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x)
[Out] int((d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x)
```

3.149 $\int \frac{(d+ex^n)^q}{x(a+bx^n+cx^{2n})}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 263

$$\int \frac{(d+ex^n)^q}{x(a+bx^n+cx^{2n})}\,dx = \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^n)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{a\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)n(1+q)} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^n)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{a\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)n(1+q)} - \frac{(d+ex^n)^{1+q}\,\text{Hypergeometric2F1}\left(1,1+q,2+q,1+\frac{ex^n}{d}\right)}{adn(1+q)}$$

```
[Out] -(d+e*x^n)^(1+q)*hypergeom([1, 1+q],[2+q],1+e*x^n/d)/a/d/n/(1+q)+c*(d+e*x^n
)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(d+e*x^n)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/
2))))*(1+b/(-4*a*c+b^2)^(1/2))/a/n/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))+c
*(d+e*x^n)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(d+e*x^n)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2))))*(1-b/(-4*a*c+b^2)^(1/2))/a/n/(1+q)/(2*c*d-e*(b+(-4*a*c+b^2)^
(1/2)))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1488, 974, 67, 844, 70}

$$\int \frac{(d+ex^n)^q}{x(a+bx^n+cx^{2n})}\,dx$$
$$=\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^n)^{q+1}\,\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^n+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{an(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$
$$+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{q+1}\,\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{2c(ex^n+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{an(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$
$$-\frac{(d+ex^n)^{q+1}\,\text{Hypergeometric2F1}\left(1,q+1,q+2,\frac{ex^n}{d}+1\right)}{adn(q+1)}$$

```
[In] Int[(d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q
, 2 + q, (2*c*(d + e*x^n))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(a*(2*c*d
- (b - Sqrt[b^2 - 4*a*c])*e)*n*(1 + q)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(d +
 e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*n*(1
 + q)) - ((d + e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^n
)/d])/(a*d*n*(1 + q))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
```

```
b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(d+ex)^q}{x(a+bx+cx^2)}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{(d+ex)^q}{ax}+\frac{(-b-cx)(d+ex)^q}{a(a+bx+cx^2)}\right)\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{(d+ex)^q}{x}\,dx,x,x^n\right)}{an}+\frac{\text{Subst}\left(\int \frac{(-b-cx)(d+ex)^q}{a+bx+cx^2}\,dx,x,x^n\right)}{an}\\
&= -\frac{(d+ex^n)^{1+q}\,{}_2F_1\left(1,1+q;2+q;1+\frac{ex^n}{d}\right)}{adn(1+q)}\\
&\quad+\frac{\text{Subst}\left(\int \left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}+\frac{\left(-c+\frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\right)\,dx,x,x^n\right)}{an}\\
&= -\frac{(d+ex^n)^{1+q}\,{}_2F_1\left(1,1+q;2+q;1+\frac{ex^n}{d}\right)}{adn(1+q)}\\
&\quad-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\,dx,x,x^n\right)}{an}\\
&\quad-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}\,dx,x,x^n\right)}{an}
\end{aligned}
$$

$$= \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^n)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{a\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)n(1+q)} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^n)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{a\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)n(1+q)} - \frac{(d+ex^n)^{1+q}\,{}_2F_1\left(1,1+q;2+q;1+\frac{ex^n}{d}\right)}{adn(1+q)}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex^n)^q}{x(a+bx^n+cx^{2n})}\,dx = \frac{(d+ex^n)^{1+q}\left(\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+q,2+q,\frac{2c(d+ex^n)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Hypergeometric2F1}\left(1,1+q,2+\dots\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\dots\right)}{an(1+q)}$$

```
[In] Integrate[(d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] ((d + e*x^n)^(1 + q)*((c*(1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 +
 q, 2 + q, (2*c*(d + e*x^n))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric
2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) - Hypergeometric2F1[1, 1 + q, 2 + q,
1 + (e*x^n)/d]/d))/(a*n*(1 + q))
```

Maple [F]

$$\int \frac{(d+e\,x^n)^q}{x\,(a+b\,x^n+c\,x^{2n})}dx$$

```
[In] int((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{(d+ex^n)^q}{x\,(a+bx^n+cx^{2n})}\,dx = \int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x}\,dx$$

```
[In] integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e*x^n + d)^q/(c*x*x^(2*n) + b*x*x^n + a*x), x)
```

Sympy [F]

$$\int \frac{(d+ex^n)^q}{x\,(a+bx^n+cx^{2n})}\,dx = \int \frac{(d+ex^n)^q}{x\,(a+bx^n+cx^{2n})}\,dx$$

```
[In] integrate((d+e*x**n)**q/x/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral((d + e*x**n)**q/(x*(a + b*x**n + c*x**(2*n))), x)
```

Maxima [F]

$$\int \frac{(d+ex^n)^q}{x\,(a+bx^n+cx^{2n})}\,dx = \int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x}\,dx$$

```
[In] integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x), x)
```

Giac [F]

$$\int \frac{(d+ex^n)^q}{x\,(a+bx^n+cx^{2n})}\,dx = \int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x}\,dx$$

```
[In] integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^q}{x\,(a+bx^n+cx^{2n})}\,dx=\int \frac{(d+e\,x^n)^q}{x\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

```
[In] int((d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x)
[Out] int((d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))), x)
```

3.150 $\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 212

$$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx = \frac{2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\text{AppellF1}\left(-\frac{1}{n},1,-q,-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x} + \frac{2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\text{AppellF1}\left(-\frac{1}{n},1,-q,-\frac{1-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x}$$

```
[Out] 2*c*(d+e*x^n)^q*AppellF1(-1/n,1,-q,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2))
,-e*x^n/d)/x/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+2*c*(d+e*x^n)
^q*AppellF1(-1/n,1,-q,(-1+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/x/
((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1570, 525, 524}

$$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx = \frac{2c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\text{AppellF1}\left(-\frac{1}{n},1,-q,-\frac{1-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)} + \frac{2c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\text{AppellF1}\left(-\frac{1}{n},1,-q,-\frac{1-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[In] Int[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2*c*(d + e*x^n)^q*AppellF1[-n^(-1), 1, -q, -((1 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x*(1
 + (e*x^n)/d)^q) + (2*c*(d + e*x^n)^q*AppellF1[-n^(-1), 1, -q, -((1 - n)/n)
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt
[b^2 - 4*a*c])*x*(1 + (e*x^n)/d)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1570

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
2*(c/r), Int[(f*x)^m*((d + e*x^n)^q/(b - r + 2*c*x^n)), x], x] - Dist[2*(c/
r), Int[(f*x)^m*((d + e*x^n)^q/(b + r + 2*c*x^n)), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(2c)\int \frac{(d+ex^n)^q}{x^2\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^n)^q}{x^2\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}\\ &= \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{x^2\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}\\ &\quad - \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{x^2\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}\end{aligned}$$

$$= \frac{2c(d+ex^n)^q \left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x} + \frac{2c(d+ex^n)^q \left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x}$$

Mathematica [F]

$$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx = \int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx$$

```
[In] Integrate[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x]
[Out] Integrate[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))), x]
```

Maple [F]

$$\int \frac{(d+e\,x^n)^q}{x^2(a+b\,x^n+c\,x^{2n})}dx$$

```
[In] int((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x)
[Out] int((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx = \int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x^2}\,dx$$

```
[In] integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e*x^n + d)^q/(c*x^2*x^(2*n) + b*x^2*x^n + a*x^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((d+e*x**n)**q/x**2/(a+b*x**n+c*x**(2*n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx = \int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x^2}\,dx$$

```
[In] integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

Giac [F]

$$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx = \int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x^2}\,dx$$

```
[In] integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx = \int \frac{(d+e\,x^n)^q}{x^2\,(a+b\,x^n+c\,x^{2\,n})}\,dx$$

```
[In] int((d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x)
[Out] int((d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))), x)
```

3.151 $\int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 210

$$\int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})}\,dx = \frac{c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\text{AppellF1}\left(-\frac{2}{n},1,-q,-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x^2} + \frac{c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\text{AppellF1}\left(-\frac{2}{n},1,-q,-\frac{2-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x^2}$$

```
[Out] c*(d+e*x^n)^q*AppellF1(-2/n,1,-q,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-
e*x^n/d)/x^2/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+c*(d+e*x^n)^q
*AppellF1(-2/n,1,-q,(-2+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/x^2/
((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1570, 525, 524}

$$\int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})}\,dx = \frac{c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\text{AppellF1}\left(-\frac{2}{n},1,-q,-\frac{2-n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x^2\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)} + \frac{c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}\text{AppellF1}\left(-\frac{2}{n},1,-q,-\frac{2-n}{n},-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x^2\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[In] Int[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (c*(d + e*x^n)^q*AppellF1[-2/n, 1, -q, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x^2*(1 +
(e*x^n)/d)^q) + (c*(d + e*x^n)^q*AppellF1[-2/n, 1, -q, -((2 - n)/n), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4
*a*c])*x^2*(1 + (e*x^n)/d)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1570

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
2*(c/r), Int[(f*x)^m*((d + e*x^n)^q/(b - r + 2*c*x^n)), x], x] - Dist[2*(c/
r), Int[(f*x)^m*((d + e*x^n)^q/(b + r + 2*c*x^n)), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(2c)\int \frac{(d+ex^n)^q}{x^3\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^n)^q}{x^3\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} \\
&= \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{x^3\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} \\
&\quad - \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{x^3\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}
\end{aligned}
$$

$$= \frac{c(d+ex^n)^q \left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x^2} + \frac{c(d+ex^n)^q \left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x^2}$$

Mathematica [F]

$$\int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})}\,dx = \int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})}\,dx$$

```
[In] Integrate[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x]
[Out] Integrate[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))), x]
```

Maple [F]

$$\int \frac{(d+e\,x^n)^q}{x^3(a+b\,x^n+c\,x^{2n})}dx$$

```
[In] int((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x)
[Out] int((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x)
```

Fricas [F]

$$\int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})}\,dx = \int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x^3}\,dx$$

```
[In] integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((e*x^n + d)^q/(c*x^3*x^(2*n) + b*x^3*x^n + a*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})}\,dx = \text{Timed out}$$

```
[In] integrate((d+e*x**n)**q/x**3/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d+ex^n)^q}{x^3\,(a+bx^n+cx^{2n})}\,dx = \int \frac{(ex^n+d)^q}{(cx^{2\,n}+bx^n+a)x^3}\,dx$$

```
[In] integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

Giac [F]

$$\int \frac{(d+ex^n)^q}{x^3\,(a+bx^n+cx^{2n})}\,dx = \int \frac{(ex^n+d)^q}{(cx^{2\,n}+bx^n+a)x^3}\,dx$$

```
[In] integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^n)^q}{x^3\,(a+bx^n+cx^{2n})}\,dx = \int \frac{(d+e\,x^n)^q}{x^3\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

```
[In] int((d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x)
[Out] int((d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))), x)
```

3.152 $\int (fx)^m (d+ex^n)^2 (a+bx^n+cx^{2n})^p \, dx$

Optimal result

Integrand size = 31, antiderivative size = 498

$$\int (fx)^m (d+ex^n)^2 (a+bx^n+cx^{2n})^p \, dx$$

$$= \frac{d^2(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2}{b-\sqrt{}}\right.}{f(1+m)}$$

$$+\frac{2dex^{1+n}(fx)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m+n}{n},-p,-p,\frac{1+m+}{n}\right.}{1+m+n}$$

$$+\frac{e^2x^{1+2n}(fx)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m+2n}{n},-p,-p,\frac{1+m-}{n}\right.}{1+m+2n}$$

```
[Out] d^2*(f*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*c
*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/f/(1+m)/((1+2*
c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+2*d
*e*x^(1+n)*(f*x)^m*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m+n)/n,-p,-p,(1+m+2*n)
/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+m+n)
/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))
^p)+e^2*x^(1+2*n)*(f*x)^m*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m+2*n)/n,-p,-p,
(1+m+3*n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2))
)/(1+m+2*n)/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b
^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1574, 1399, 524, 20}

$$\int (fx)^m (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx$$
$$= \frac{d^2(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{m+1}{n},-p,-p,\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right.}{f(m+1)}$$
$$+\frac{2dex^{n+1}(fx)^m\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{m+n+1}{n},-p,-p,\frac{m+2n+1}{n}\right.}{m+n+1}$$
$$+\frac{e^2x^{2n+1}(fx)^m\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{m+2n+1}{n},-p,-p,\frac{m+3n+1}{n}\right.}{m+2n+1}$$

```
[In] Int[(f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (d^2*(f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1
 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (2*d*e*x^(1 + n)*(f*x)^m*(a + b*x^n + c*x
^(2*n))^p*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + n)*(1 +
(2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m +
 2*n)/n, -p, -p, (1 + m + 3*n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + 2*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1574

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(d^2(fx)^m \left(a+bx^n+cx^{2n}\right)^p + 2dex^n(fx)^m \left(a+bx^n+cx^{2n}\right)^p + e^2x^{2n}(fx)^m \left(a+bx^n+cx^{2n}\right)^p\right) dx \\
&= d^2\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p dx + (2de)\int x^n(fx)^m \left(a+bx^n+cx^{2n}\right)^p dx \\
&\quad + e^2\int x^{2n}(fx)^m \left(a+bx^n+cx^{2n}\right)^p dx \\
&= \left(2dex^{-m}(fx)^m\right)\int x^{m+n}\left(a+bx^n+cx^{2n}\right)^p dx \\
&\quad + \left(e^2x^{-m}(fx)^m\right)\int x^{m+2n}\left(a+bx^n+cx^{2n}\right)^p dx \\
&\quad + \left(d^2\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\right)\int (fx)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^p dx
\end{aligned}$$

$$= \frac{d^2(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(\frac{1+m}{n};-p,-p;\frac{1+m+n}{n};-\frac{2c}{b-\sqrt{b}}\right.}{f(1+m)}$$

$$+\left(2dex^{-m}(fx)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n + cx^{2n})^p\right)\int x^{m+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^p dx$$

$$+\left(e^2x^{-m}(fx)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n + cx^{2n})^p\right)\int x^{m+2n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^p dx$$

$$= \frac{d^2(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(\frac{1+m}{n};-p,-p;\frac{1+m+n}{n};-\frac{2c}{b-\sqrt{b}}\right.}{f(1+m)}$$
$$+\frac{2dex^{1+n}(fx)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(\frac{1+m+n}{n};-p,-p;\frac{1+m+2}{n}\right.}{1+m+n}$$
$$+\frac{e^2x^{1+2n}(fx)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(\frac{1+m+2n}{n};-p,-p;\frac{1+m+3}{n}\right.}{1+m+2n}$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.79

$$\int (fx)^m (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p dx$$
$$= \frac{x(fx)^m\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\left(d^2(1+m^2+3n+2n^2+m(2+3n))\,\mathrm{A}\right.}{}$$

```
[In] Integrate[(f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + x^n*(b + c*x^n))^p*(d^2*(1 + m^2 + 3*n + 2*n^2 + m*(2 + 3*n
))*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m)*x^n*(2*d*(1 + m + 2*
n)*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m + n)*x^n*Appell
F1[(1 + m + 2*n)/n, -p, -p, (1 + m + 3*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/((1 + m)*(1 + m + n)*(1 + m +
2*n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sq
rt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int (fx)^m (d+e\,x^n)^2 \left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

```
[In] int((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
[Out] int((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int (fx)^m (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (ex^n+d)^2 \left(cx^{2n}+bx^n+a\right)^p (fx)^m \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m,
 x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(d+e*x**n)**2*(a+b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (fx)^m (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (ex^n+d)^2 \left(cx^{2n}+bx^n+a\right)^p (fx)^m \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((e*x^n + d)^2*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (fx)^m (d+ex^n)^2 (a+bx^n+cx^{2n})^p \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-128,[1,0,5,3,0,6,4,1,6,0,2]%%%}+%%%{512,[1,0,5,3,0,6,4,0,7,1
,1]%%%
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^n)^2 (a+bx^n+cx^{2n})^p \, dx = \int (f\,x)^m\,(d+e\,x^n)^2\,\left(a+b\,x^n+c\,x^{2\,n}\right)^p \,dx$$

```
[In] int((f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p, x)
```

3.153 $\int (fx)^m (d+ex^n) (a+bx^n+cx^{2n})^p \, dx$

Optimal result

Integrand size = 29, antiderivative size = 323

$$\int (fx)^m (d+ex^n) (a+bx^n+cx^{2n})^p \, dx$$
$$= \frac{d(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2c}{b-\sqrt{b}}\right.}{f(1+m)}$$
$$+\frac{ex^{1+n}(fx)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{1+m+n}{n},-p,-p,\frac{1+m+2n}{n}\right.}{1+m+n}$$

```
[Out] d*(f*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*c*x
^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/f/(1+m)/((1+2*c*
x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+e*x^(
1+n)*(f*x)^m*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m+n)/n,-p,-p,(1+m+2*n)/n,-2*
c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+m+n)/((1+2
*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used

$= \{1574, 1399, 524, 20\}$

$$\int (fx)^m (d+ex^n) \left(a+bx^n+cx^{2n}\right)^p \, dx$$
$$= \frac{d(fx)^{m+1} \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p} (a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{m+1}{n}, -p, -p, \frac{m+n+1}{n}, -\frac{2cx^n}{b-\sqrt{b^2-}}\right.}{f(m+1)}$$
$$+ \frac{ex^{n+1}(fx)^m \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p} (a+bx^n+cx^{2n})^p \operatorname{AppellF1}\left(\frac{m+n+1}{n}, -p, -p, \frac{m+2n+1}{n},\right.}{m+n+1}$$

```
[In] Int[(f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (d*(f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 +
 m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e*x^(1 + n)*(f*x)^m*(a + b*x^n + c*x^(2*n)
)^p*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + n)*(1 + (2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1574

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\text{integral} = \int \left(d(fx)^m \left(a+bx^n+cx^{2n}\right)^p + ex^n(fx)^m \left(a+bx^n+cx^{2n}\right)^p\right) dx$$

$$= d\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p dx + e\int x^n(fx)^m \left(a+bx^n+cx^{2n}\right)^p dx$$

$$= \left(ex^{-m}(fx)^m\right)\int x^{m+n}\left(a+bx^n+cx^{2n}\right)^p dx + \left(d\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\right)\int (fx)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^p dx$$

$$= \frac{d(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+m}{n};-p,-p;\frac{1+m+n}{n};-\frac{2c}{b-\sqrt{b}}\right.}{f(1+m)} + \left(ex^{-m}(fx)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\right)\int x^{m+n}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^p dx$$

$$= \frac{d(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+m}{n};-p,-p;\frac{1+m+n}{n};-\frac{2c}{b-\sqrt{b}}\right.}{f(1+m)} + \frac{ex^{1+n}(fx)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+m+n}{n};-p,-p;\frac{1+m+2n}{n}\right.}{1+m+n}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.85

$$\int (fx)^m (d+ex^n) \left(a+bx^n+cx^{2n}\right)^p \, dx$$
$$= \frac{x(fx)^m \left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p} (a+x^n(b+cx^n))^p \left(d(1+m+n)\,\text{AppellF1}\left(\frac{1+m}{n},-p,-p,\right.\right.}{(1+m)(}$$

```
[In] Integrate[(f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + x^n*(b + c*x^n))^p*(d*(1 + m + n)*AppellF1[(1 + m)/n, -p, -
p, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])] + e*(1 + m)*x^n*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n
)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
]))/((1 + m)*(1 + m + n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int (fx)^m \left(d+e\,x^n\right) \left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

```
[In] int((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int (fx)^m (d+ex^n) \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (ex^n+d)\left(cx^{2\,n}+bx^n+a\right)^p (fx)^m \, dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m (d + ex^n) \left(a + bx^n + cx^{2n}\right)^p dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(d+e*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (fx)^m (d + ex^n) \left(a + bx^n + cx^{2n}\right)^p dx = \int (ex^n + d)\left(cx^{2n} + bx^n + a\right)^p (fx)^m dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Giac [F]

$$\int (fx)^m (d + ex^n) \left(a + bx^n + cx^{2n}\right)^p dx = \int (ex^n + d)\left(cx^{2n} + bx^n + a\right)^p (fx)^m dx$$

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d + ex^n) \left(a + bx^n + cx^{2n}\right)^p dx = \int (f\,x)^m \left(d + e\,x^n\right) \left(a + b\,x^n + c\,x^{2\,n}\right)^p dx$$

```
[In] int((f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x)
[Out] int((f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p, x)
```

3.154 $\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 158

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \frac{(fx)^{1+m} \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p} \left(a+bx^n+cx^{2n}\right)^p \text{AppellF1}\left(\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2cx^n}{b-\sqrt{b^2-4\cdots}}\right.}{f(1+m)}$$

```
[Out] (f*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*c*x^n
/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/f/(1+m)/((1+2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \frac{(fx)^{m+1} \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p} \left(a+bx^n+cx^{2n}\right)^p \text{AppellF1}\left(\frac{m+1}{n},-p,-p,\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{b^2-4\cdots}}\right.}{f(m+1)}$$

```
[In] Int[(f*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m
 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\text{integral} = \left(\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\right)\int (fx)^m\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^p\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^p dx$$

$$= \frac{(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+m}{n};-p,-p;\frac{1+m+n}{n};-\frac{2cx}{b-\sqrt{b^2}}\right.}{f(1+m)}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

$$\int (fx)^m\left(a+bx^n+cx^{2n}\right)^p\,dx$$

$$= \frac{x(fx)^m\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p \operatorname{AppellF1}\left(\frac{1+m}{n},-p,-p,\frac{1+m+n}{n},-\frac{2c}{b+\sqrt{}}\right.}{1+m}$$

```
[In] Integrate[(f*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + x^n*(b + c*x^n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/
n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])
/((1 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]

$$\int (fx)^m \left(a+b\,x^n+c\,x^{2n}\right)^p dx$$

```
[In] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
[Out] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

Fricas [F]

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (cx^{2\,n}+bx^n+a)^p (fx)^m \, dx$$

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \int (cx^{2\,n}+bx^n+a)^p (fx)^m \, dx$$

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Giac [F]

$$\int (fx)^m \left(a + bx^n + cx^{2n}\right)^p \, dx = \int \left(cx^{2n} + bx^n + a\right)^p (fx)^m \, dx$$

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(a + bx^n + cx^{2n}\right)^p \, dx = \int (f\,x)^m \left(a + b\,x^n + c\,x^{2\,n}\right)^p dx$$

```
[In] int((f*x)^m*(a + b*x^n + c*x^(2*n))^p,x)
[Out] int((f*x)^m*(a + b*x^n + c*x^(2*n))^p, x)
```

3.155 $\int \frac{(fx)^m(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{(fx)^m\,(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx=\text{Int}\left(\frac{(fx)^m\,(a+bx^n+cx^{2n})^p}{d+ex^n},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\,(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx=\int \frac{(fx)^m\,(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx$$

```
[In] Int[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]
```

Rubi steps

$$\text{integral}=\int \frac{(fx)^m\,(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{d + ex^n} dx = \int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{d + ex^n} dx$$

```
[In] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x]
[Out] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b\,x^n + c\,x^{2n})^p}{d + e\,x^n} dx$$

```
[In] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
[Out] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{d + ex^n} dx = \int \frac{(cx^{2n} + bx^n + a)^p (fx)^m}{ex^n + d} dx$$

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{d + ex^n} dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p/(d+e*x**n),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m (a+bx^n+cx^{2n})^p}{d+ex^n}\,dx = \int \frac{(cx^{2n}+bx^n+a)^p (fx)^m}{ex^n+d}\,dx$$

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m (a+bx^n+cx^{2n})^p}{d+ex^n}\,dx = \int \frac{(cx^{2n}+bx^n+a)^p (fx)^m}{ex^n+d}\,dx$$

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)
```

Mupad [N/A]

Not integrable

Time = 8.70 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m (a+bx^n+cx^{2n})^p}{d+ex^n}\,dx = \int \frac{(f\,x)^m\,(a+b\,x^n+c\,x^{2\,n})^p}{d+e\,x^n}\,dx$$

```
[In] int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x)
[Out] int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x)
```

3.156 $\int \frac{(fx)^m(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{(fx)^m\,(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx = \mathrm{Int}\left(\frac{(fx)^m\,(a+bx^n+cx^{2n})^p}{(d+ex^n)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\,(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(fx)^m\,(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

```
[In] Int[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x]
[Out] Defer[Int][((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(fx)^m\,(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx = \int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx$$

```
[In] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x]
[Out] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b\,x^n + c\,x^{2n})^p}{(d + e\,x^n)^2} dx$$

```
[In] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
[Out] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx = \int \frac{(cx^{2n} + bx^n + a)^p (fx)^m}{(ex^n + d)^2} dx$$

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e^2*x^(2*n) + 2*d*e*x^n + d^2),
 x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**2,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx = \int \frac{(cx^{2n} + bx^n + a)^p (fx)^m}{(ex^n + d)^2} dx$$

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx = \int \frac{(cx^{2n} + bx^n + a)^p (fx)^m}{(ex^n + d)^2} dx$$

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 9.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx = \int \frac{(f\,x)^m\,(a + b\,x^n + c\,x^{2\,n})^p}{(d + e\,x^n)^2} dx$$

```
[In] int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x)
[Out] int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                   end if
               else #result contains complex but optimal is not
                   if debug then
                           print("result contains complex but optimal is not");
                   fi;
                   return "C","Result contains complex when optimal does not.";
               fi;
           else # result do not contain complex
                # this assumes optimal do not as well. No check is needed here.
               if debug then
                      print("result do not contain complex, this assumes optimal do not as well")
               fi;
               if leaf_count_result<=2*leaf_count_optimal then
                   if debug then
                       print("leaf_count_result<=2*leaf_count_optimal");
                   fi;
                   return "A"," ";
               else
                   if debug then
                       print("leaf_count_result>2*leaf_count_optimal");
                   fi;
                   return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
               fi;
           fi;
       else #ExpnType(result) > ExpnType(optimal)
           if debug then
               print("ExpnType(result) > ExpnType(optimal)");
           fi;
           return "C",cat("Result contains higher order function than in optimal. Order ",
                          convert(ExpnType_result,string)," vs. order ",
                          convert(ExpnType_optimal,string),".");
       fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```